Uncoated Cubes for GNSS Satellites

by

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International Technical Laser Workshop on SLR Tracking of GNSS Constellations

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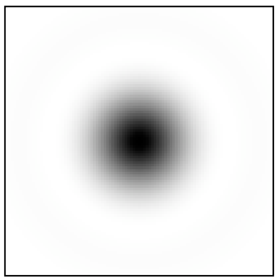
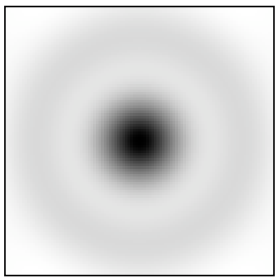
Metsovo, Greece

Basic Concepts

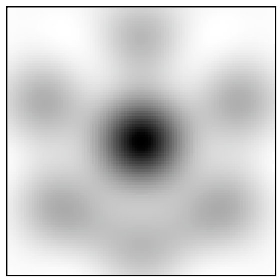
- Velocity aberration
- Diffraction pattern
- Dihedral angle offset
- Coated vs uncoated cubes



Coated no dihedral

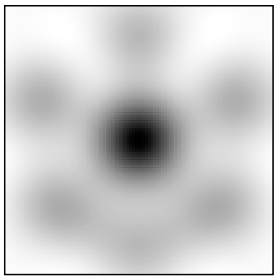


Coated 0.8 dihedral

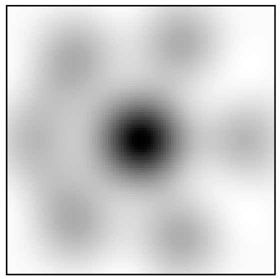


Uncoated no dihedral

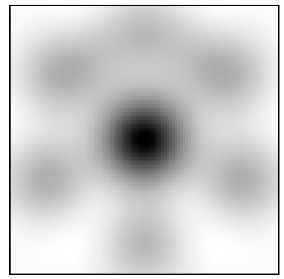
Different orientations
No dihedral offset



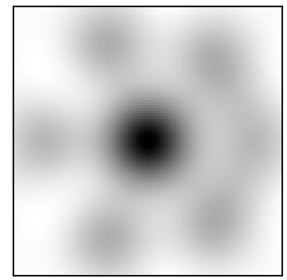
0



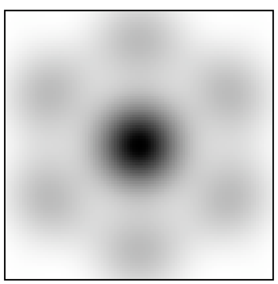
30



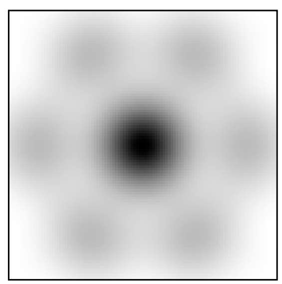
60



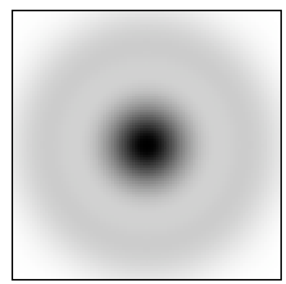
90



0 + 60

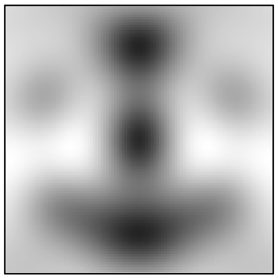


30 + 90

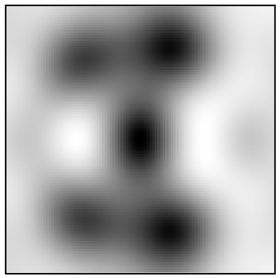


All

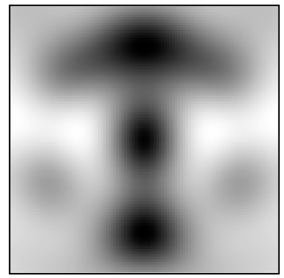
Different Orientations
0.75 arcsec offset



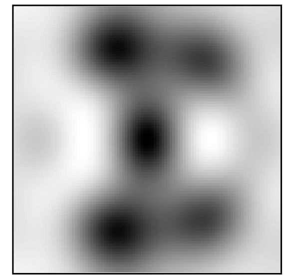
0



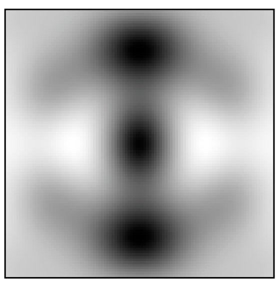
30



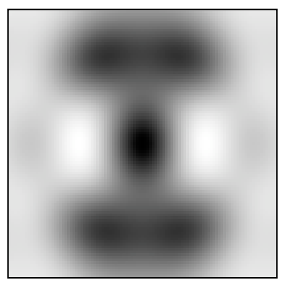
60



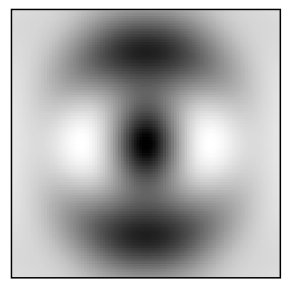
90



0 + 60



30 + 90



All

Conclusions

- Uncoated cubes have a natural beam spread due to phase changes from total internal reflection. No dihedral offset is needed.
- Orienting sets of 4 cubes at 0, 30, 60, & 90 degrees produces a smooth circular pattern.
- Adding a dihedral angle offset causes an asymmetry that cannot be removed. This gives inconsistent signal strength.

GFZ

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DEUTSCHES GEOFORSCHUNGSZENTRUM

Far Field Measurements of Cube Corner Prisms

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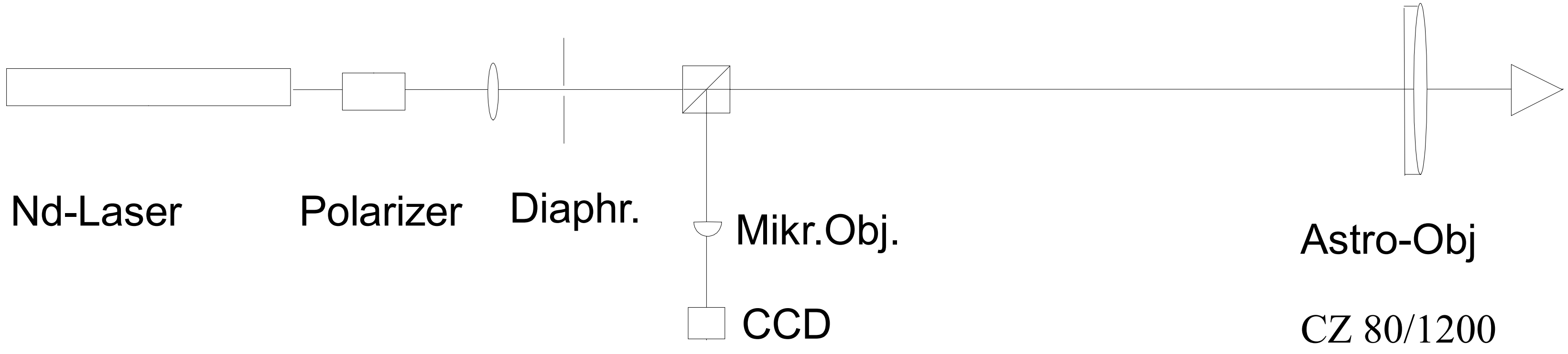
The Optical Set Up

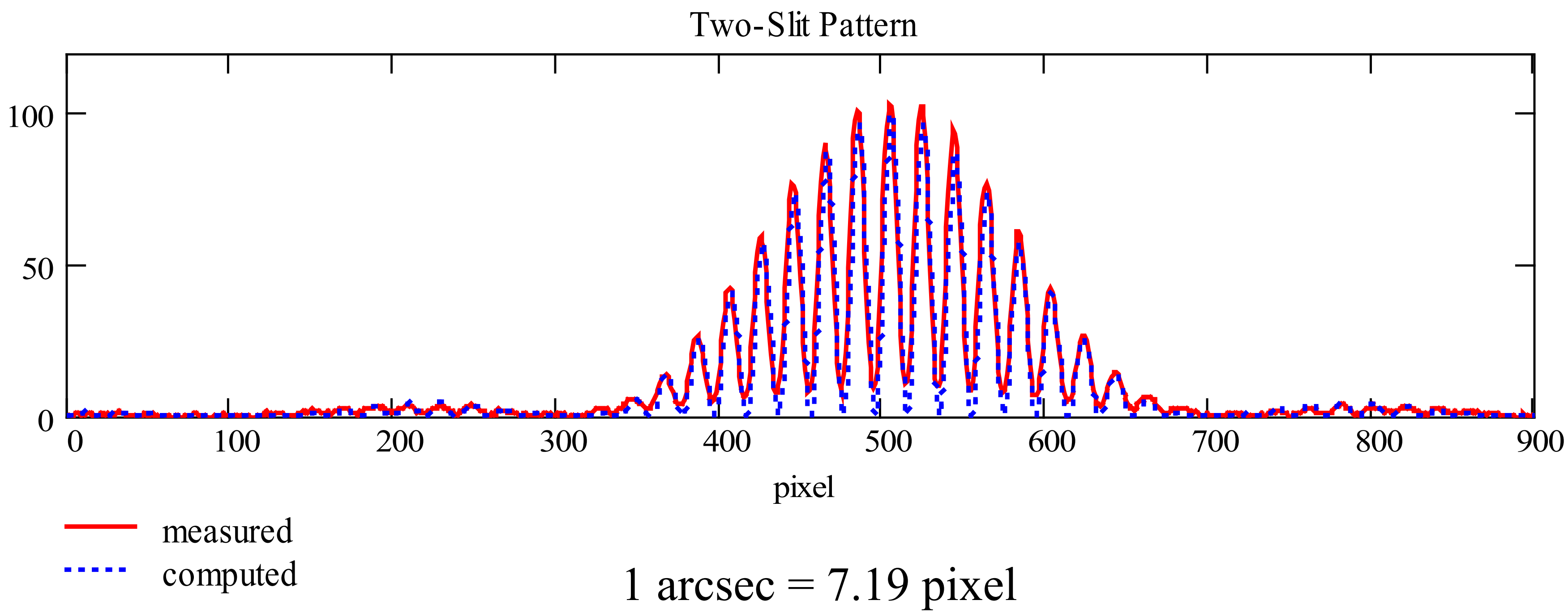


GFZ Helmholtz-Zentrum Potsdam

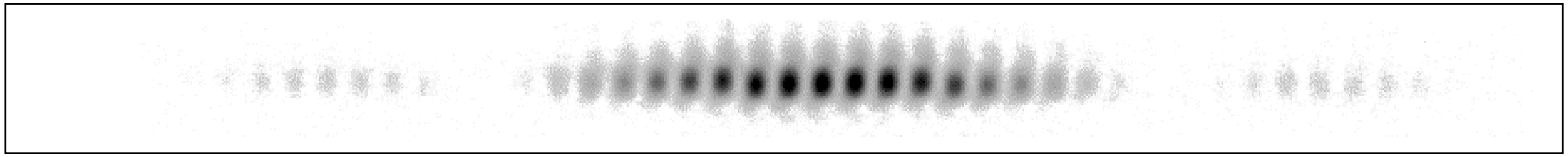
Helmholtz-Zentrum Potsdam Deutsches GeoForschungsZentrum

Calibration Using Two Slit Pattern

Spacing: 40mm, Slit width: 4mm



1 arcsec = 7.19 pixel



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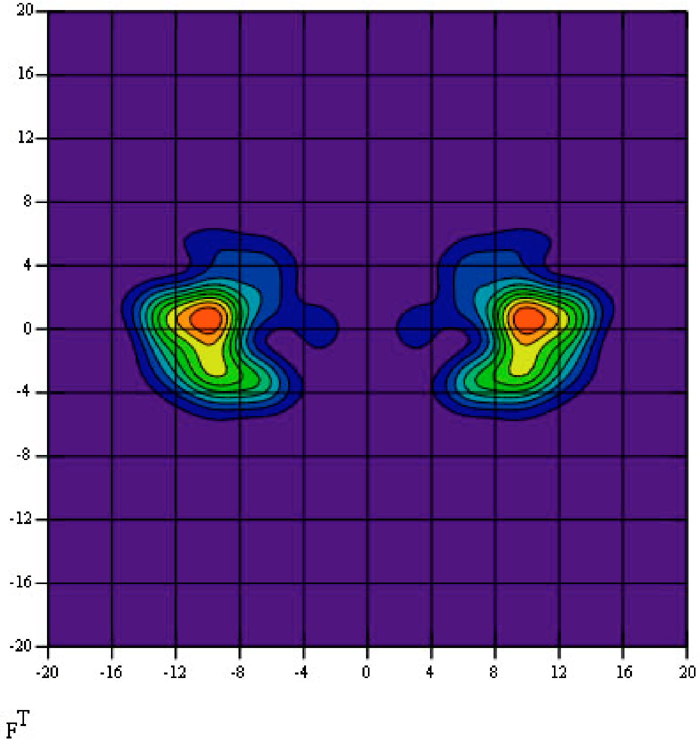
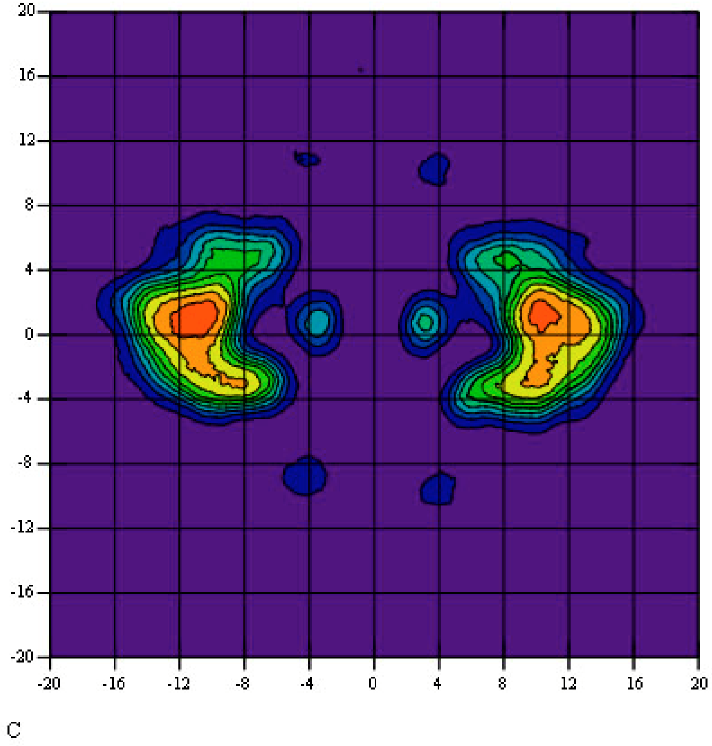
CHAMP Prism Uncoated

38mm Diam. 3.8“offset

Theory



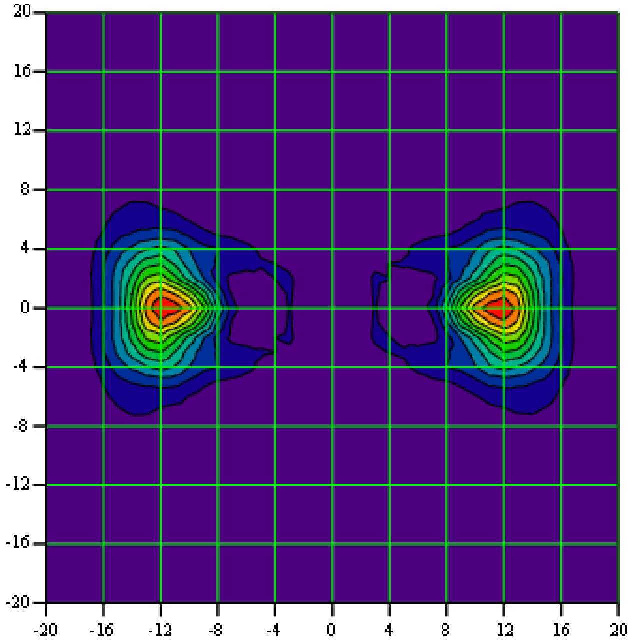
Observed



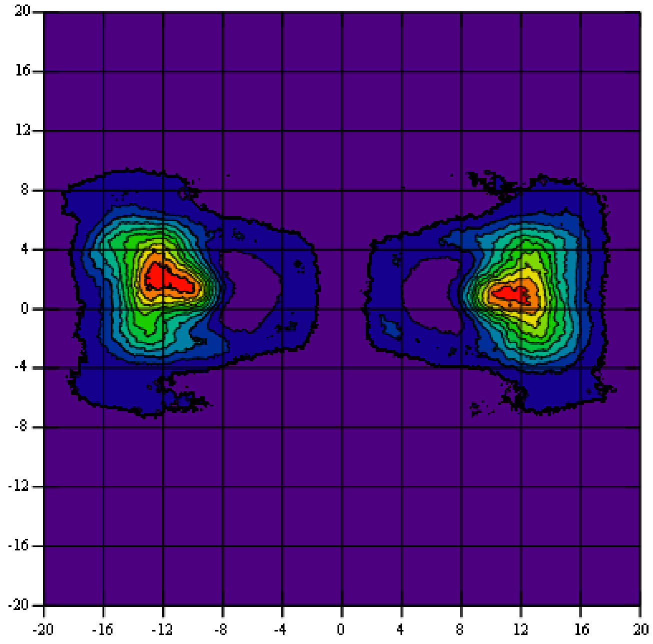
Far Field of a Prism Dedicated for SWARM
(coated and with spherical front face)

Theory

Observed



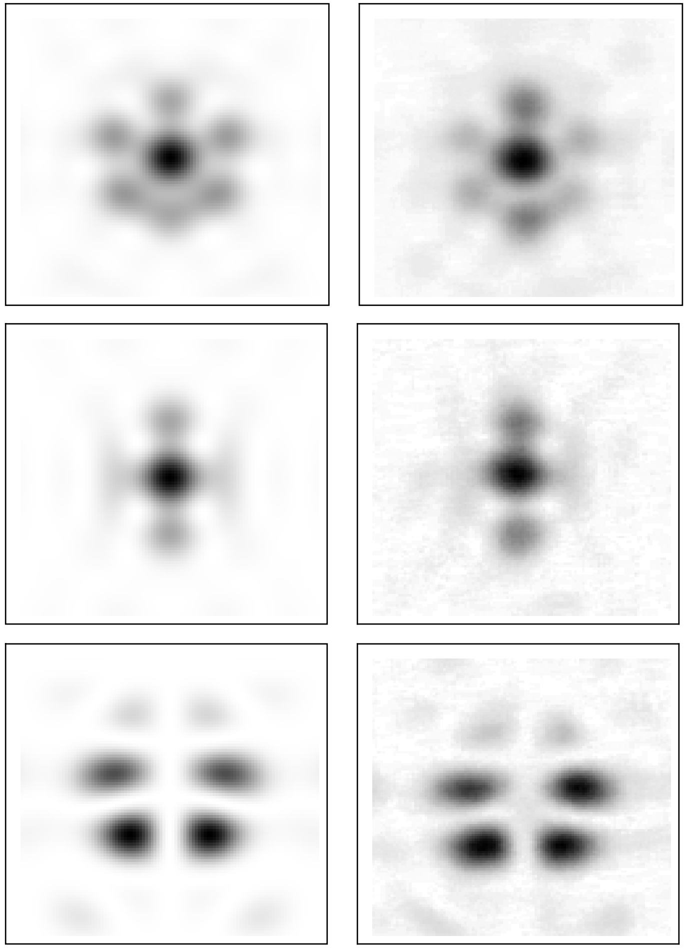
A



C

Confirming the Negative Sign of the Phase Shift at Total Internal Reflection using an almost perfect cube corner of 1.5inch diam.

Theory | Observed

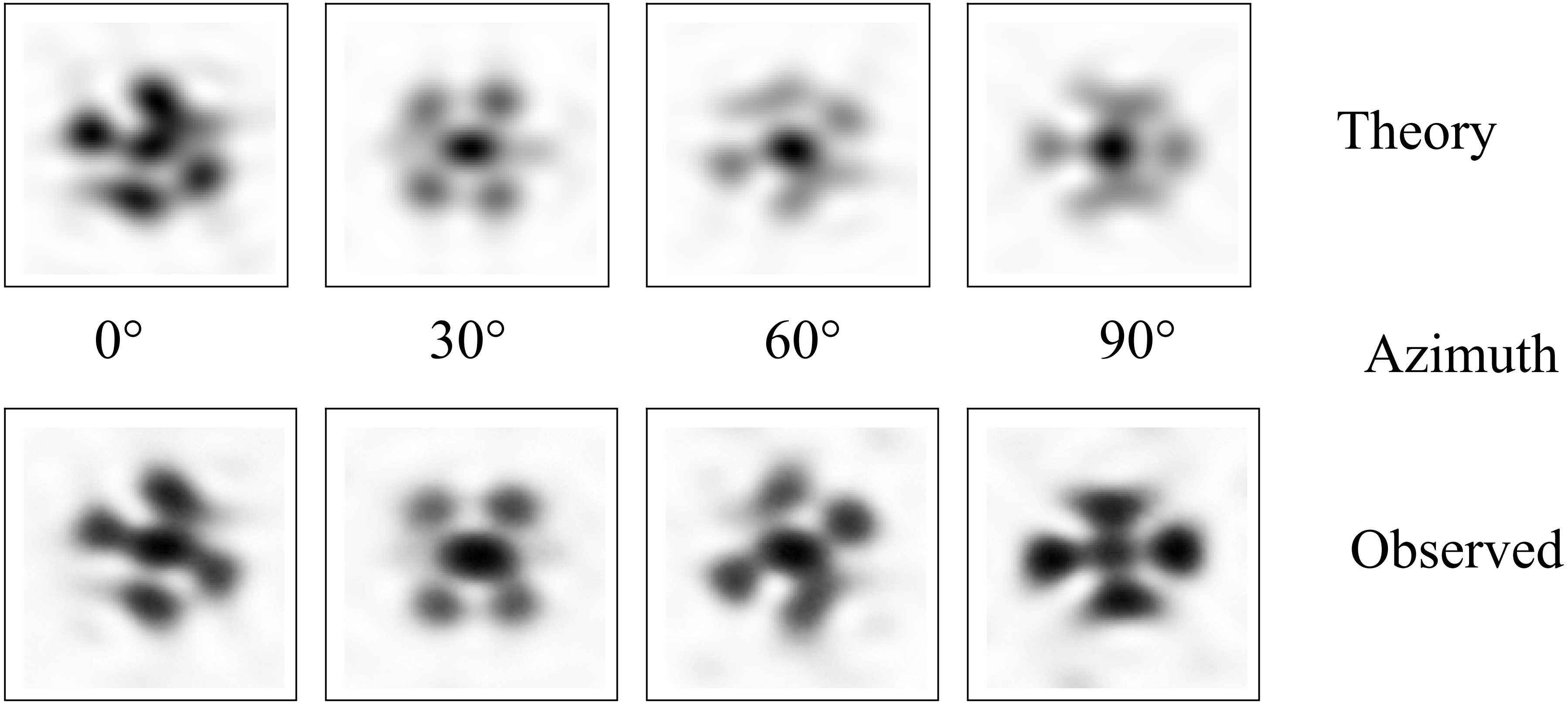


Total Energy

Vertical Component

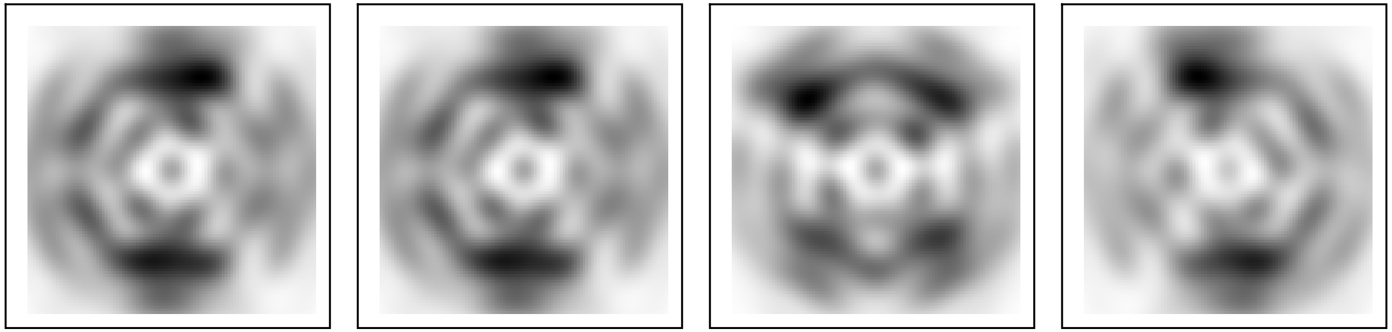
Horizontal Component

Far Field Patterns (total energy) at 14° Inclination.



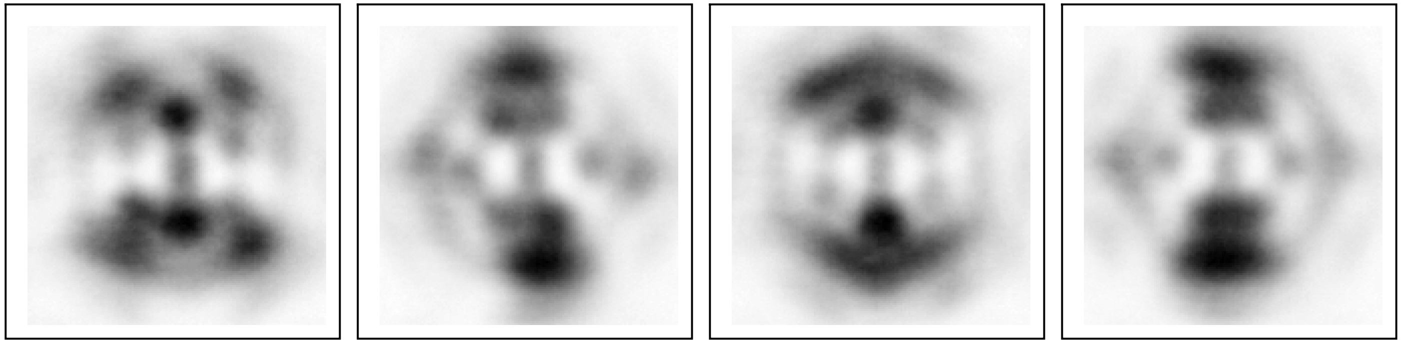
LARES Prism

(measured offset angles: 1.5“ 1.3“ 1.7“)



Theory

0° 30° 60° 90° Azimuth



Observed

Galileo uncoated reflector with or without dihedral angle offset?

2009-03-13

Laser-ranging retroreflectors carried on most of Earth-orbiting satellites have dihedral angle offsets, from 0.5 to 2 arcseconds. Motivated by David Arnold, we ask "is the dihedral angle offset re necessary?" This note discusses the Galileo (altitude = 23916km, inclination = 56deg) case.

A simple orbit calculation gives the range of velocity aberration and angle of incidence as follows:

angle of incidence: 0 to 11.5 degrees

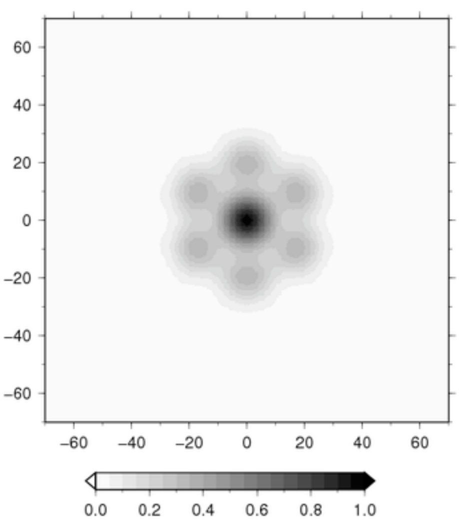
velocity aberration: 21.2 to 25.3 microradians

Note that Galileo satellites orbit higher than other GNSS satellites, which results in narrower angles of incidence and smaller velocity aberration than GPS and GLONASS.

We assume a 38-mm-diameter, circular-face, uncoated, n=1.46 reflector for this study. Laser wavelength is set to 532 nm, and the average of two polarizations is taken. A far-field diffraction p is generated for every possible two-dimensional angle of incidence with 2 degrees' interval. A huge number of diffraction patterns are generated per reflector.

Here is one of far-field diffraction patterns that a **zero-dihedral-angle reflector** generates when **the angle of incidence is zero**:

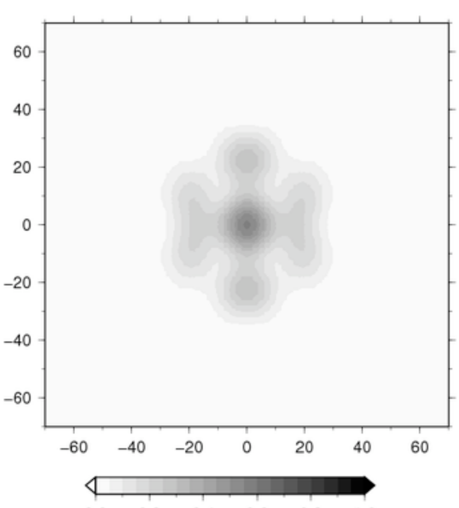
[Figure 1]



(x and y axis: in microradians)

The pattern gets distorted as the angle of incidence becomes larger. For instance, the pattern becomes like below when **the angle of incidence is 8 degrees**.

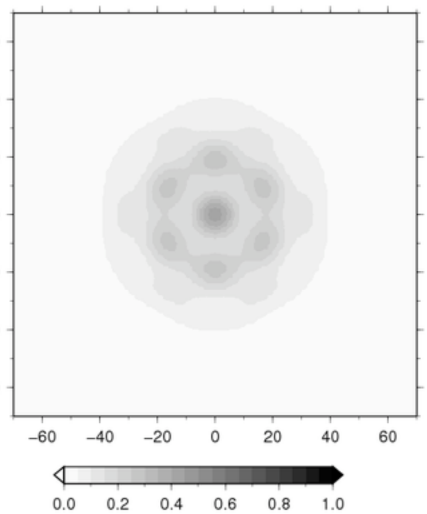
[Figure 2]



Note these are just one of the computed patterns as we assumed a number of azimutal angles (longitude) of incidence.

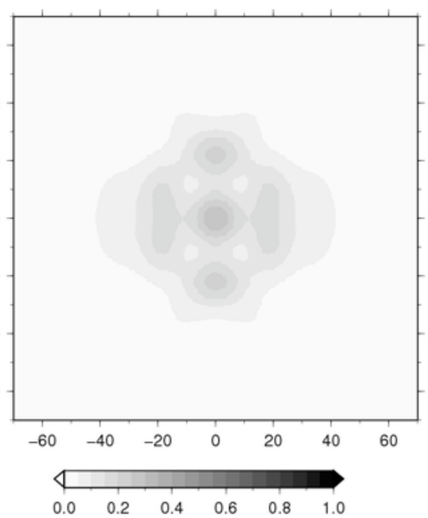
Now, let us assume a small, **0.75 arcsecond diheadral angle**. The pattern is simulated as below when **the angle of incidence is zero**.

[Figure 3]



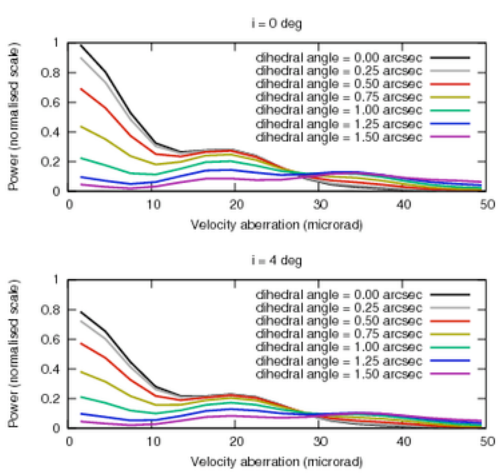
Compared with Figure 1, it's clearly seen that the energy is more scattered. However, it is hard to tell which one performs bettern within the 21-to-25-microradian ring. Here is the case of **8 degrees of angle of incidence** with this reflector.

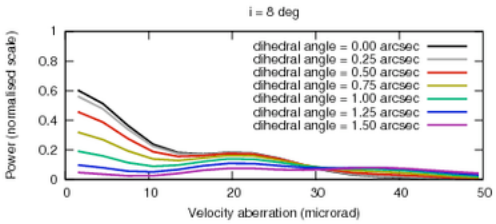
[Figure 4]

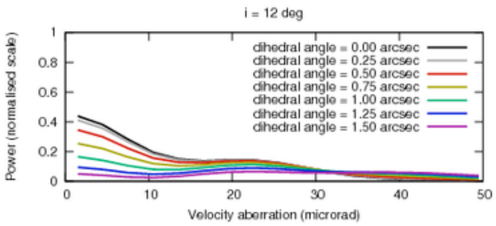


which should be compared with Figure 2.

Only 2 reflectors are shown above, but we modelled 7 dihedral angles: 0.00, 0.25, 0.50, 0.75, 1.00, 1.25 and 1.50 arcseconds,and, by averaging over azimuth angle cases, the average intensity po angle of incidence is given as a function of velocity aberration.







Roughly speaking, a small dihedral angle performs better below 30-microradian velocity aberration, and a large dihedral angle performs better over 30-microradian velocity aberration.

Looking into the 21-to-25 microradians region, the average intensity over 0 to 12 degrees of angle of incidence is calculated, relative to the zero dihedral angle case, as:

dihedral angle = 0.00 arcsec: 100% (baseline)

dihedral angle = 0.25 arcsec: 99%

dihedral angle = 0.50 arcsec: 96%

dihedral angle = 0.75 arcsec: 91%

dihedral angle = 1.00 arcsec: 77%

dihedral angle = 1.25 arcsec: 60%

dihedral angle = 1.50 arcsec: 45%

This result tells us that there is no advantage by having dihedral angle offset in this case, although there is no large difference among small dihedral angle cases.

(Of course, the result could be various with different assumptions on orbit, reflector size, coating, etc.)

(Toshimichi Otsubo t.otsubo@srv.cc.hit-u.ac.jp, 2009-03-13)

Far Fields of an Uncoated LARES Cube Corner for Normal Incidence

Circular front face 1.5 inch diam.
Measured offset angles : 1.51” , 1.31”, 1.72” (ZYGO interferometer)

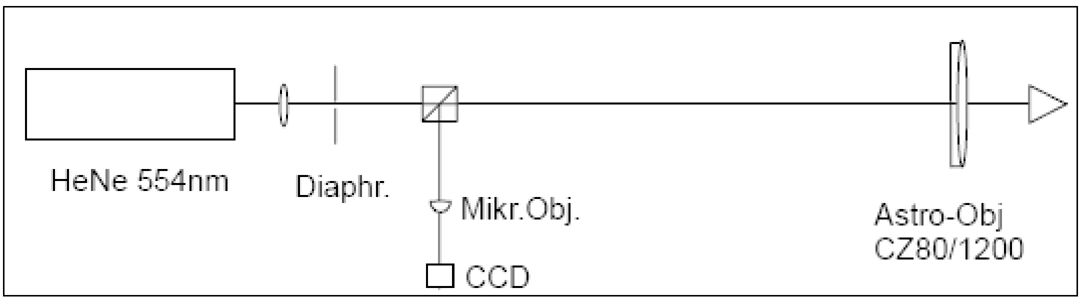
The theoretical patterns were computed using Arnold’s program “difract201”
An example input file of the program (“for001”) is as follows:

```
  101    2    50.0    .05       0.0      0.0     1.51     1.31     1.72
 1.0610   1.5000      0.        0.       1.       0.     .5320    1.461 0.
                      0.        0.       1.       0.
00
```

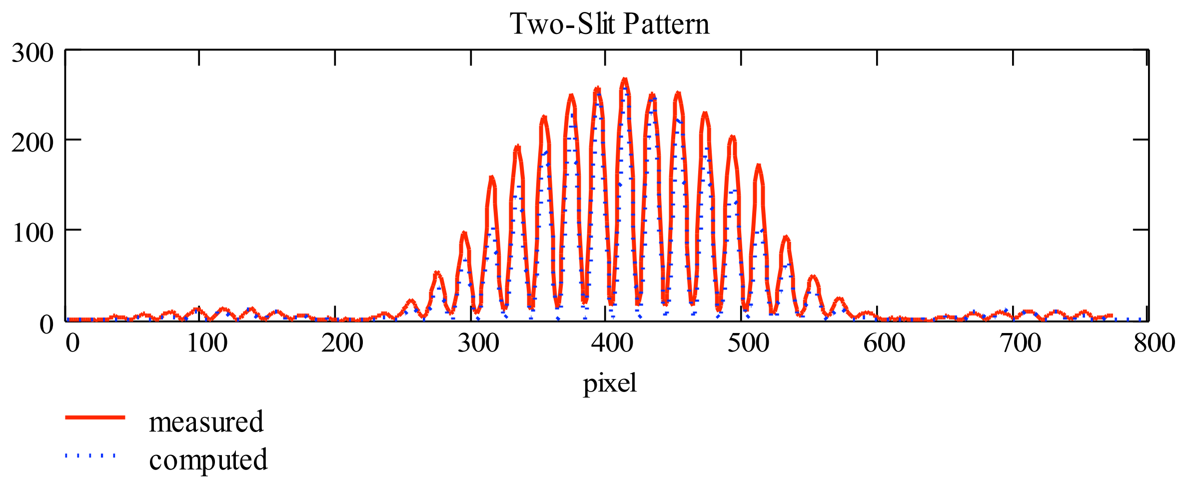
In the case of zero Azimuth the edge No. 1 (1.51” offset) was oriented vertical upward.
The accuracy of the measured offset angles (ZYGO interferometer) should be better than 0.1”.

The far field patterns are visualised as looking from the laser to the reflector. The photographed patterns must be side inverted because there is a mirror in the in the set up.

The far field patterns have been observed using the following simple set up. The He-Ne-Laser has been replaced by a Nd-Laserpointer (532nm wavelength)

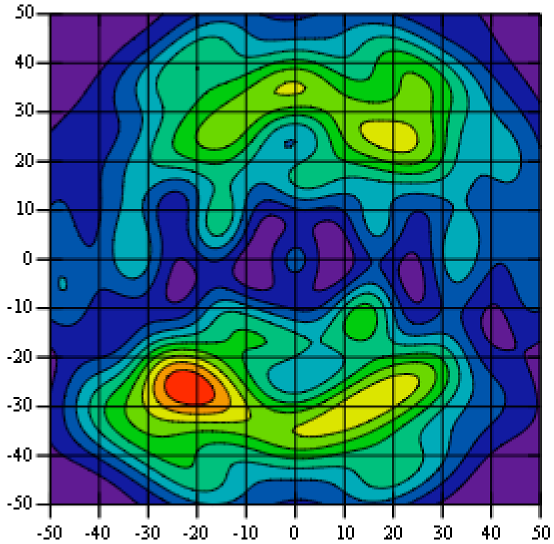


The scale has been calibrated by observing the far field of a double slit of 40mm spacing and comparing it with theoretical pattern. Best fit was obtained assuming 1µrad = 1.5 pixel.

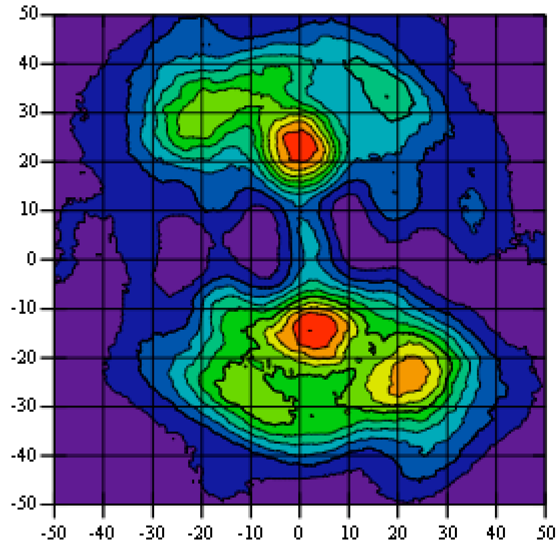


Theory

Measurement

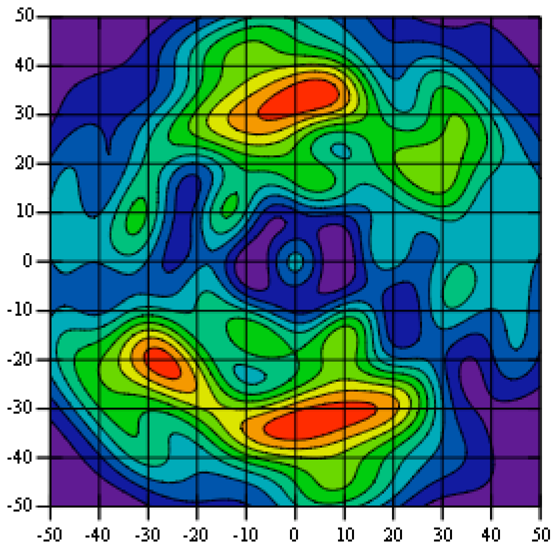


F^T

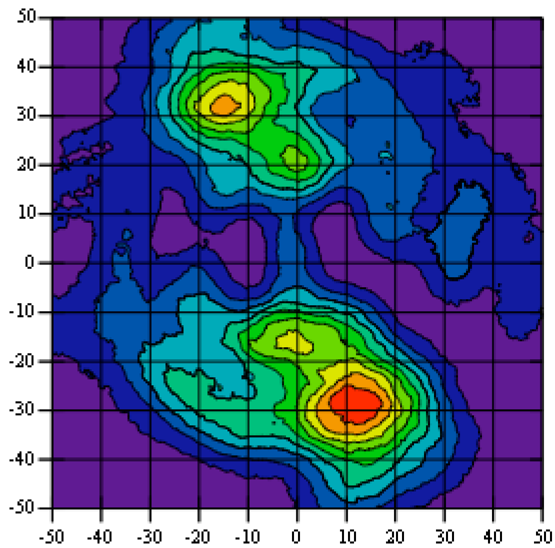


C

Az = 0°



F^T

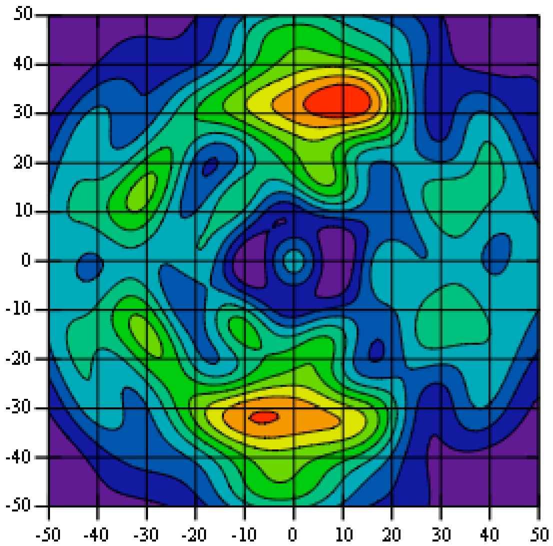


C

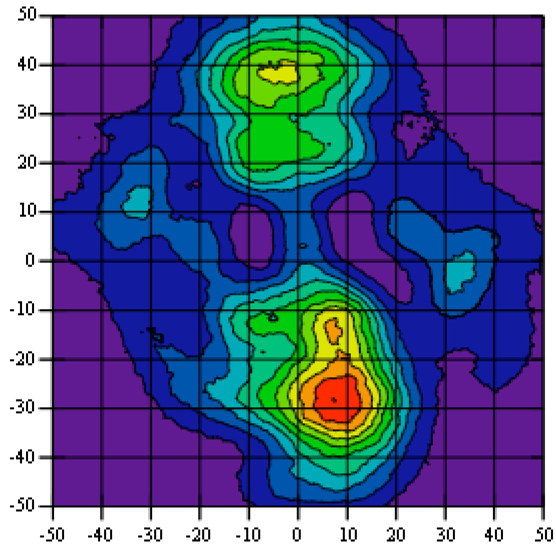
Az = 15°

Theory

Measurement

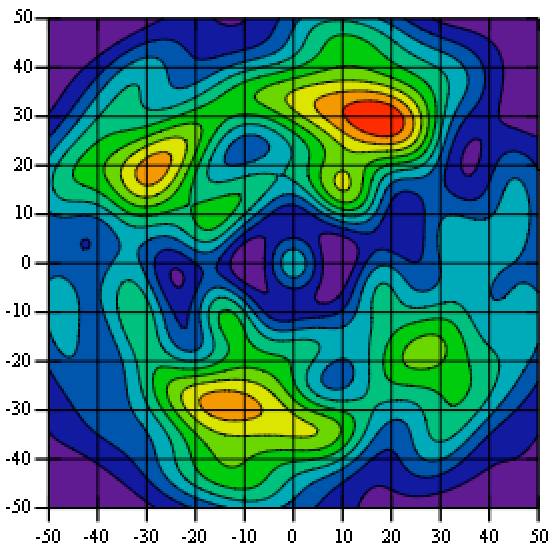


F^T

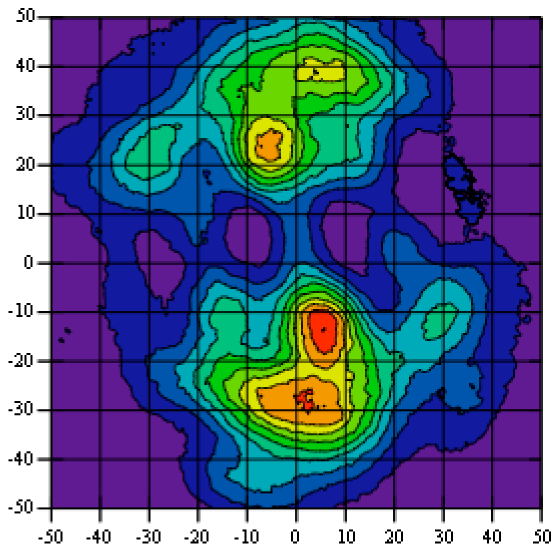


C

Az = 30°



F^T

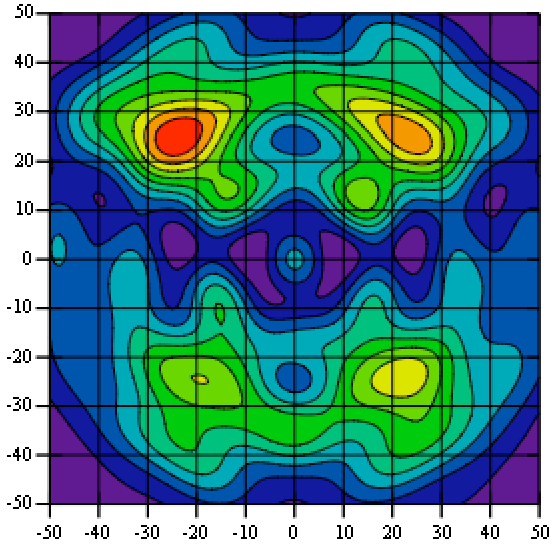


C

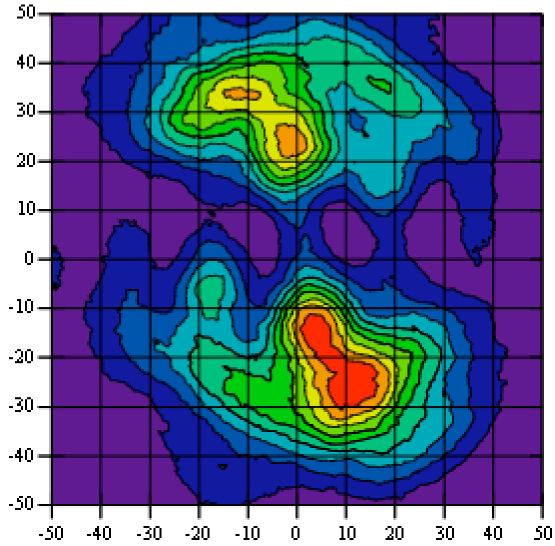
Az = 45°

Theory

Measurement

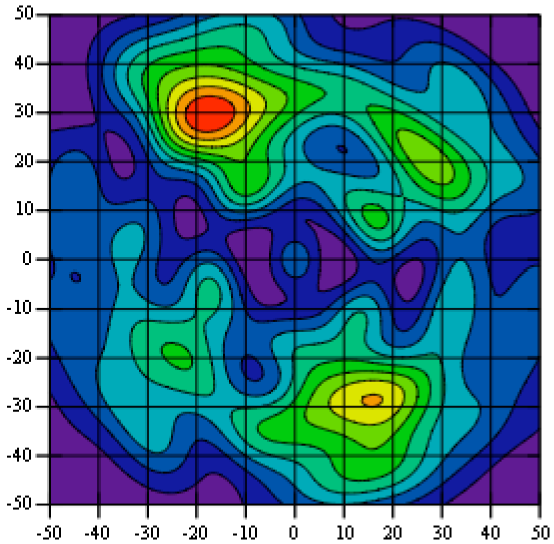


F^T

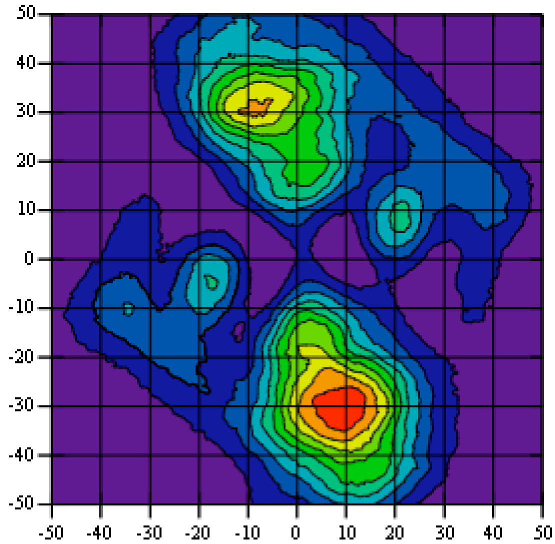


C

Az = 60°



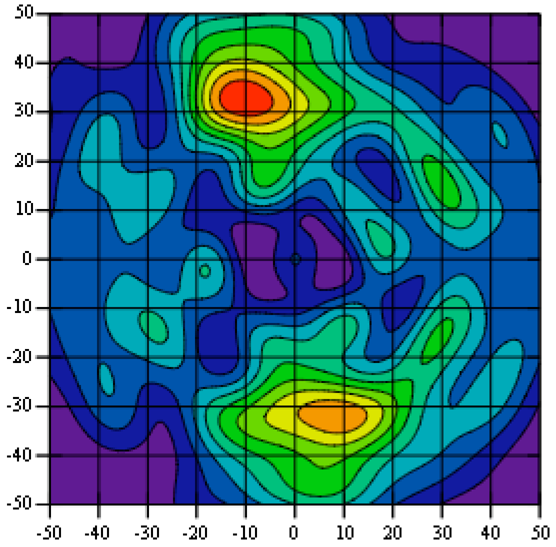
F^T



C

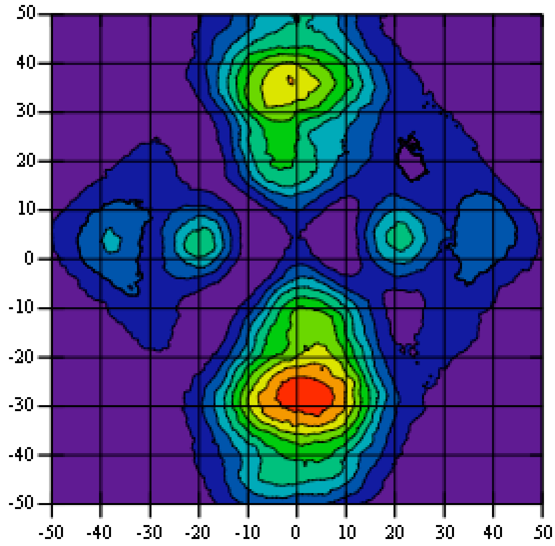
Az = 75°

Theory



F^T

Measurement



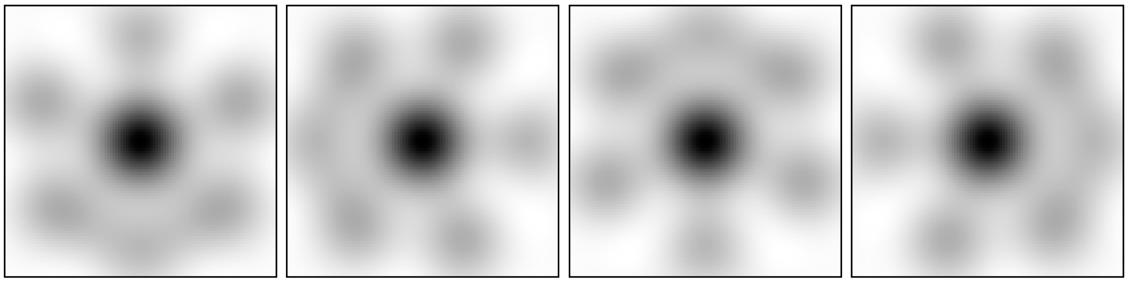
C

Az = 90°

Conclusion:

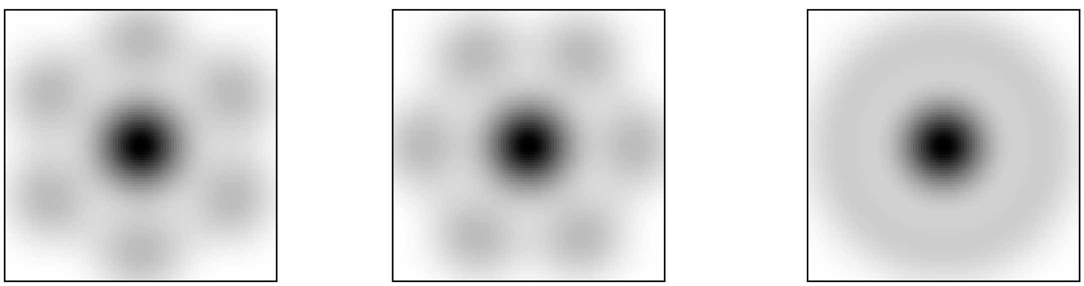
The measured far field patterns are similar but not fully equal to the expectation.
Most of the energy is concentrated near to the vertical axis , parallel to the polarization of the incident light, as expected.

1.5 inch circular uncoated cube
No dihedral angle offset
Scale -25 to +25 μrad
Linear vertical polarization



(a) θ = 0 (b) θ = 30 (c) θ = 60 (d) θ = 90

Figure 1. Diffraction patterns at different cube orientations.



(a) Average 0 & 60 (b) Average 30 & 90 (c) Average all

Figure 2. Averages of the diffraction patterns in figure 1.

θ is the rotation angle of the cube. θ = 0 is with one axis pointing up. All of the patterns have a slight asymmetry as a result of using linear polarization. The pattern rotates by 90 degrees every time the cube is rotated by 30 degrees. Rotating the cube by 60 degrees is equivalent to rotating the cube 180 degrees. Averaging the pairs 0 & 60, and 30 & 90 gives hexagonal patterns. The patterns in figures 2(a) and 2(b) are rotated 30 degrees from each other. Averaging all 4 patterns gives circular symmetry.

The units are such that the intensity of the Airy peak is unit. The intensity at the center of the pattern is .248 relative to the Airy peak. Polarization effects reduce the intensity of the central peak by about a factor of 4. The average intensity on the rings at 19 μrad is .067 relative to the Airy peak. Figures 3 and 4 show the intensity around a circle of radius 19 μrad for each pattern. The azimuth angle is measured counterclockwise from the +X axis (pointing to the right).

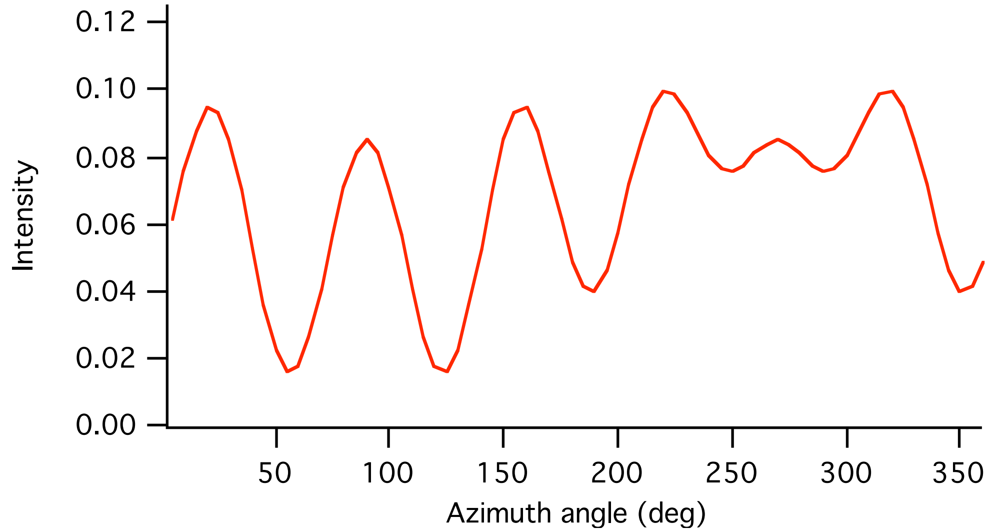


Figure 3(a) Orientation θ = 0 deg
Minimum 0.0161, average 0.0669, maximum 0.0992

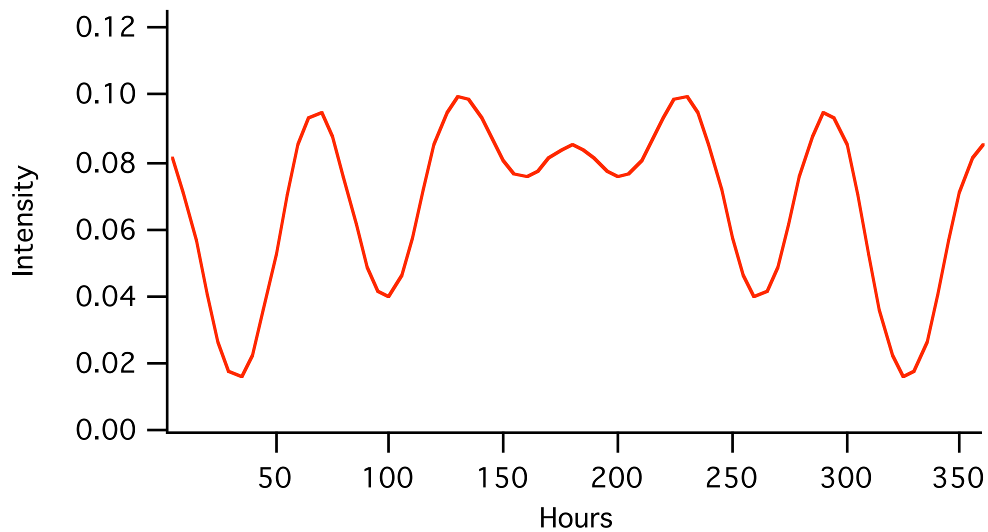


Figure 3(b) Orientation θ = 30 deg
Minimum 0.0161, average 0.0669, maximum 0.0992

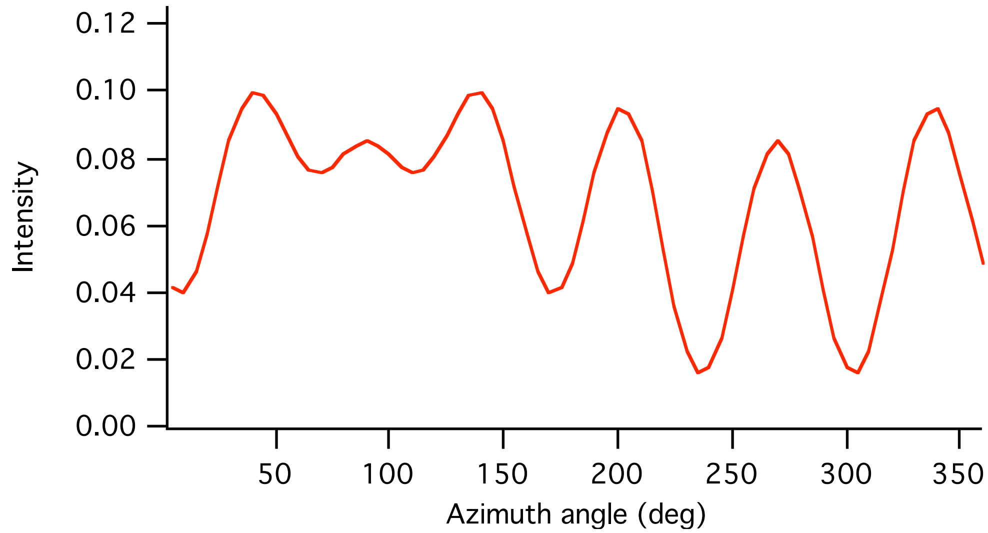


Figure 3(c) Orientation θ = 60 deg
Minimum 0.0161, average 0.0669, maximum 0.0992

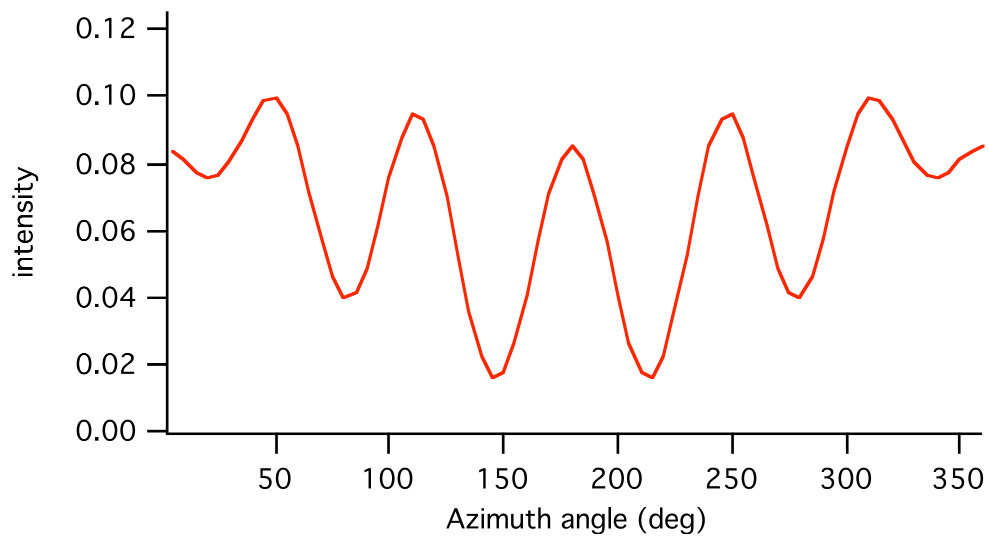


Figure 3(d) Orientation θ = 90 deg
Minimum 0.01612, average, 0.0669, maximum 0.0992

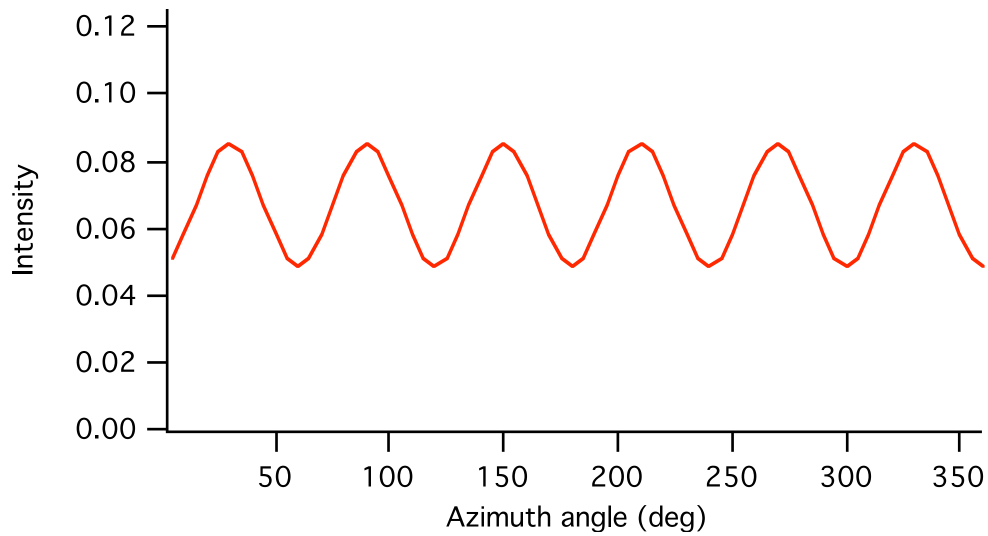


Figure 4(a) Average of 0 and 60 deg
Minimum, 0.0488, average 0.0669, maximum 0.0850

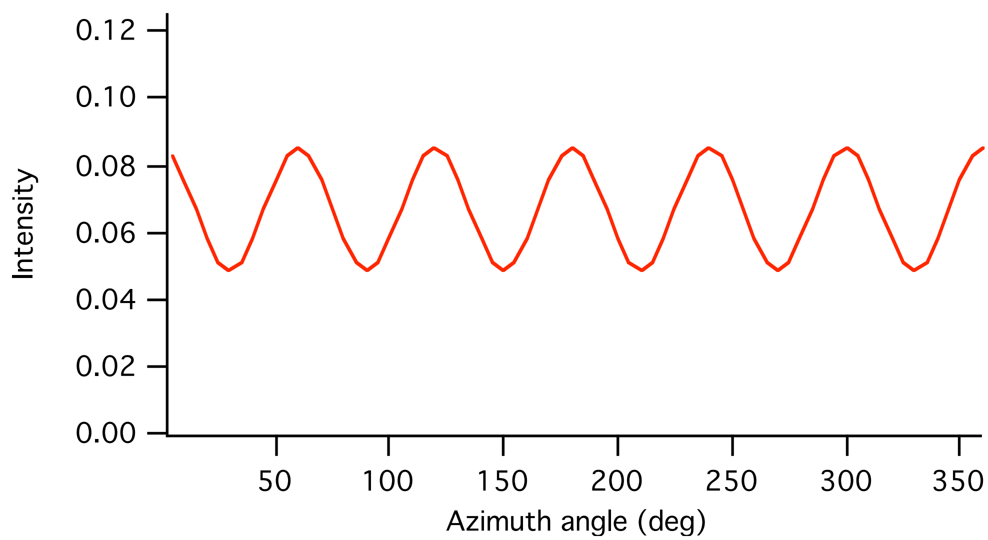


Figure 4(b) Average of 30 and 90 deg
Minimum 0.0488, average 0.0669, maximum 0.0850

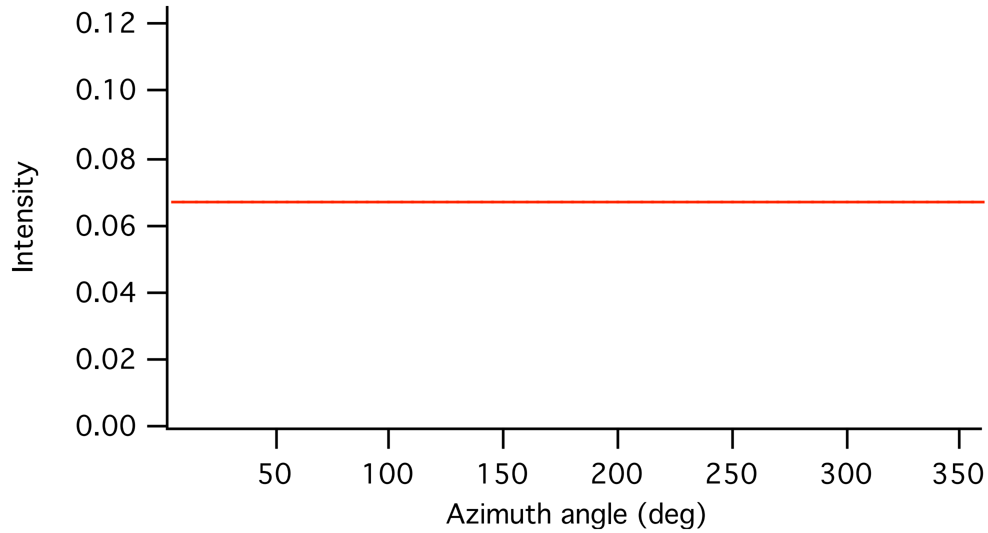


Figure 4(c) Average of all 4 orientations
Minimum 0.0669, average 0.0669, maximum 0.0669

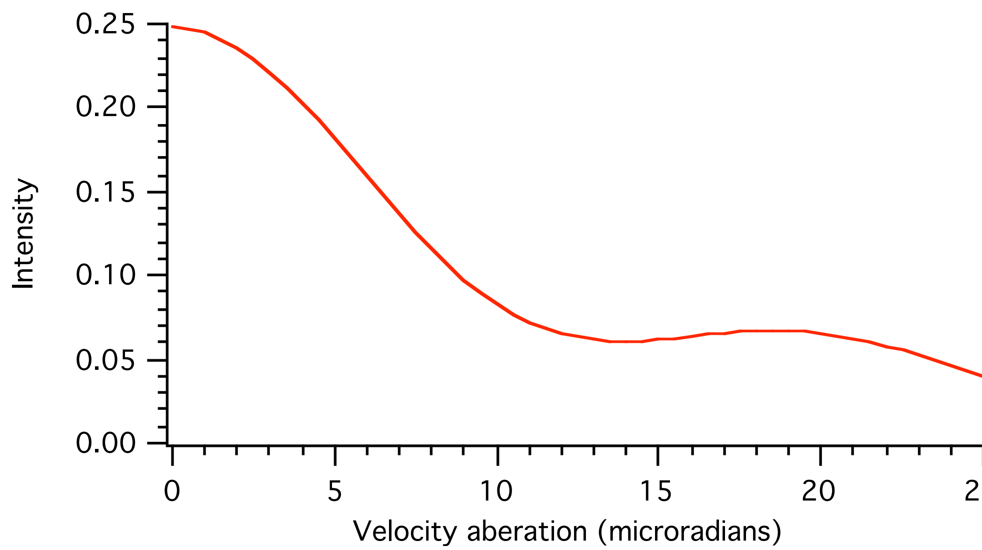
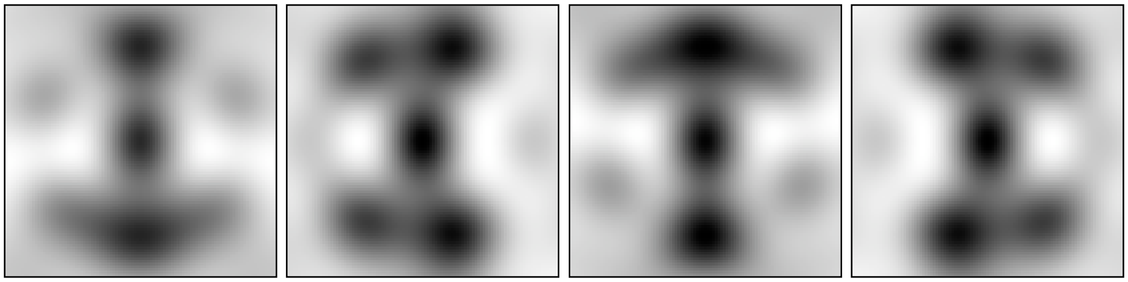


Figure 5. Intensity vs radius for the average of all 4 orientations. Intensity at 19 μrad is 0.0669

Summary of zero offset cases

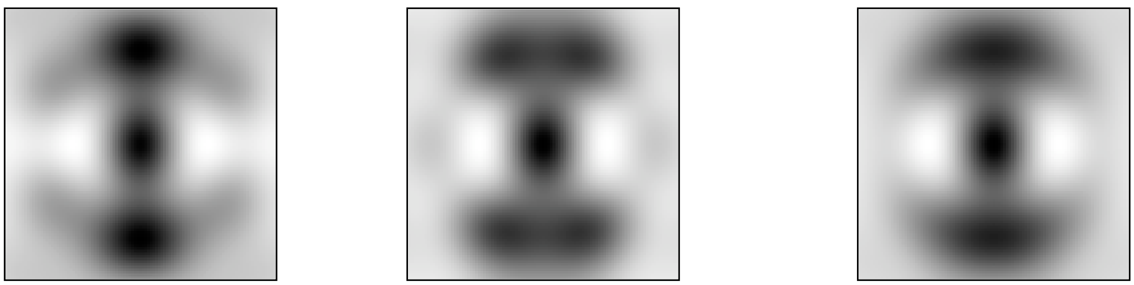
For each orientation of the cube the range of values around a circle of radius 19 μrad is
Minimum 0.01612, average, 0.0669, maximum 0.0992.
For the average of the pairs 0 and 60 deg, and 30 and 90 deg the range of values is
Minimum 0.0488, average 0.0669, maximum 0.0850
For the average of all 4 orientations the intensity is constant at 0.0669.

1.5 inch circular uncoated cube
Dihedral angle offset 0.75 arcsec
Scale -25 to +25 μrad
Linear vertical polarization



(a) θ = 0 (b) θ = 30 (c) θ = 60 (d) θ = 90

Figure 6. Diffraction patterns at different cube orientations.



(a) Average 0 & 60 (b) Average 30 & 90 (c) Average all

Figure 7. Averages of the diffraction patterns in figure 6.

Figures 8 and 9 show the intensity around a circle of radius 19 μrad for each pattern. The azimuth angle is measured counterclockwise from the +X axis (pointing to the right).

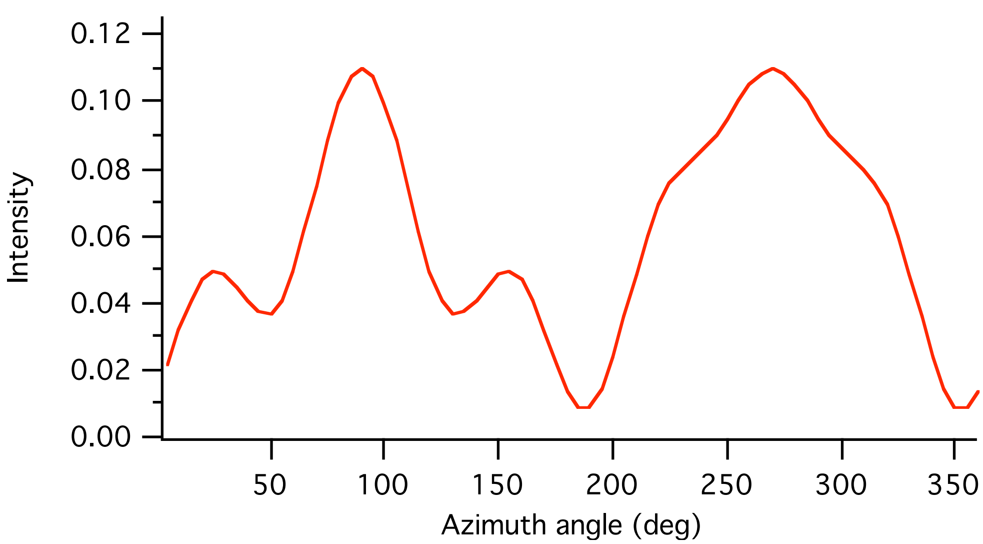


Figure 8(a) Orientation θ = 0 deg
Minimum 0.0086, average 0.0592, maximum 0.1099

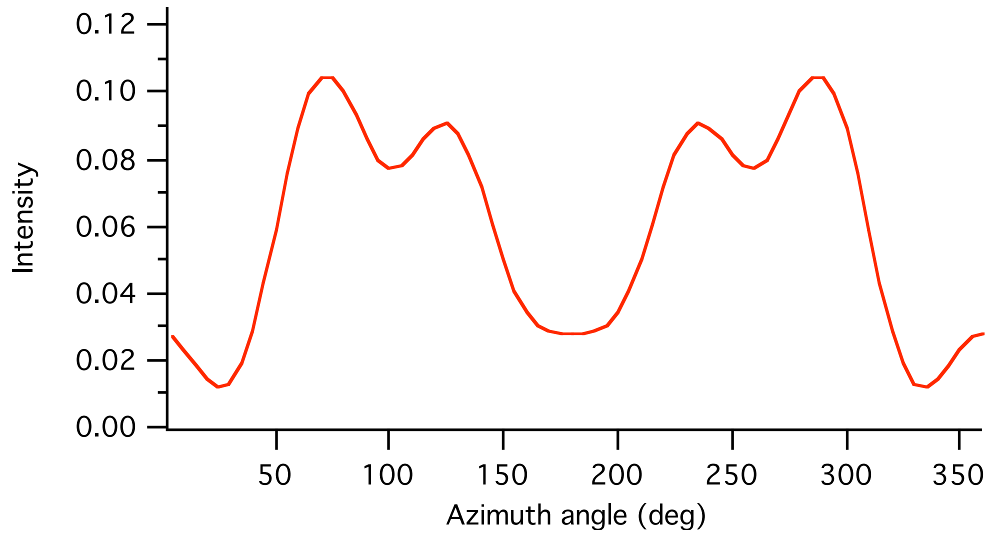


Figure 8(b) Orientation θ = 30 deg
Minimum 0.0116, average 0.0592, maximum 0.1046

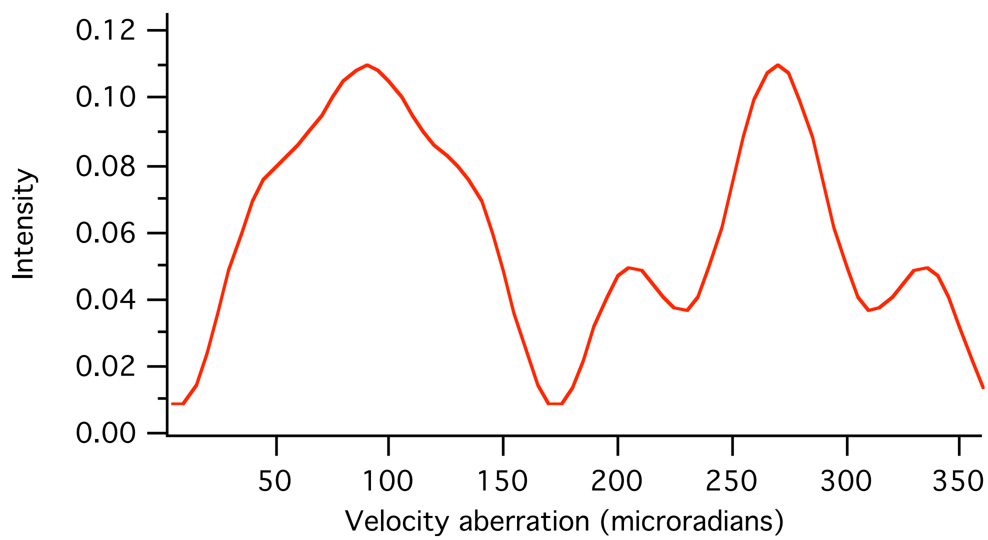


Figure 8(c) Orientation θ = 60 deg
Minimum 0.0086, average 0.0592, maximum 0.1099

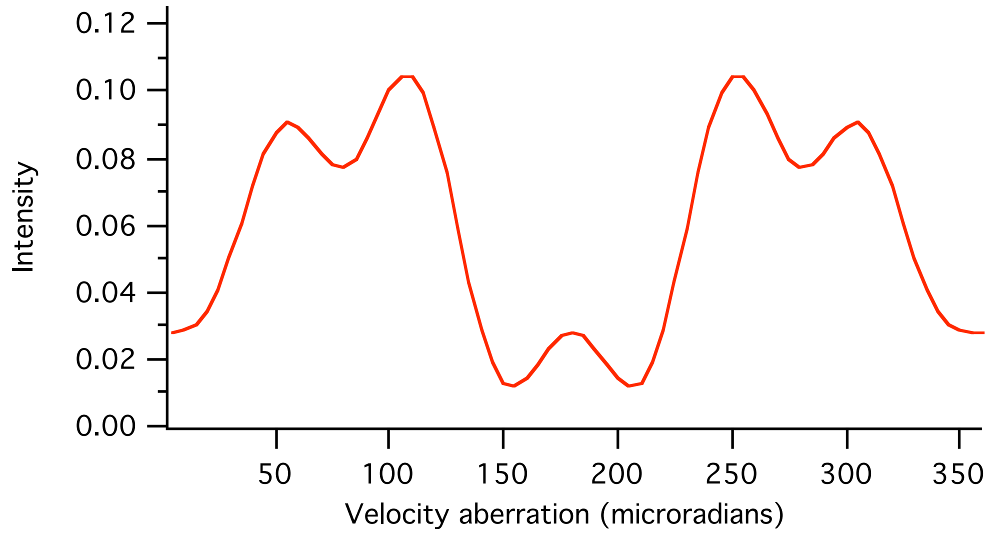


Figure 8(d) Orientation θ = 90 deg
Minimum 0.0116, average 0.0592, maximum 0.1046

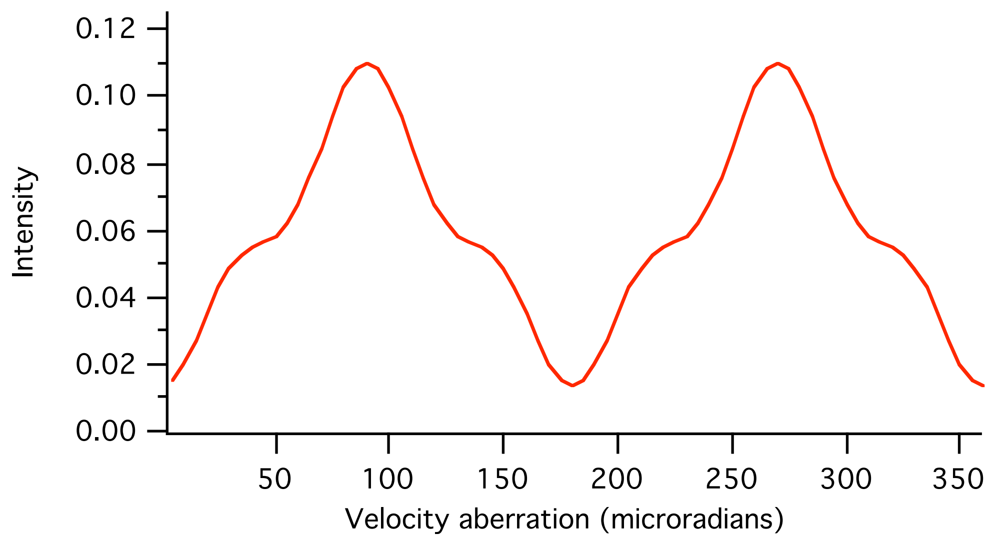


Figure 9(a) Average of 0 and 60 deg
Minimum 0.0133, average 0.0592, maximum 0.1099

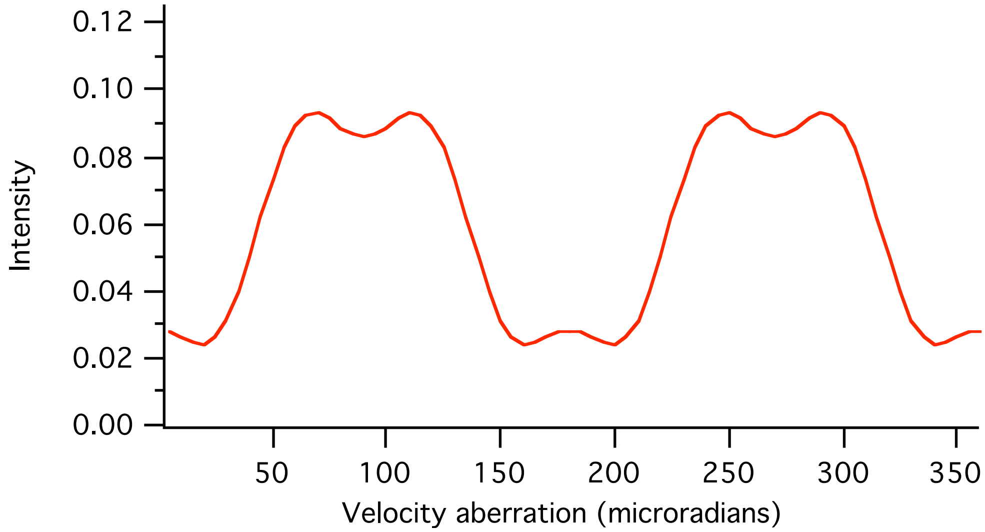


Figure 9(b) Average of 30 and 90 deg
Minimum 0.0240, average 0.0592, maximum 0.0930

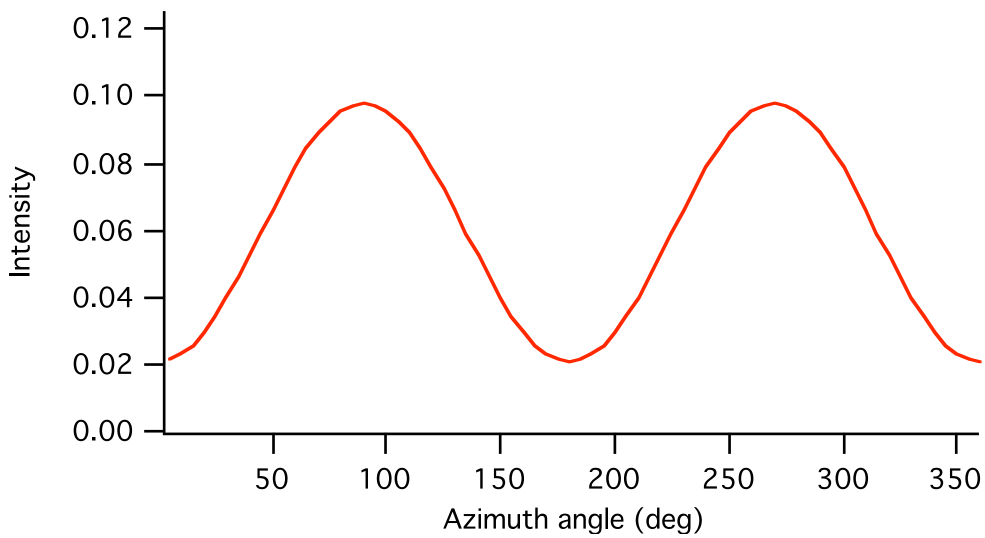


Figure 9(c) Average of all 4 orientations
Minimum 0.0207, average 0.0592, maximum 0.0977

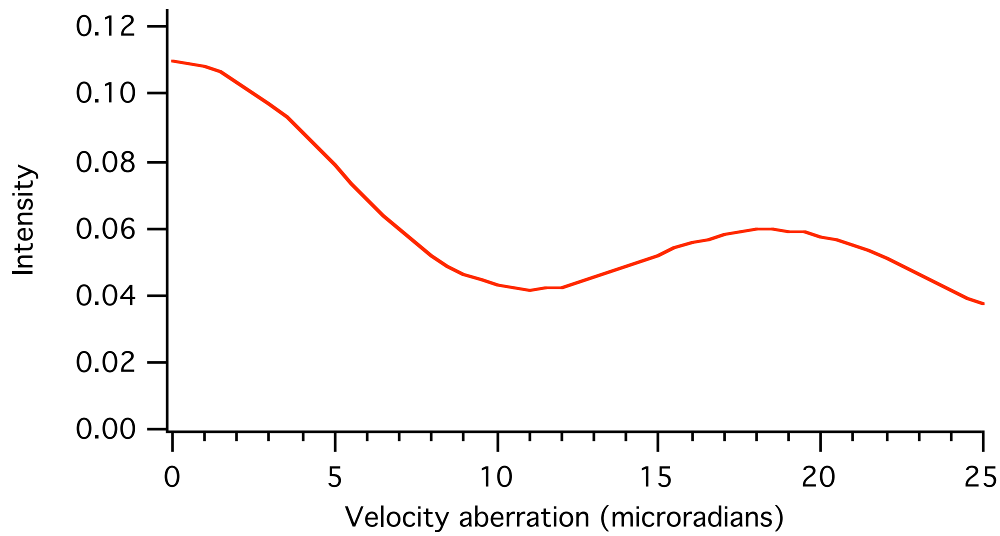


Figure 10. Intensity vs radius for the average of all 4 orientations. Intensity at 19 µrad is 0.0592. This is 88% of the intensity with no dihedral angle offset.

Summary of 0.75 arcsec offset cases

For each orientation of the cube the range of values around a circle of radius 19 µrad are
Minimum 0.0086, average 0.0592, maximum 0.1099 at 0 and 60 deg
Minimum 0.0116, average 0.0592, maximum 0.1046 at 30 and 90 deg

For the average of pairs the range of values is
Minimum 0.0133, average 0.0592, maximum 0.1099 for 0 and 60 deg
Minimum 0.0240, average 0.0592, maximum 0.0930 for 30 and 90 deg

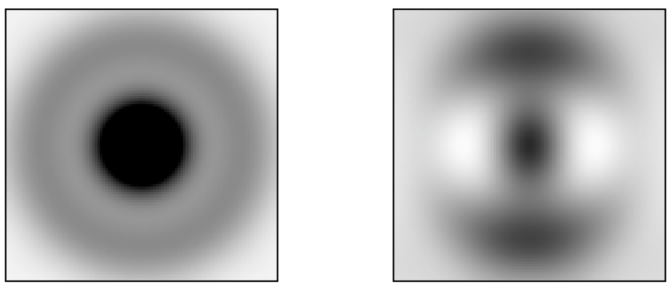
For the average of all 4 orientations the variation of the intensity is
Minimum 0.0207, average 0.0592, maximum 0.0977

Summary

With no dihedral angle offset it is possible to create a pattern with circular symmetry (figure 2c) by averaging the patterns of cubes rotated at 0, 30, 60, and 90 degrees. The intensity around a circle at 19 μrad is constant at 0.0669 relative to the Airy peak.

With a 0.75 dihedral angle offset it is not possible to obtain circular symmetry. The average of patterns at 0, 30, 60, and 90 degrees has bright spots aligned with the polarization vector (figure 7c). The average intensity at 19 μrad is 88% of the intensity with no dihedral angle offset. The variation of the intensity around a circle at 19 μrad is: Minimum 0.0207, average 0.0592, maximum 0.0977

In order to directly compare the intensity between the two cases figures 2(c) and 7(c) have been re-plotted at the same intensity scale from 0 to .125 units. The central peak with intensity .248 is off-scale in figure 11(a).



(a) no offset (b) 0.75 arcsec

Figure 11. Average pattern with no dihedral angle offset (a) and a 0.75 arcsec dihedral angle offset (b). Each figure is the sum of 4 patterns divided by 4. The cubes are rotated by 0, 30, 60, and 90 deg.