

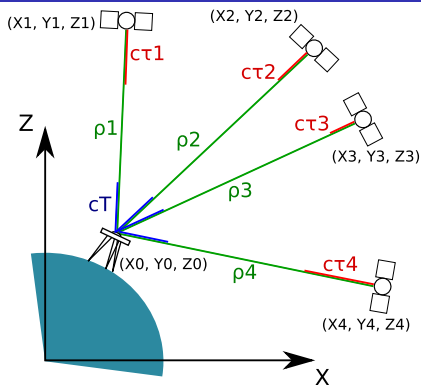
Apparent Geocenter Oscillations in GNSS Solutions Caused by the Ionospheric Effect of Second Order

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GNSS trilateration: finding the receiver's coordinates



Where

- ρ_i — geometric distance between the satellite i and receiver
- τ_i — clock error of the satellite i
- T — receiver clock error
- $R_i = \rho_i + c\tau_i + cT$ — pseudorange

Known parameters: X_i, Y_i, Z_i, τ_i

Measured parameters: R_i

Unknown parameters: T, X_0, Y_0, Z_0

Realistic observation equation and error budget

$$\sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} = c\tau_i - cT$$
$$\delta_i + R_i + G_i + \epsilon_i + \Psi_i +$$
$$R_i - I_i + S_i -$$
$$\Phi - \Delta - M - \varepsilon - V -$$

...

Where

- Satellite-specific

- τ_i — Clock bias
- δ_i — Differential bias
- R_i — Orbital error (2 cm-...)
- G_i — Relativistic effect
- ϵ_i — Transmitter noise (1 cm)
- Ψ_i — Antenna phase centre variation (0–20 cm)

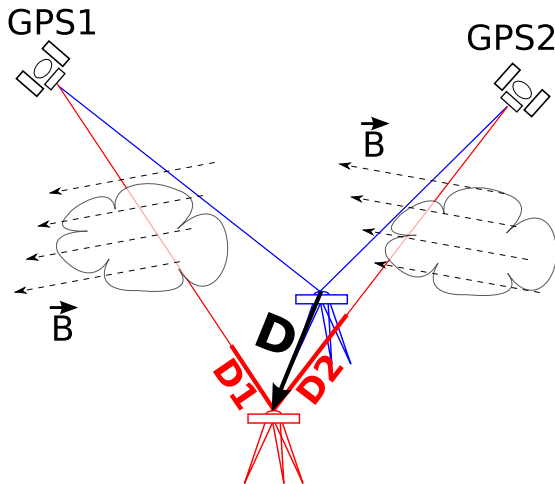
- Propagation errors

- I_i — Ionospheric refraction (up to 200 m)
- S_i — Tropospheric refraction (~ 2 m)

- Receiver-specific

- Φ — Antenna phase centre variation (1–2 cm)
- T — Clock bias
- Δ — Differential bias
- M — Multipath (up to 5 cm)
- ε — Receiver noise (1 cm)
- V — Phase windup (5 cm)

What happens if we fail to account for some contribution?



Refraction index of plasma: Appleton-Hartree equation

$$n^2 = 1 - \frac{X}{1 - iZ - \frac{Y_T^2}{2(1 - X - iZ)} \pm \sqrt{\frac{Y_T^4}{4(1 - X - iZ)^2 + Y_L^2}}}$$

where

$$X = \left(\frac{f_N}{f}\right)^2, \quad Z = \frac{\nu}{2\pi f}$$

$$Y = \frac{f_H}{f} = \frac{eB_0}{2\pi f m_e}, \quad Y_T = Y \sin \theta, \quad Y_L = Y \cos \theta,$$

$$f_N = \frac{1}{2\pi} \sqrt{\frac{Ne^2}{m\epsilon_0}} \quad f_H = \frac{eB}{2\pi m_e}$$

Refraction index of plasma: Appleton-Hartree equation

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$$f_N = \frac{1}{2\pi} \sqrt{\frac{Ne^2}{m\epsilon_0}} \quad f_H = \frac{eB}{2\pi m_e}$$

Total Electron Content (TEC) and path delay: the “textbook approach”

$$\Delta\rho = \int (n - 1) ds = \underbrace{\frac{e^2}{8\pi^2 \epsilon_0 m_e}}_{40.308 \text{ m}^3\text{s}^{-2}} f^{-2} \int N ds$$

Path delay for L1

$$\Delta\rho = 0.162 \frac{\text{m}}{\text{TECU}}$$

Path delay for L2

$$\Delta\rho = 0.267 \frac{\text{m}}{\text{TECU}}$$

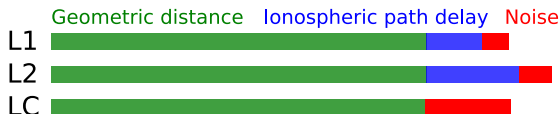
TEC from the measurements

$$\text{TEC} = \frac{1}{40.308} \frac{f_1^2 f_2^2}{f_1^2 - f_2^2} (R_2 - R_1)$$

TECU = TEC Unit = 10^{16} electrons/ m^2 .

Geometric linear combination, LC

$$LC = \frac{1}{f_1^2 - f_2^2} (f_1^2 L_1 - f_2^2 L_2) \quad PC = \frac{1}{f_1^2 - f_2^2} (f_1^2 P_1 - f_2^2 P_2)$$



Properties of LC

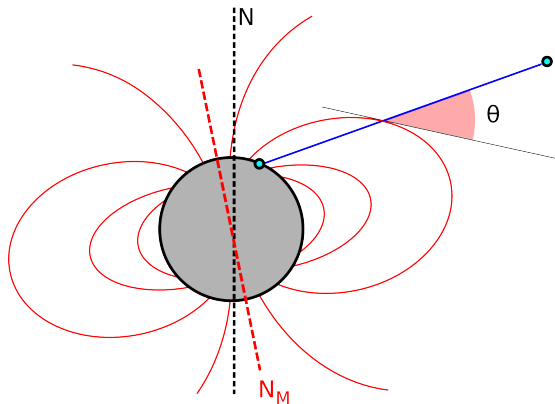
LC is not influenced by the 'classical ionosphere'. Therefore, still unaccounted remain other propagation terms:

- tropospheric refraction;
- higher-order effects.

Noise amplitude of LC is greater by factor of 3, compared to the same of L_1 .

Second order effect

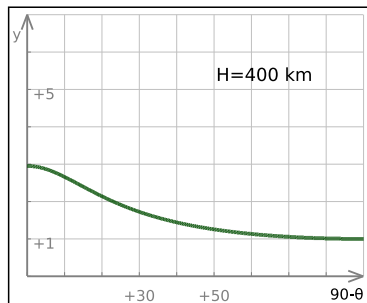
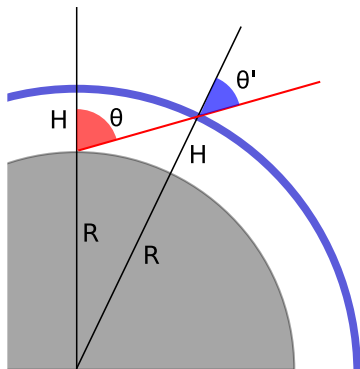
$$\Delta\rho = \int \frac{f_H f_N^2}{f^3} \cos\theta dl = \frac{7527c}{2} \int \frac{NB_0}{f^3} \cos\theta dl$$



International Geomagnetic Reference Field (IGRF) model is used further.

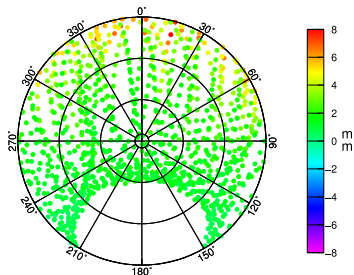
Thin layer geometry

$$\text{TEC} = \frac{\text{VTEC}}{\cos \theta'}, \quad \sin \theta' = \frac{R}{R+H} \sin \theta$$

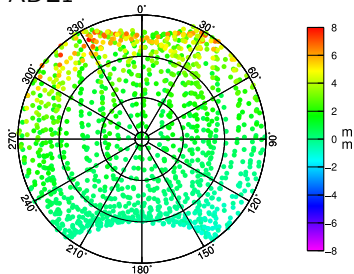


International Reference Ionosphere (IRI2007) model is used further.

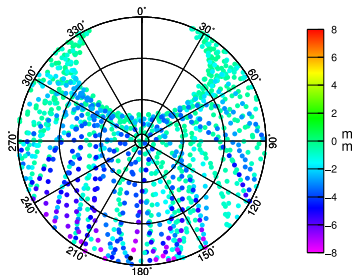
Sky map of 2-nd order delay



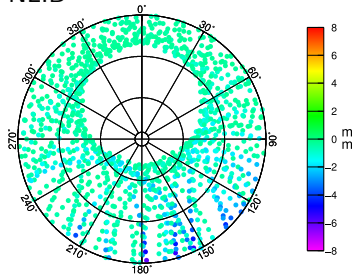
ADE1



ASC1

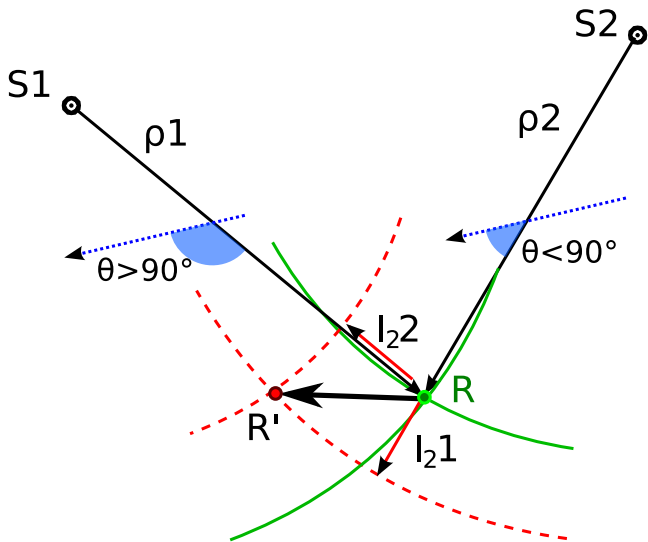


NLIB

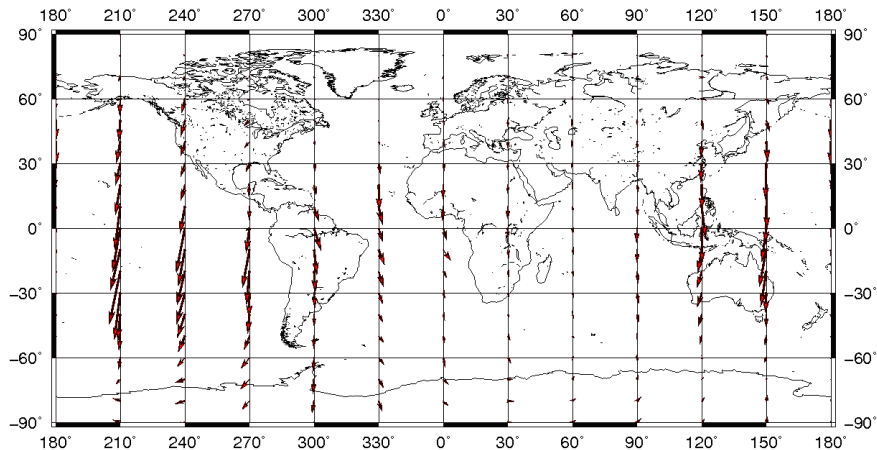


TROM

Geometry of the apparent displacement



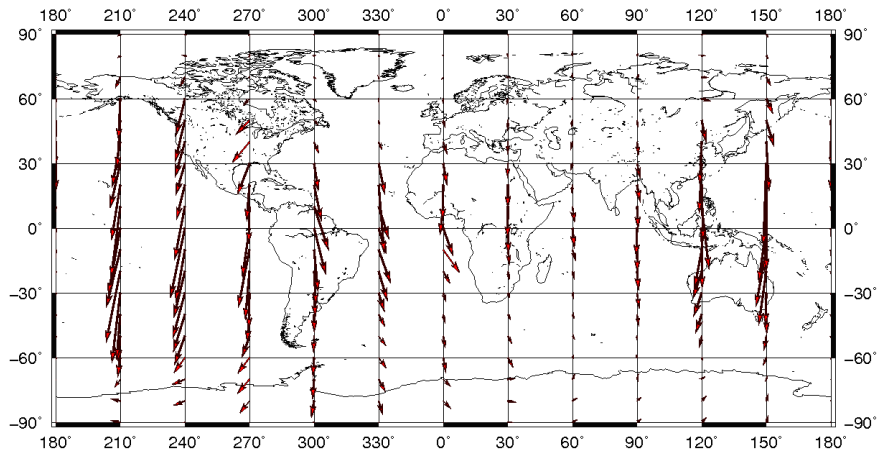
Apparent error: dependence on the Sun



1 cm

1 January 2005, 00UT

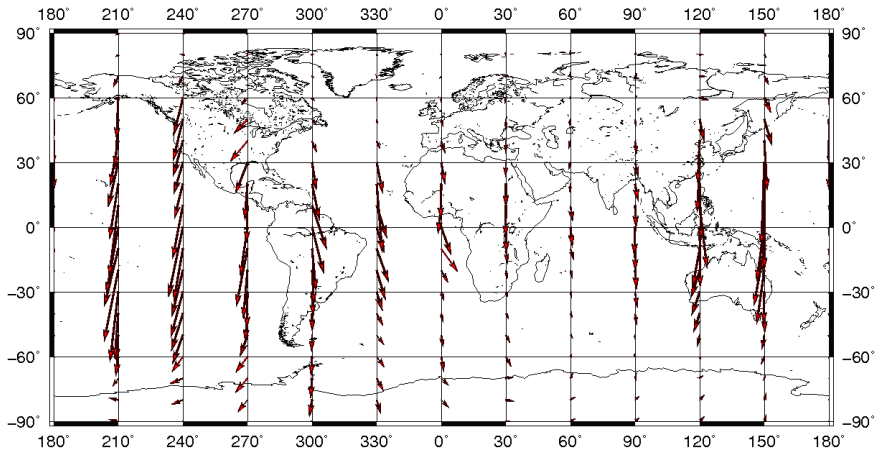
Apparent error: dependence on the Sun



1 cm

1 January 2000, 00UT

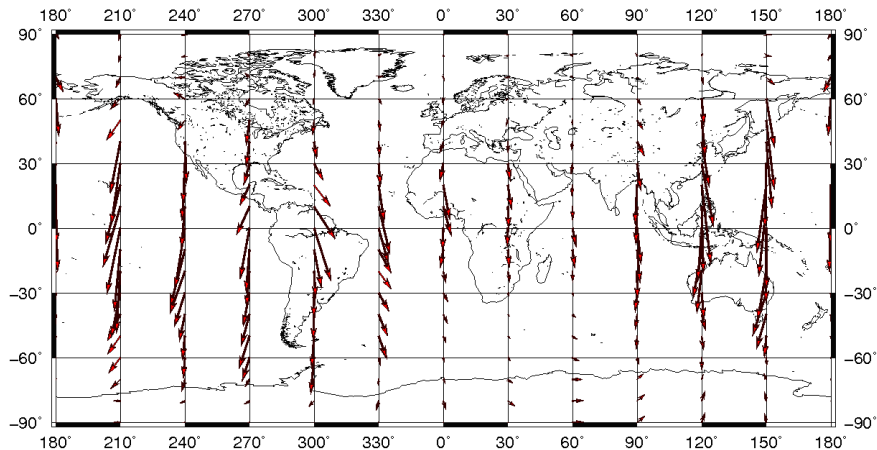
Using IRI: apparent horizontal errors



1 cm

1 January 2000, 00UT

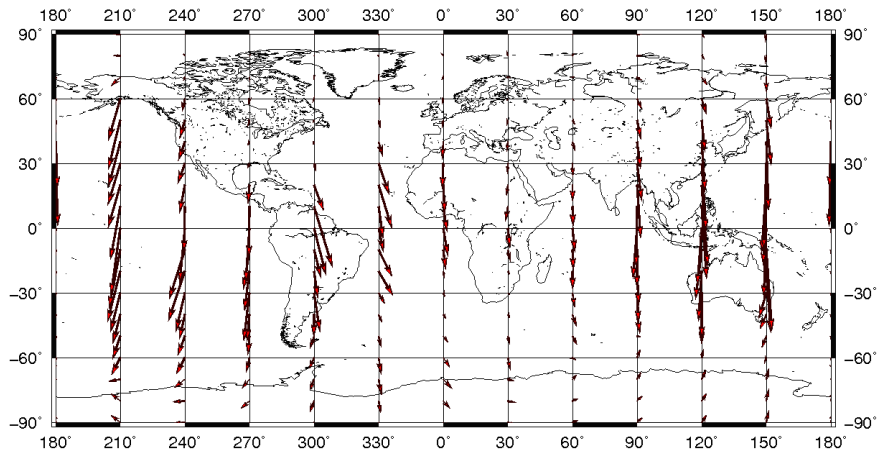
Using IRI: apparent horizontal errors



1 cm

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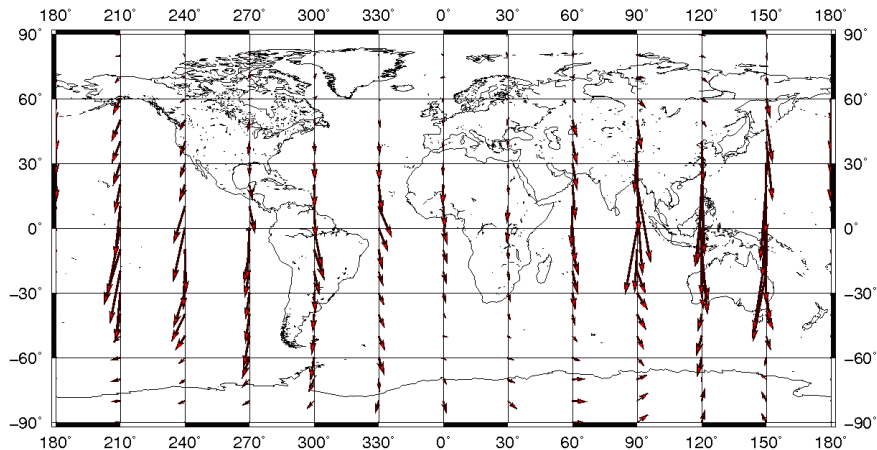
Using IRI: apparent horizontal errors



1 cm

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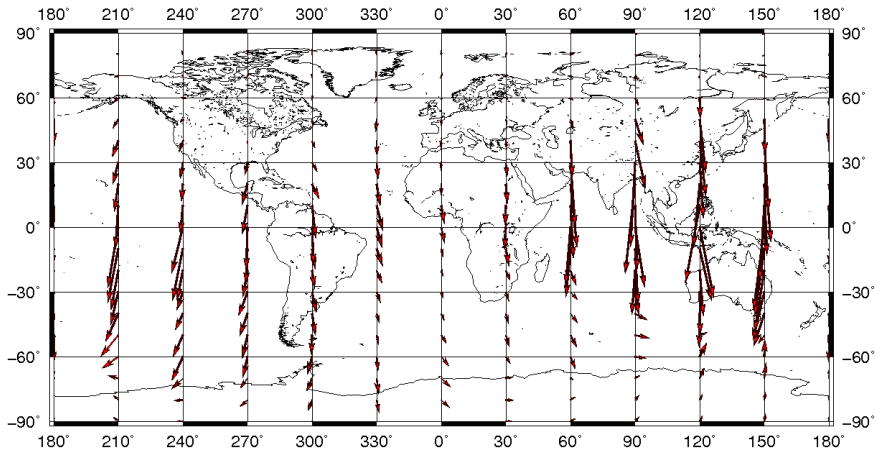
Using IRI: apparent horizontal errors



1 cm

1 January 2000, 03UT

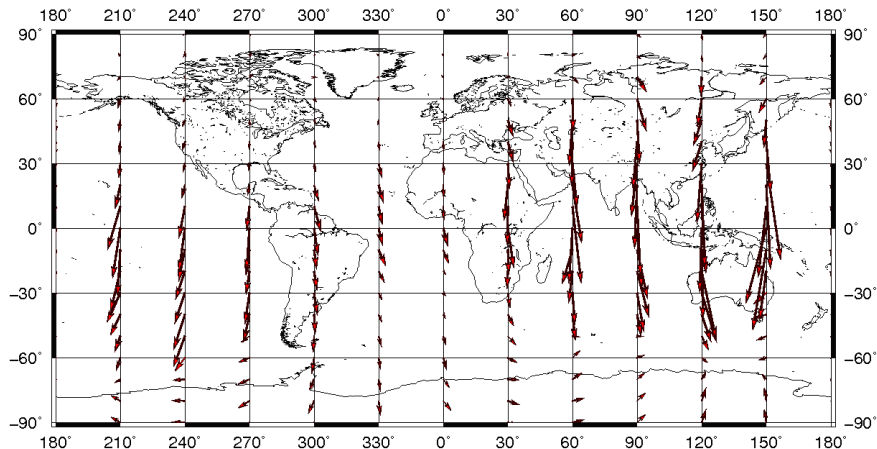
Using IRI: apparent horizontal errors



1 cm

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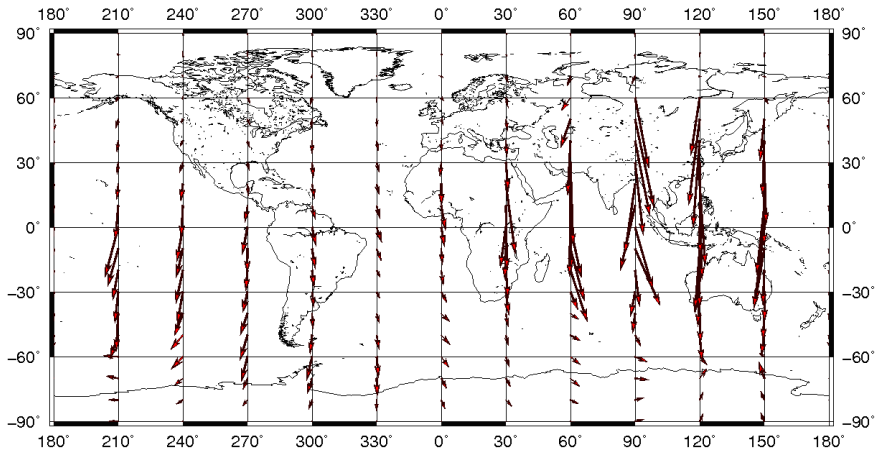
Using IRI: apparent horizontal errors



1 cm

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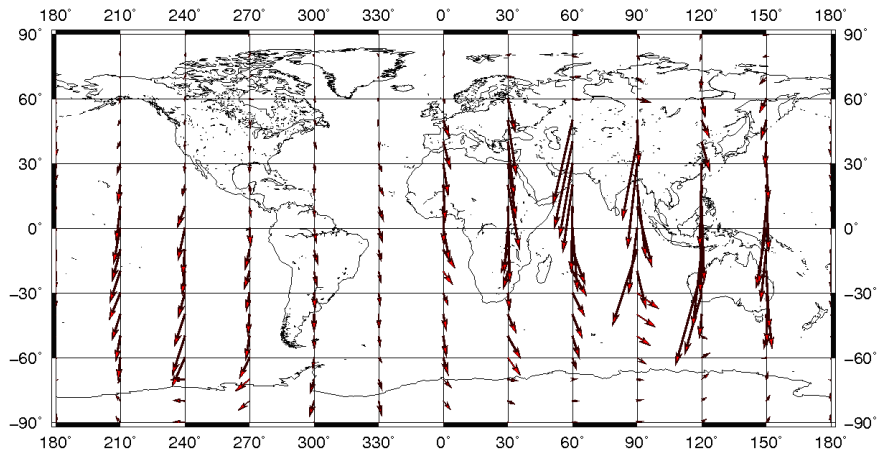
Using IRI: apparent horizontal errors



1 cm

1 January 2000, 06UT

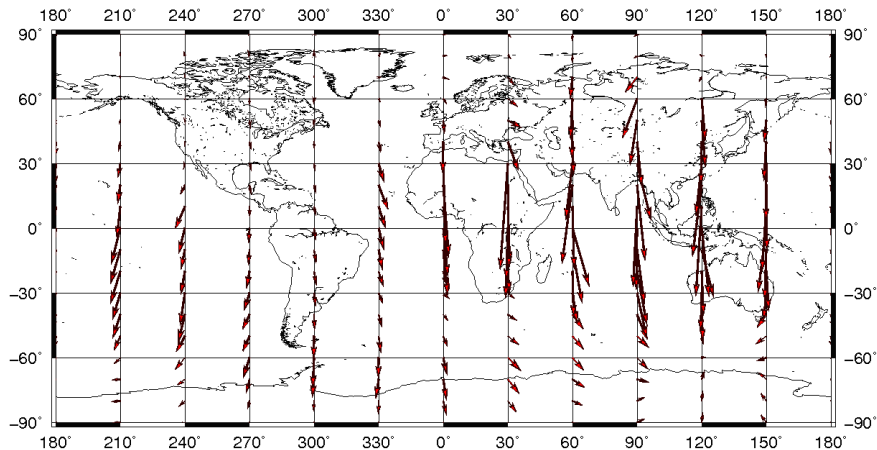
Using IRI: apparent horizontal errors



1 cm

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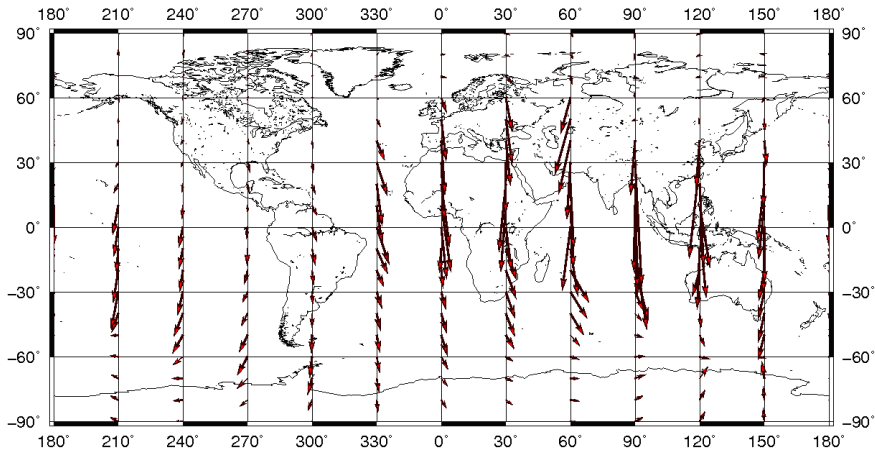
Using IRI: apparent horizontal errors



1 cm

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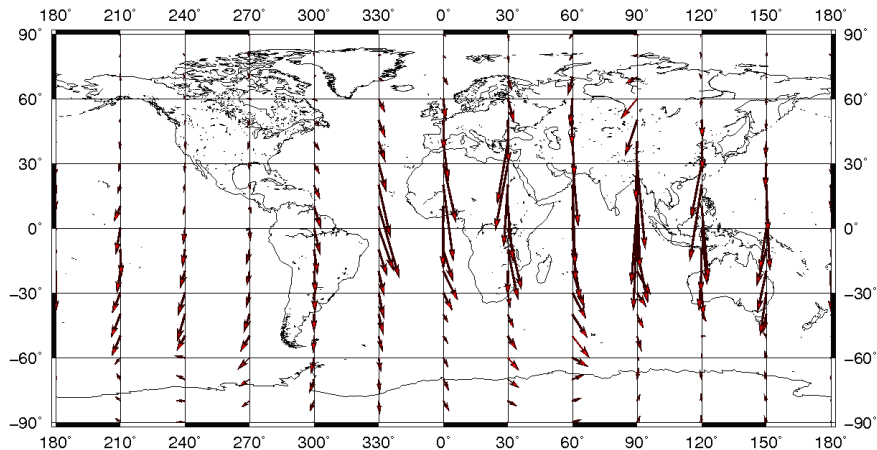
Using IRI: apparent horizontal errors



1 cm

1 January 2000, 09UT

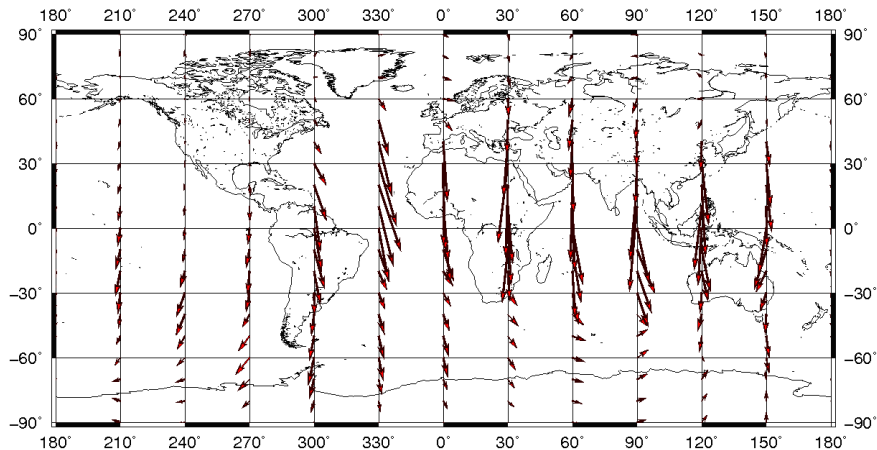
Using IRI: apparent horizontal errors



1 cm

1 January 2000, 10UT

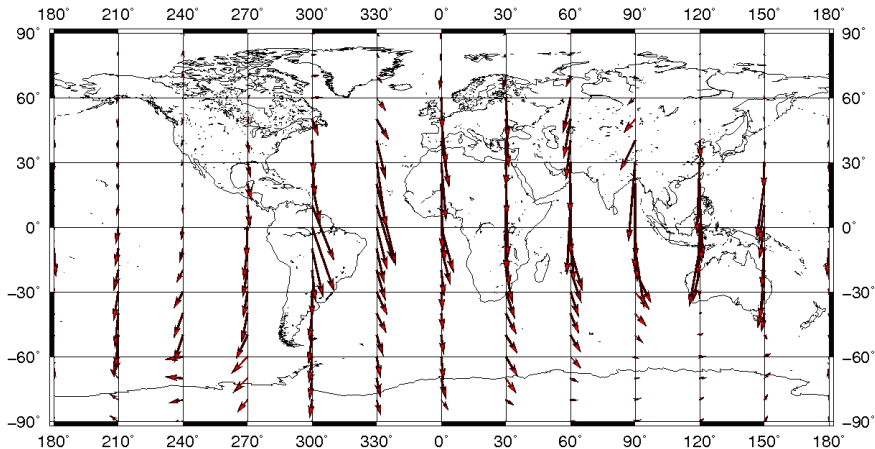
Using IRI: apparent horizontal errors



1 cm

1 January 2000, 11UT

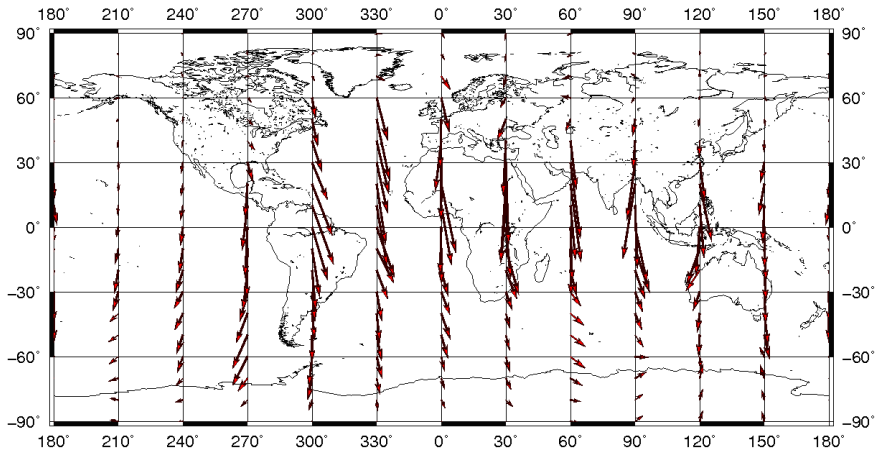
Using IRI: apparent horizontal errors



1 cm

1 January 2000, 12UT

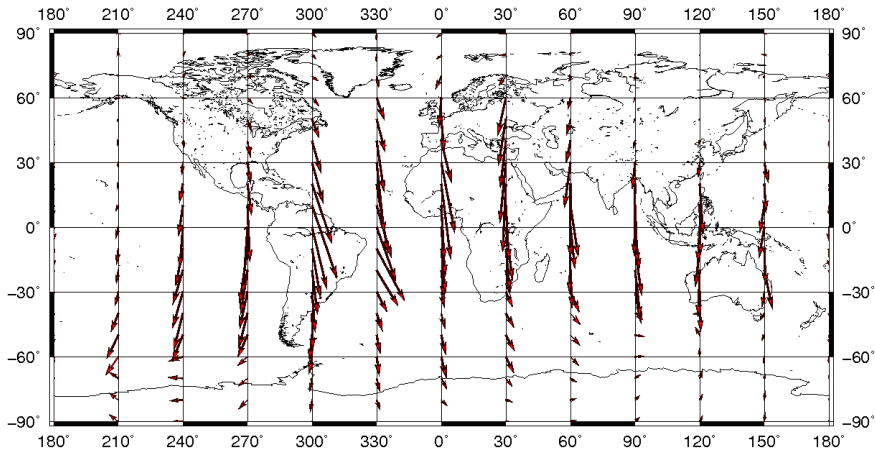
Using IRI: apparent horizontal errors



1 cm

1 January 2000, 13UT

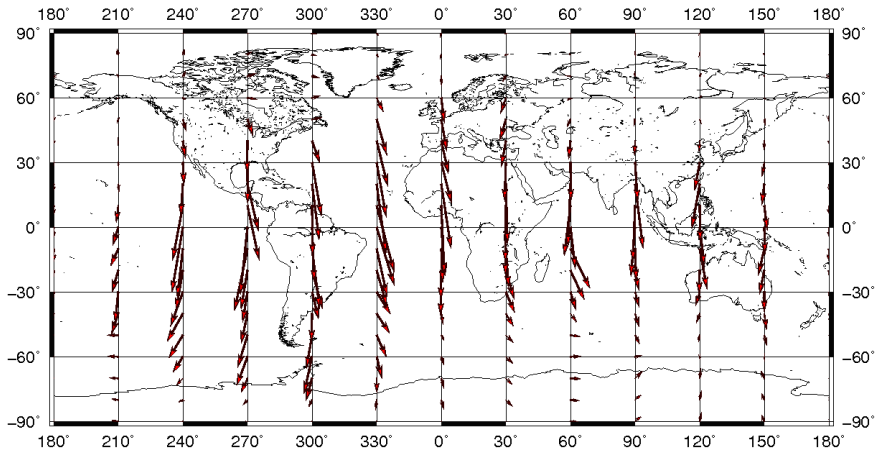
Using IRI: apparent horizontal errors



1 cm

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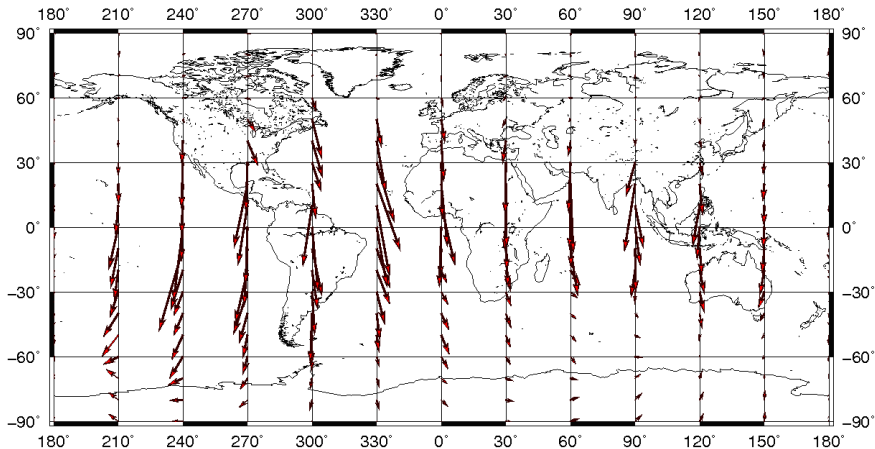
Using IRI: apparent horizontal errors



1 cm

1 January 2000, 15UT

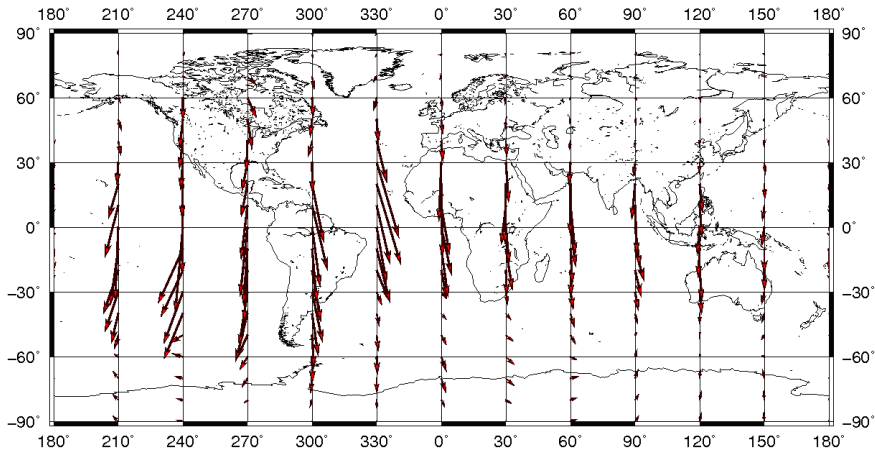
Using IRI: apparent horizontal errors



1 cm

1 January 2000, 16UT

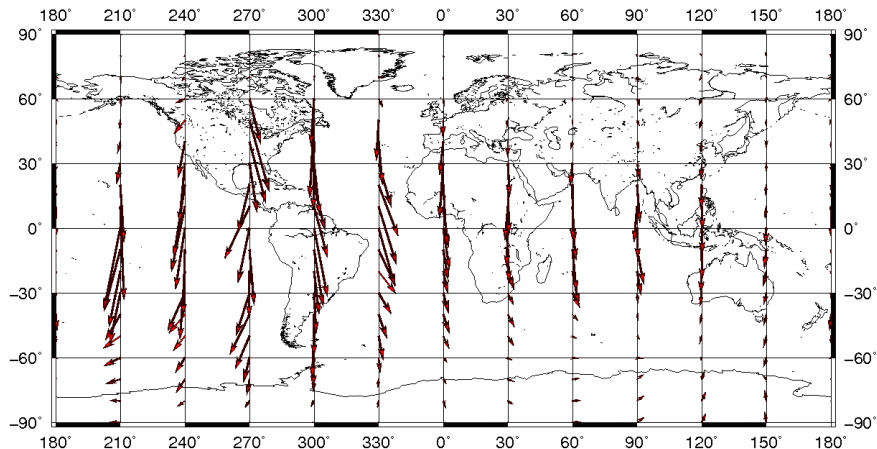
Using IRI: apparent horizontal errors



1 cm

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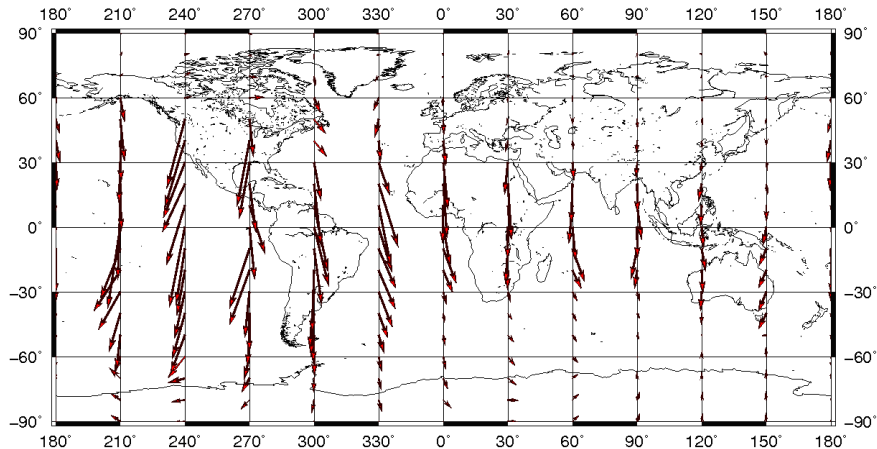
Using IRI: apparent horizontal errors



1 cm

1 January 2000, 18UT

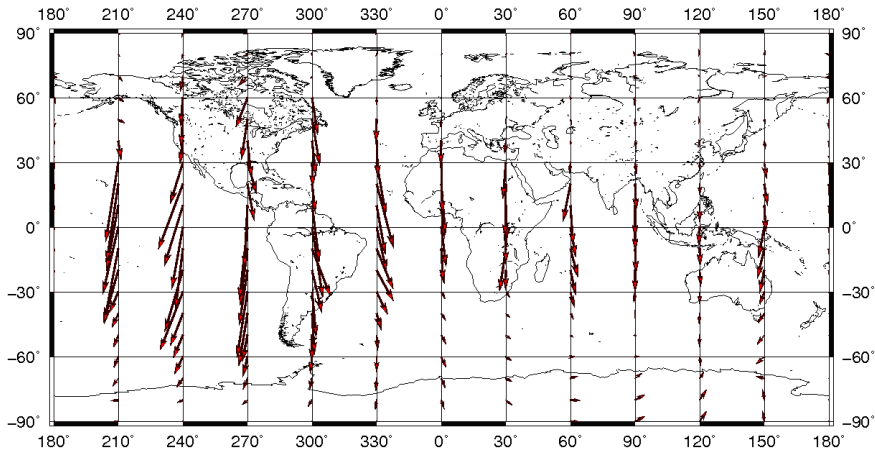
Using IRI: apparent horizontal errors



1 cm

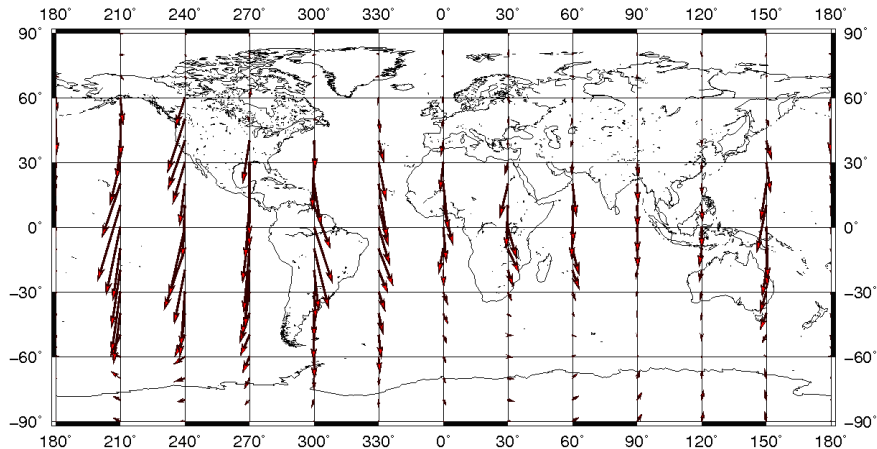
1 January 2000, 19UT

Using IRI: apparent horizontal errors



1 January 2000, 20UT

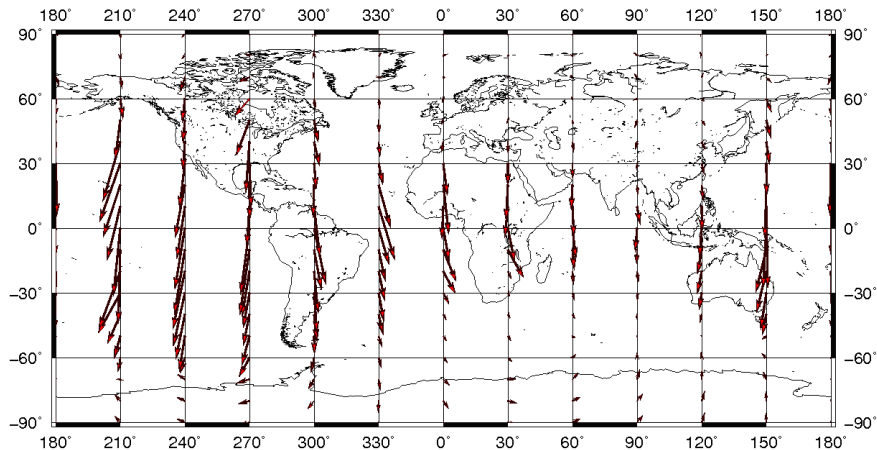
Using IRI: apparent horizontal errors



1 cm

1 January 2000, 21UT

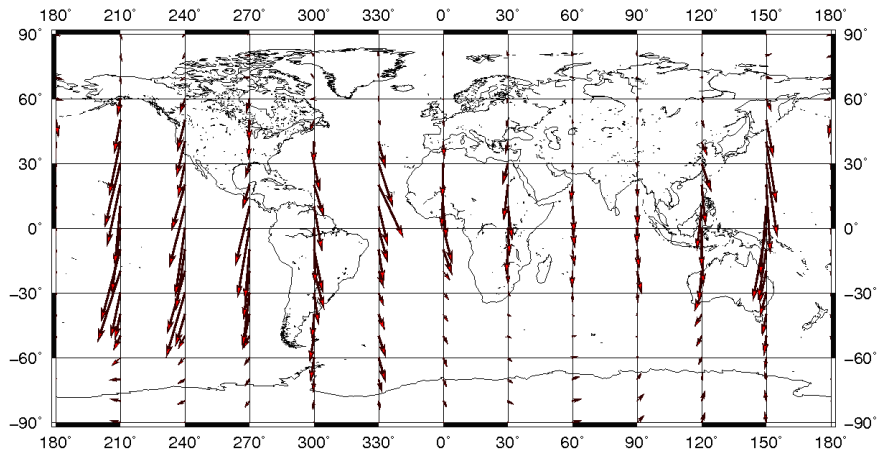
Using IRI: apparent horizontal errors



1 cm

1 January 2000, 22UT

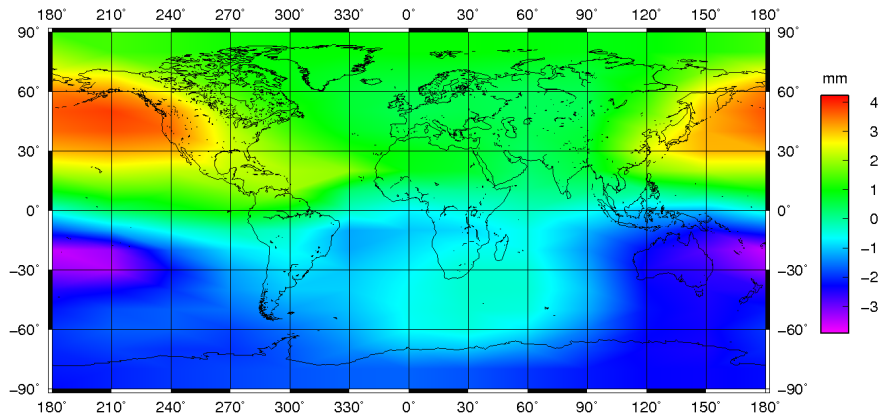
Using IRI: apparent horizontal errors



1 cm

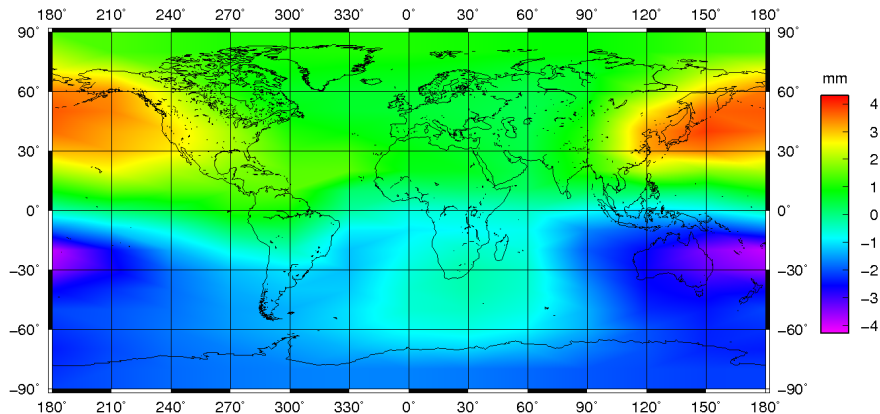
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Using IRI: apparent vertical errors



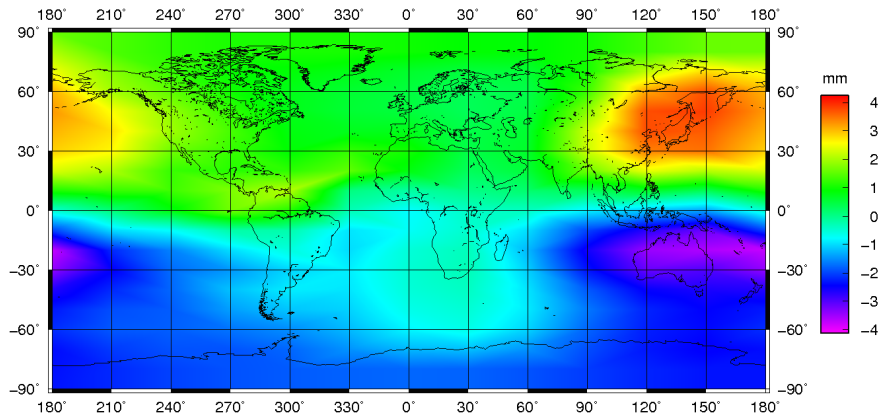
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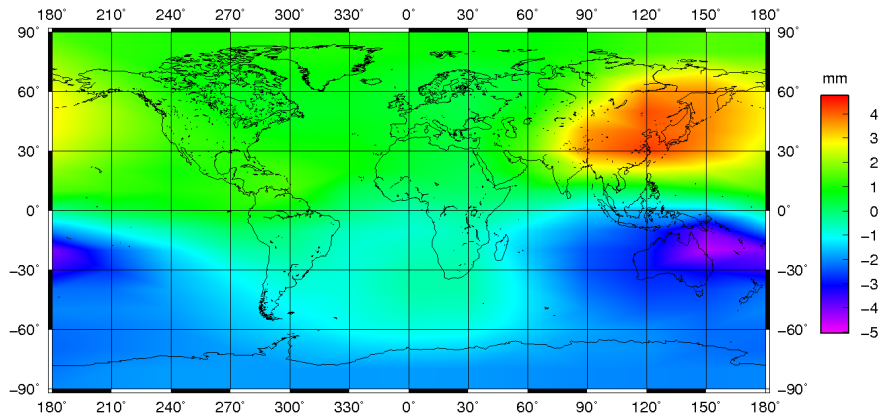
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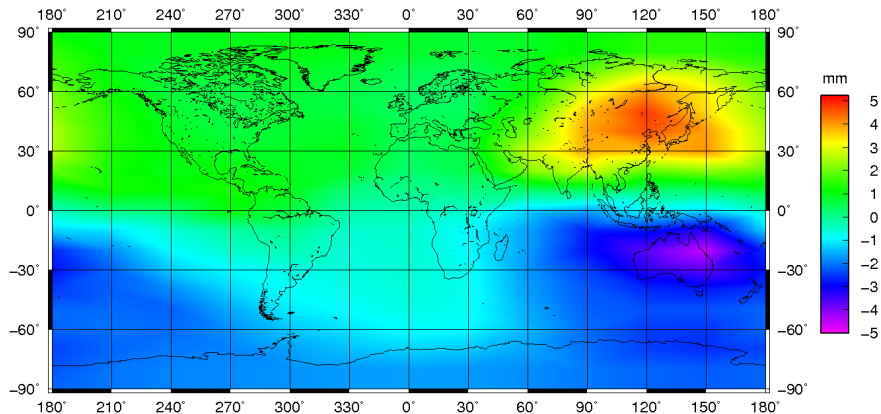
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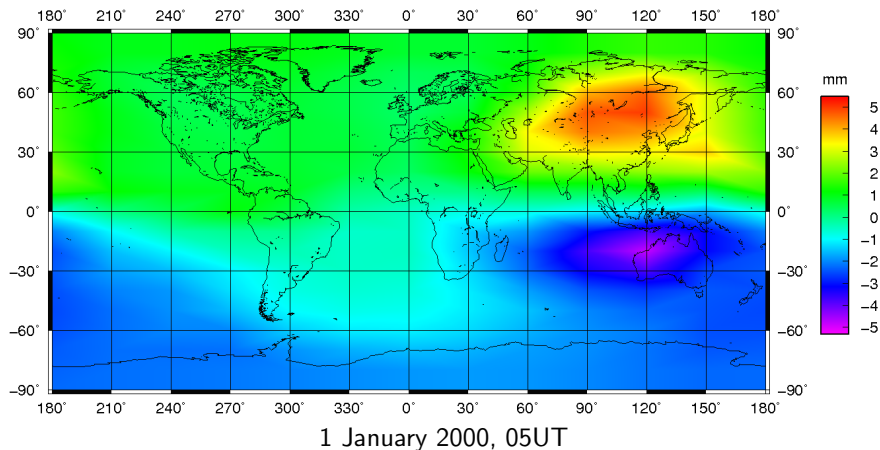
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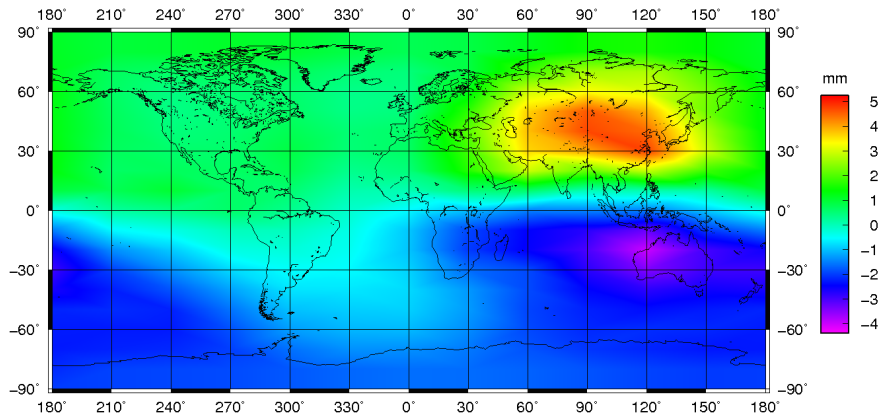


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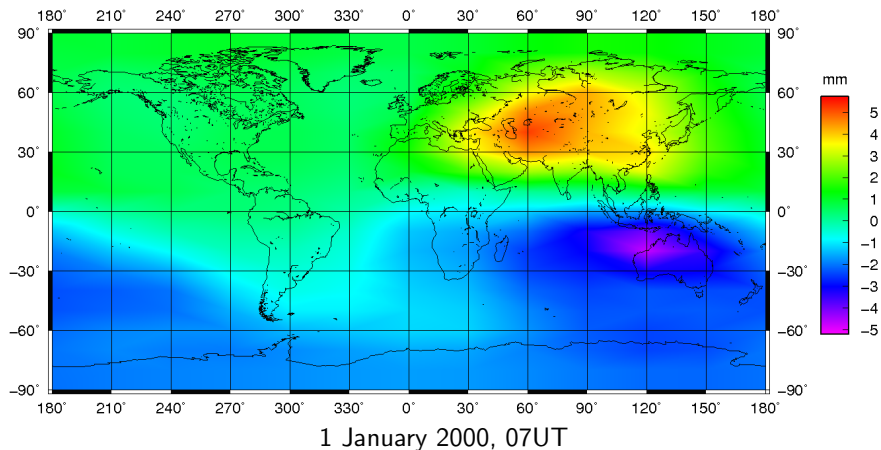


Using IRI: apparent vertical errors

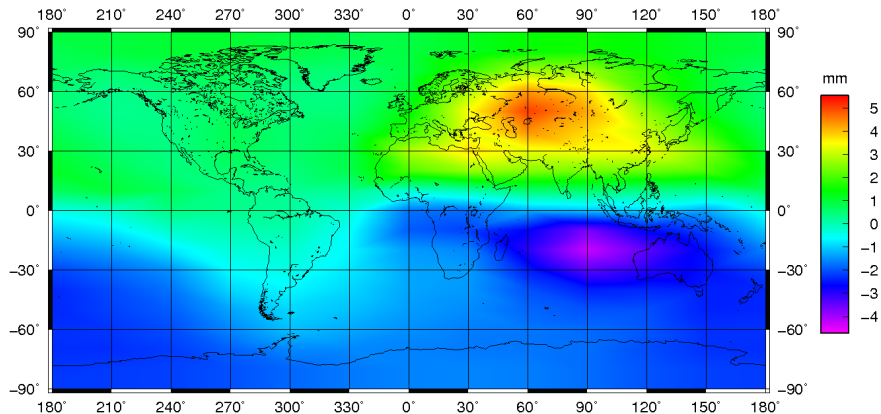


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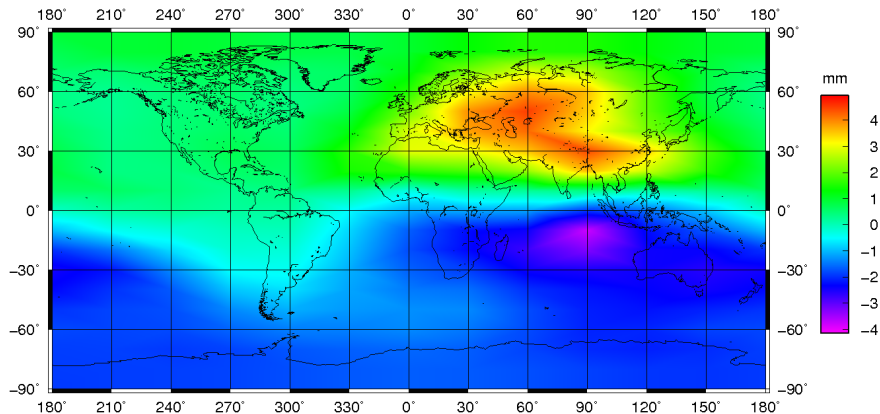


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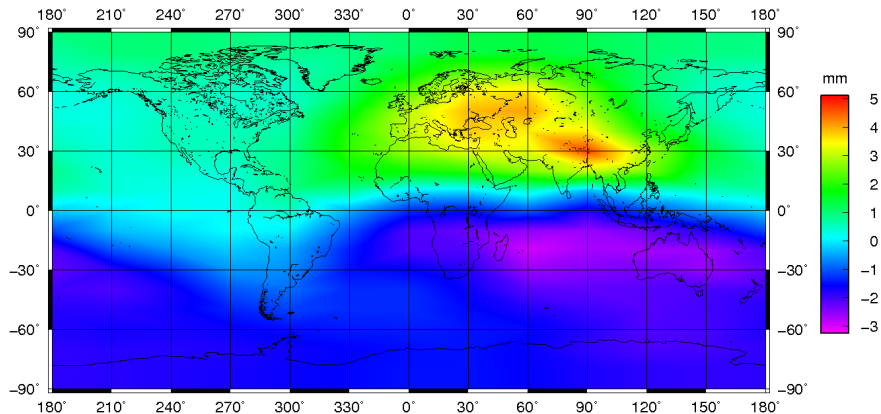
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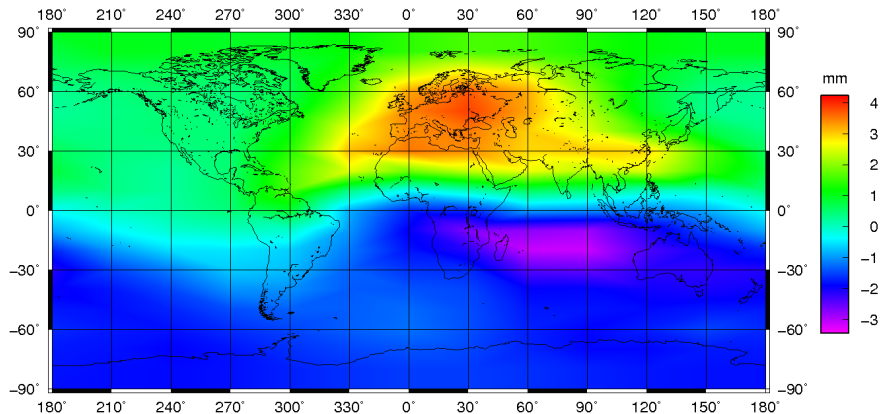
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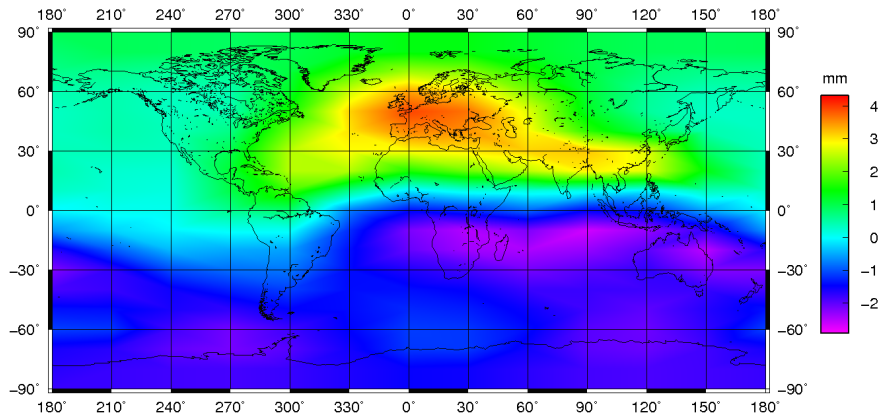
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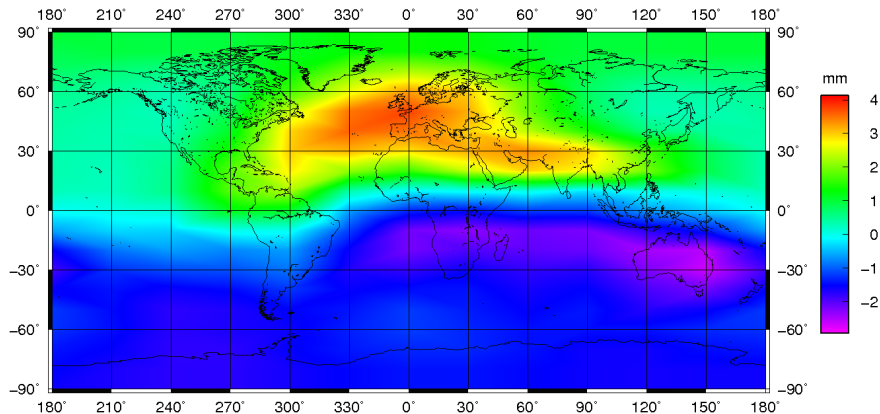
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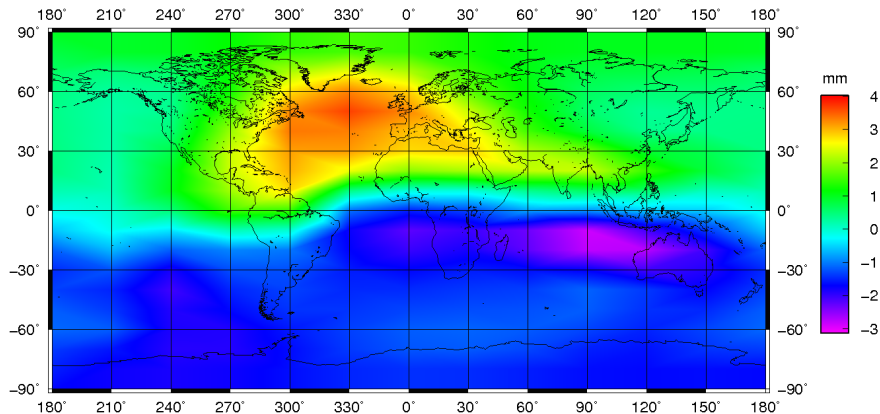
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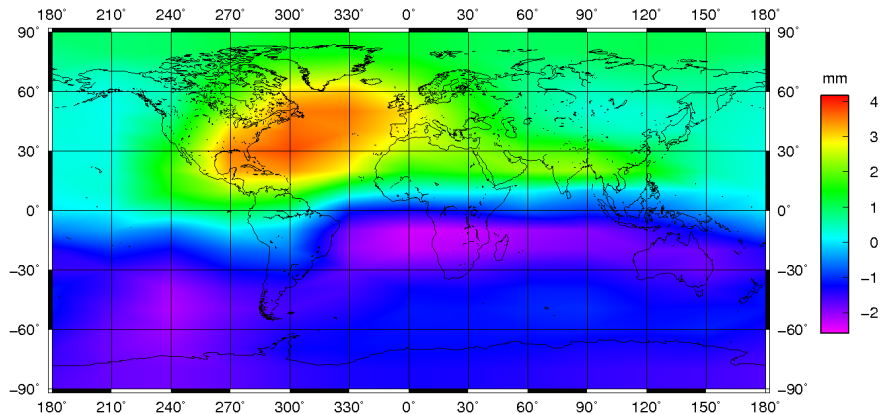
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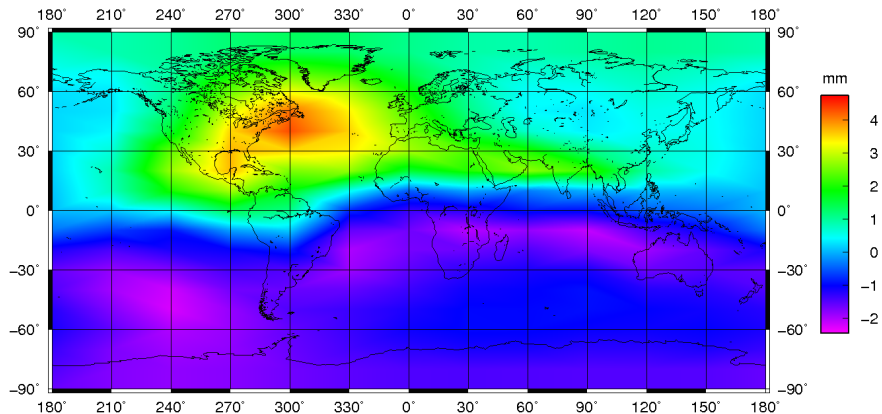
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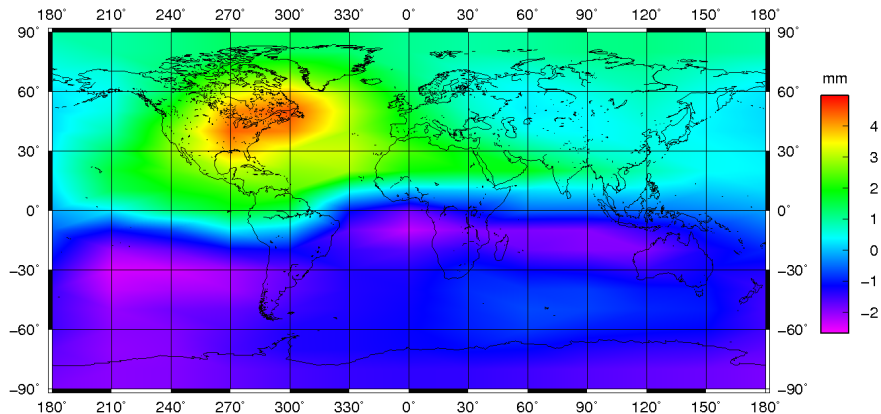
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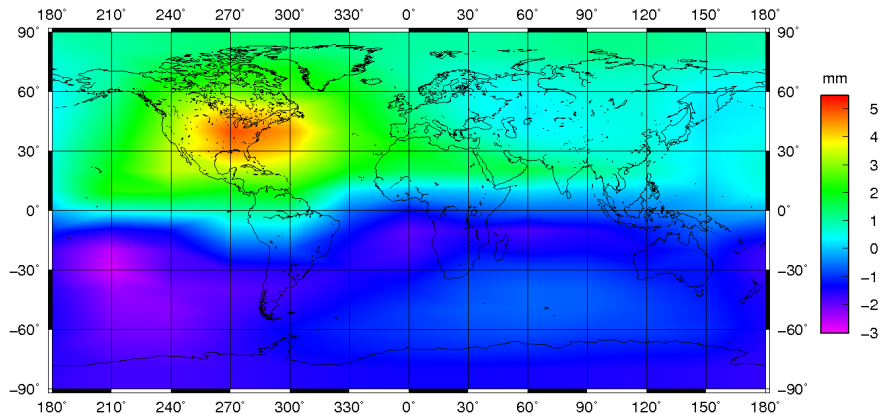
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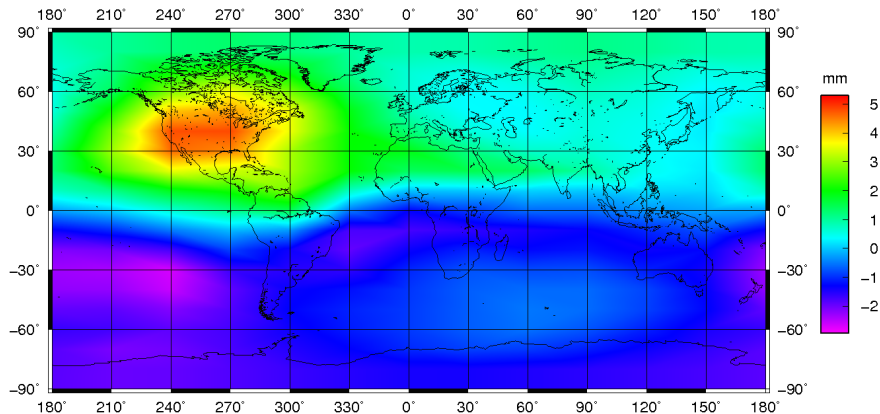
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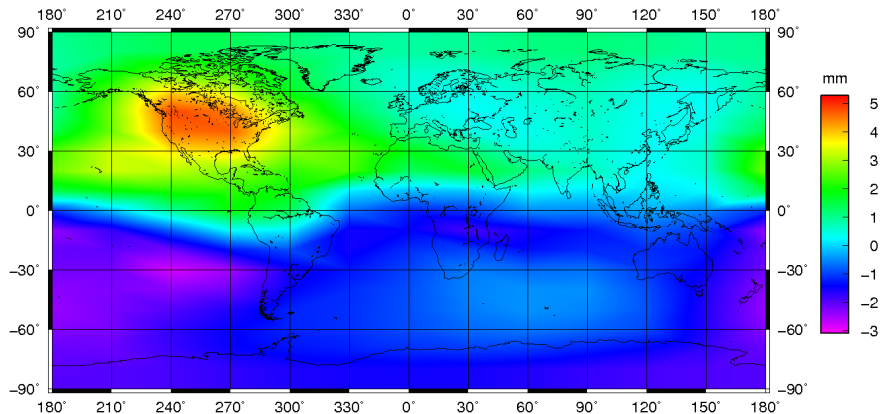
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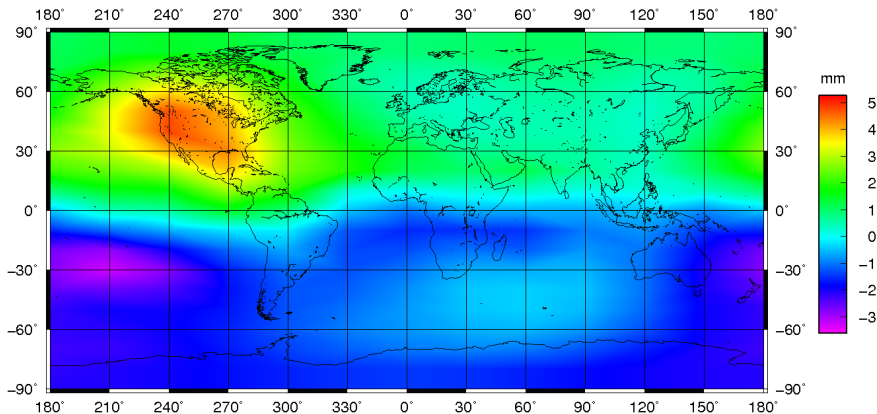
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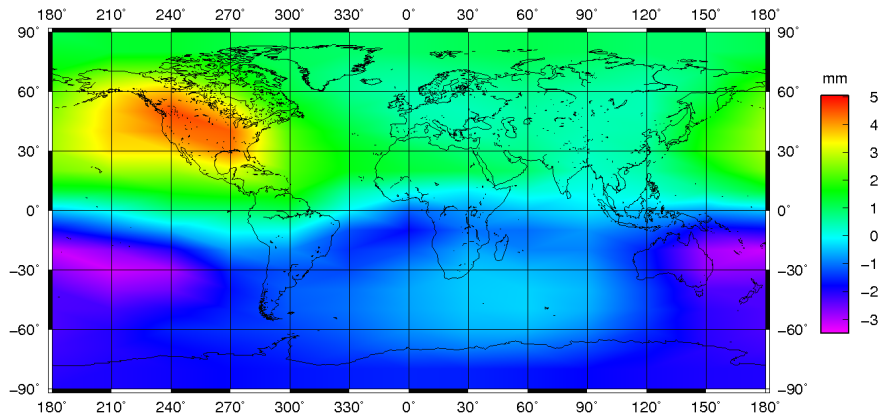
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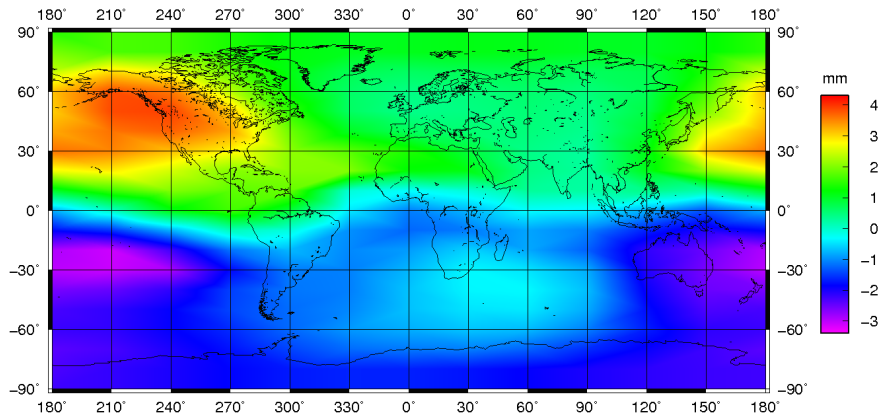
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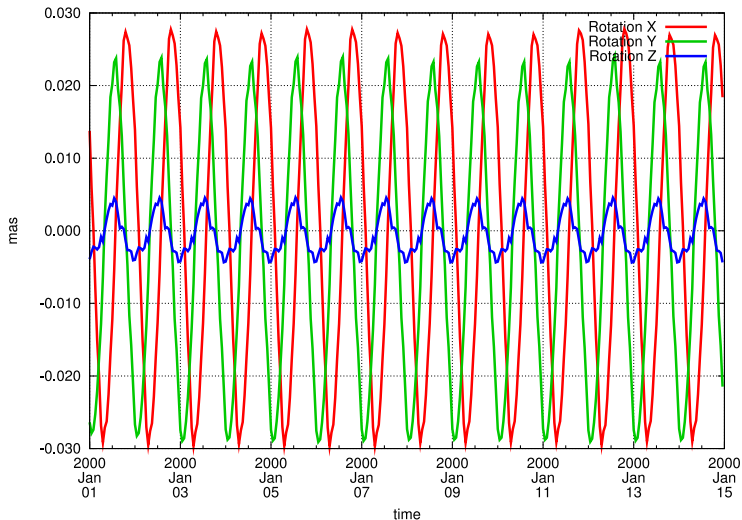
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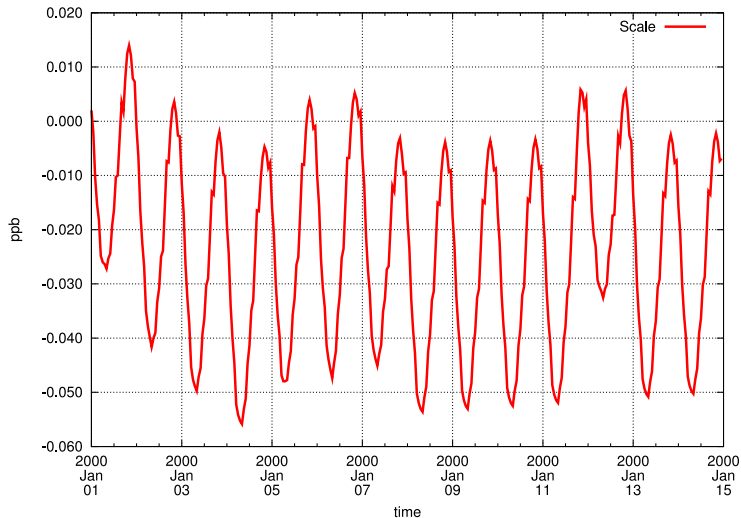


1 January 2000, 23UT

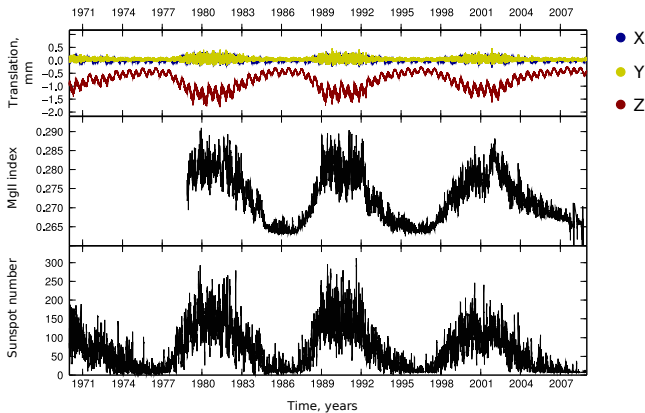
Apparent Reference Frame Instabilities: subdaily rotations



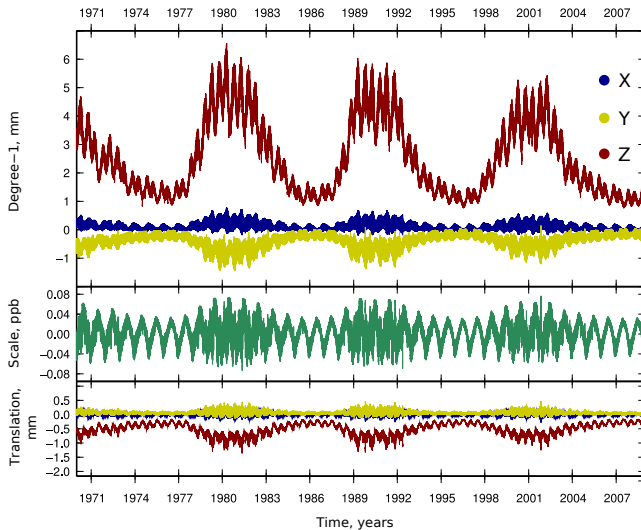
Apparent Reference Frame Instabilities: subdaily scale

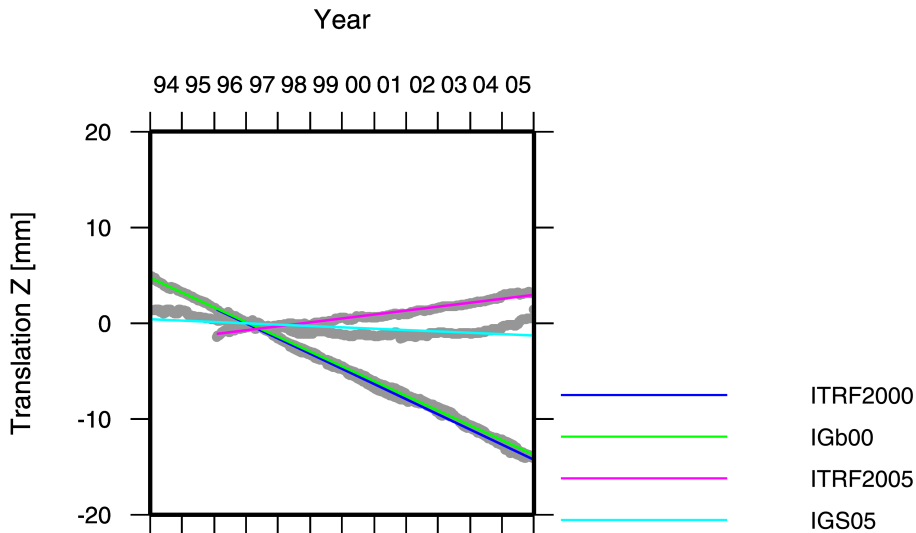


Apparent Geocentre oscillations and solar activity: Helmert Transform



Apparent Geocentre oscillations and solar activity: Degree-1





Conclusions

- GNSS is reliable and precise geodetic tool even in the presence of geomagnetic disturbances;
- But the accuracy and precision of space geodetic techniques has reached so high levels, that accounting for the 2-nd order ionospheric effect becomes necessary.
- Ionospheric effect of 2-nd order causes systematic errors in determination of coordinates of sites, satellite orbits, geocenter position, other parameters. This error has seasonal and 11-year variability and can propagate into the systematic error in satellite oceanic and ice altimetry, if the orbits of the altimeters are being determined using GNSS.
- Fortunately, satellite altimetry is not influenced by to 2-nd order error because of much higher sounding frequency. The same is true for SAR, but SAR is even less prone because of local character of measurements.

- Satellite orbits as determined from an analysis lacking 2-nd order model!!!
- Studies of spatial and temporal variability of 2-nd order effect using real TEC data from GNSS.
- More detailed comparison of geodetic solutions using radiotechnical data (subject to 2-nd order effect) and alternative technics, namely SLR;
- Real GNSS data processing with and without accounting for the 2nd order term: comparison.