Adaptive Likehood Estimator for Forecasting Ionospheric Component on Synthetic Aperture Radar Interferometry (InSAR) Technique

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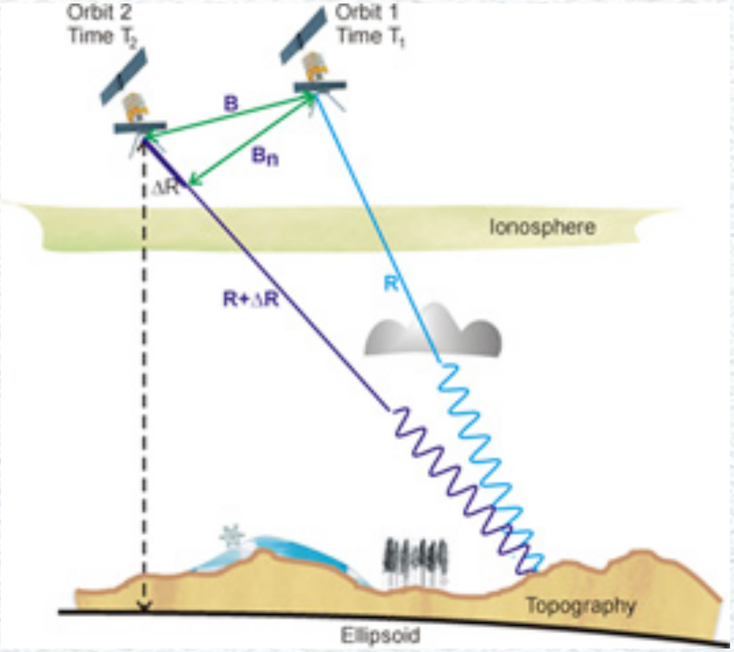
SAR Interferometry (InSAR)

SAR Interferometry (InSAR) consists of interfering two SAR images of an area, which have been acquired from two slightly different positions in space or time. When a target is viewed under two slightly different angles, the **elevation** can be precisely recovered using the **phase information** thus allowing a **3-dimensional reconstruction** of the viewed scene.



SAR Interferometry (InSAR) Overview

The signal received by the radar from a target at distance R has an amplitude (A) related to the scattering strength of the target and a phase (φ) related to the two-way traveling path of the wave between the radar and the target

$$g = A \cdot e^{-j\varphi}$$

The traveling path is given by the distance between the radar and the target

There are phase effects arising in the resolution cell because of the different scatterers located at different positions within the cell ($\varphi_{scatter}$)

A further phase contribution arises if the wave travels through a medium with dielectric properties different than vacuum,
e.g. water vapor or wet snow layer (φ_{delay})

$$\varphi = -\frac{4\pi}{\lambda} R + \varphi_{scatter} + \varphi_{delay}$$

SAR Interferometry (InSAR) Product

If we take the difference between the phases of a target viewed from two slightly different positions in space having slant range distance R_2 and R_1 we obtain

$$\phi = \Delta\varphi = \varphi_2 - \varphi_1 = -\frac{4\pi}{\lambda}(R_2 - R_1) + \Delta\varphi_{scatter} + \Delta\varphi_{delay}$$

, where $(\Delta R = R_2 - R_1)$

The phase difference, ϕ, is related to the

- **path length difference**
- variations in the speckle pattern (repeat-pass case or unstable targets)
- variations of the medium properties (water vapor or electron density in the ionosphere - $\Delta\varphi_{delay}$)

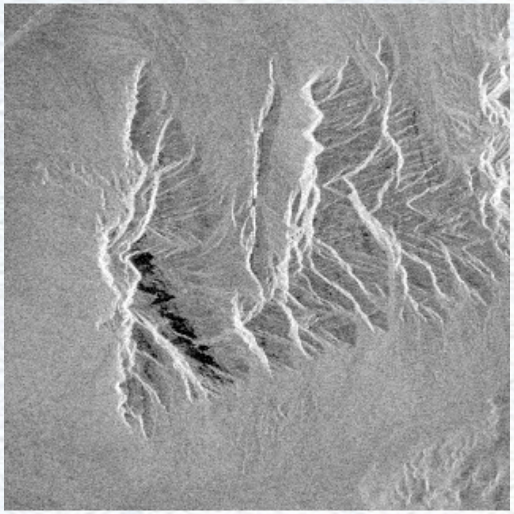
The phase difference can be obtained if we take the product of one SAR image, g_1, and the complex conjugate of the other SAR image, g_2

$$s_i = g_{1,i} g^*_{2,i} = (A_{1,i} A_{2,i}) \cdot e^{j\left[-\frac{4\pi}{\lambda}(R_2 - R_1) + \Delta\varphi_{scatter,i} + \Delta\varphi_{medium,i}\right]}$$

The result of the product is called the **interferogram**.

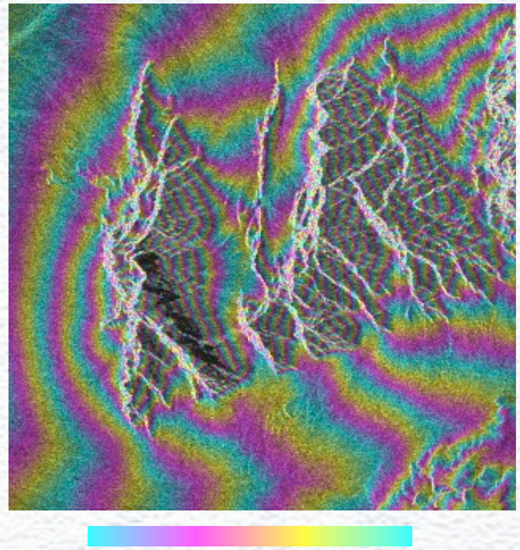
InSAR images: example from ERS-1 over Death Valley

SAR Intensity



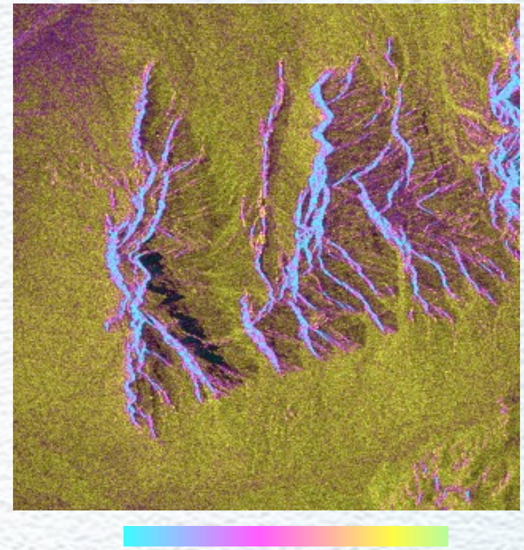
Repeat Interval: 35 days

Interferometric Phase



0 2π

Coherence



0 0.9

An interferogram is a complex image with

- magnitude given by the product of the SAR amplitudes
- phase (the InSAR phase) given by the path length difference, as well as variations of the scattering properties and the medium conditions

InSAR phase measures the phase difference of a target with respect to the radar - The coherence depends on surface properties and SAR frequency

Atmospheric phase

Changes in the atmospheric conditions (water vapour, electron density in the ionosphere - TEC variations) or dielectric (coherent) variations of a volume on the ground induce a delay of the propagation of the microwave

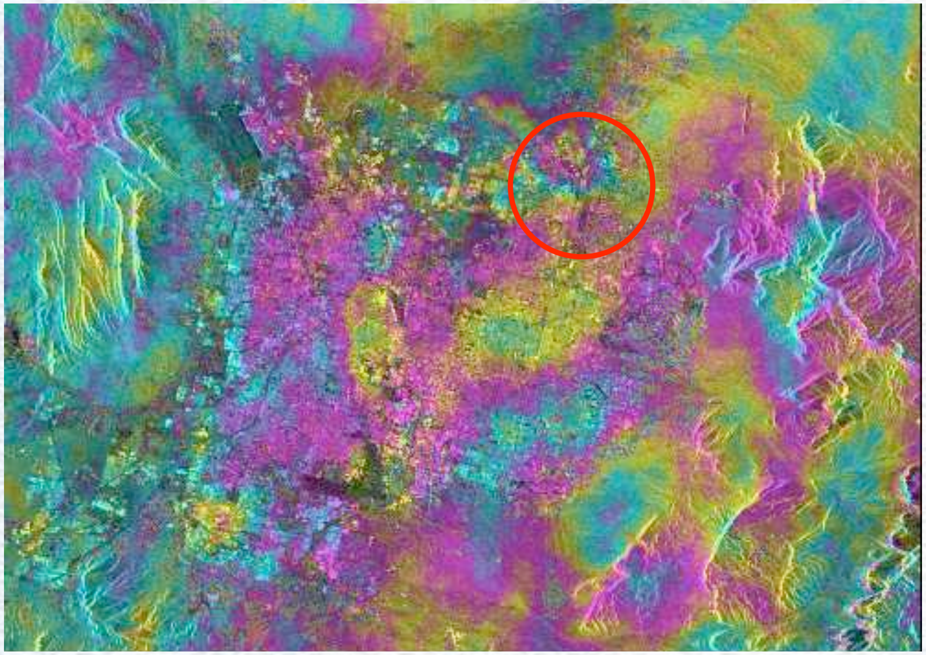
There is an additional path length between the SAR and the surface elements
The additional phase:

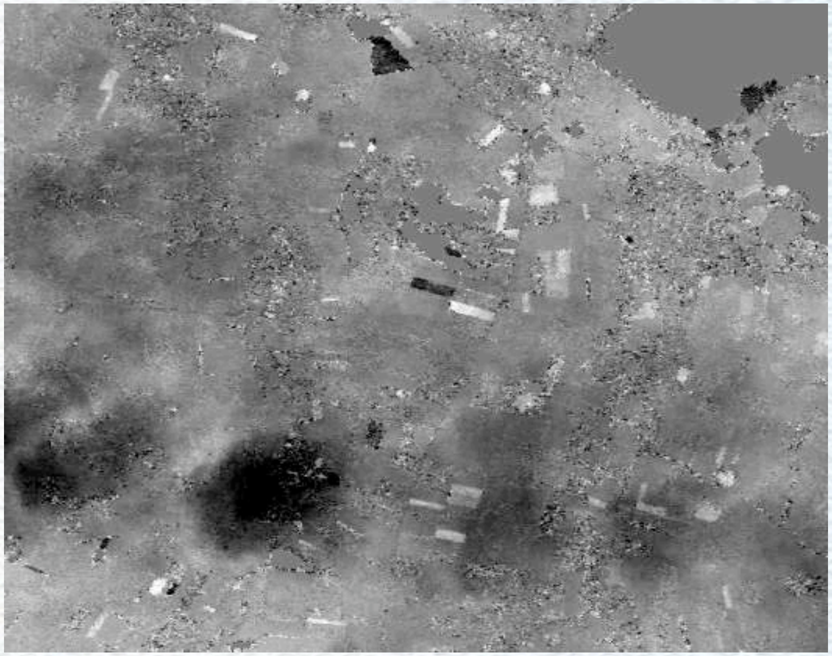
$$\phi_{path} = \frac{4\pi}{\lambda} R_{path}$$

- The phase due to atmospheric artifacts does not depend on the baseline
- The sensitivity of the phase to the atmosphere is related to the wavelength
- Longer wavelengths are less sensitive to atmospheric distortions
- A propagation delay (R_{path}) of 2 cm would result in an additional phase of an almost full fringe at C-band but only of 1/6th of a fringe at L-band

Effects of atmospheric phase on interferogram

Path delays can strongly alter the information content of the interferogram





The only area affected by displacements (subsidence) is indicated in the circle. All other fringes are due to path delays in the atmosphere (ERS interferogram, Las Vegas, Nevada)

Cloud-shaped atmospheric phase effects appearing in the lower half can easily be distinguished by its characteristic shape from the vegetation growth related effects. The shape of the phase change follows the field boundaries, indicating that this phase effect depends on the surface type

Source: Gamma Remote Sensing AG

Phase Difference

- In general the phase difference is a function of the 2D space and time
- We can assume that the values of the phase difference at a given position is a function of the values of neighbors positions or the value at a given time instance depends on the values of previous time instances.
  - That is

$$\phi(t \mid n,m) \sim g(\phi(t-1 \mid n,m), \phi(t-2 \mid n,m), ...)$$

  - Or

$$\phi(t \mid n,m) \sim g(\phi(t \mid n-1,m), \phi(t \mid n-2,m), \phi(t \mid n,m-1) ....)$$

  - Or a combination between them

Non-Linear AutoRegressive Model (RNAR)

- The drawbacks of the aforementioned model is that it expresses a linear relationship, which in practice is not true.
- In the most general case the value at the current moment n is related with a highly non-linear way with the previous values

$$\phi\ (q) = g(\phi(q-1), \phi(q-2), ..., \phi(q-p)) + e(q)$$

- The main difficulty of the previous equation is that function $g(\cdot)$ is actually unknown

Function Modeling

- Using functional analysis, we can approximate any continuous non-linear function with known functional components

$$\hat{\phi}(q) = \mathbf{v}^T \cdot \mathbf{u}(\boldsymbol{\varphi}(q-1)) + \theta$$

- Where

$$\mathbf{u}(\boldsymbol{\varphi}(q-1)) = \begin{bmatrix} u_1(\boldsymbol{\varphi}(q-1)) \\ \vdots \\ u_l(\boldsymbol{\varphi}(q-1)) \end{bmatrix} = \begin{bmatrix} f(\mathbf{w}_1^T \cdot \boldsymbol{\varphi}(q-1)) \\ \vdots \\ f(\mathbf{w}_l^T \cdot \boldsymbol{\varphi}(q-1)) \end{bmatrix} = \mathbf{f}(\mathbf{W}^T \cdot \boldsymbol{\varphi}(q-1))$$

- Vector $\boldsymbol{\varphi}(q-1)$ contains all the p previous values of the phase difference

The model

- Using this approximation, one can derive an on-line dynamic and highly prediction of the phase difference
- In particular, we assume that the function approximation is accomplished from the coefficients **v** and **w** respectively
- Therefore, the target is to estimate these coefficients
- Initially, these coefficients are estimated through a training set
  - However, the non-linear function is dynamically change through time
  - Thus, a recursive estimation scheme is required, meaning that $g_{(t)}$

The recursive Approach

- In order to make the model adaptable we make the assumptions that the new coefficients are related with the previous ones using the following relation

$$\mathbf{w}_a = \mathbf{w}_b + \Delta\mathbf{w}$$

- Where W_a refers to the coefficients after the adaption, while W_b to the coefficients before

- Using this assumption we can estimate the coefficients in a recursive framework

The recursive Approach-First Condition

- First Condition
  - Assume that the model trust the current samples as much as possible, that is

$$\text{Estimate } w_a \quad \text{so that} \quad \hat{\varphi}_{w_a}(q) \equiv \varphi(q)$$

- This condition can be written as a linear equation system if we assume that

$$\mathbf{w}_a = \mathbf{w}_b + \Delta\mathbf{w}$$

$$b = \mathbf{a}^T \cdot \Delta\mathbf{w}$$

- where vector **a** depends on the previous weights

The recursive Approach-Second Condition

- Second Condition
  - The error over the initial training set is minimized as much as possible

$$\text{Estimate } w_a \quad \text{so that} \quad \min \varepsilon_{w_a}$$

  - Where e_{wa} is the mean square error

- Using the aforementioned assumptions one can conclude to the following equation

$$E = \frac{1}{2} \Delta \mathbf{w}^T \cdot \mathbf{J}^T \cdot \mathbf{J} \cdot \Delta \mathbf{w}$$

- Where J is the Jacobian matrix of the error

The Algorithm
Recursive Non-Linear AutoRegressive Models
RNARs

- Step 1
  - Use A training set to estimate the initial coefficients
  - Or use the traditional linear approach of the Autoregressive model
- Step 2
  - Perform the prediction algorithm and estimate future values
  - Calculate the error among the predicted value and the original one
- Step 3
  - Modify the coefficient of the model so that the predicted value is minimized and the error over the initial training set
- Step 4
  - Go to Step 3

Conclusions

- We introduce a novel recursive non-linear model for predicting the phase difference
- The model is computational efficient
  - One linear equation as constraint and a minimization of a convex function
- As future work we need to evaluate the model in real life conditions and against other linear or non-linear mode

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