Using LLR Data for Testing General Relativity with Focus on the Equivalence Principle

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Determination of „Relativistic“ Parameters with LLR

Strategy: Newtonian effects modelled accurately enough or determined simultaneously

Estimation of selected relativistic parameters

– metric parameters (e.g. space time non-linearity, preferred frames)

– **test of the equivalence principle (Nordtvedt effect, dark matter)**

– **time-variable gravitational constant**

– $1/r^2$-law (Yukawa terms)

– relativistic rotations (e.g. geodetic precession)

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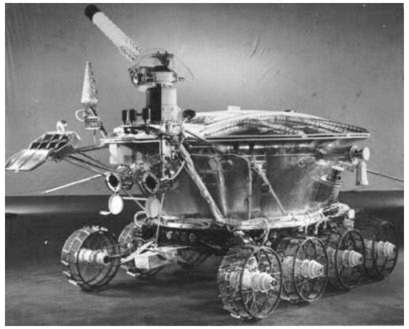
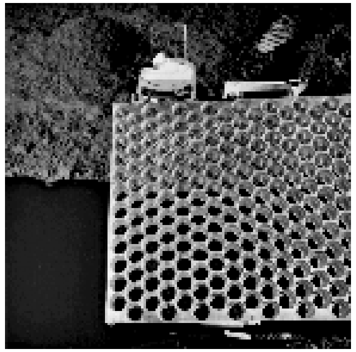
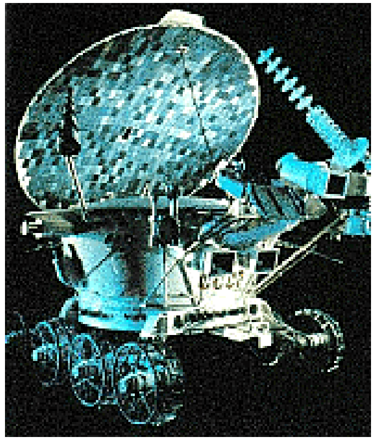
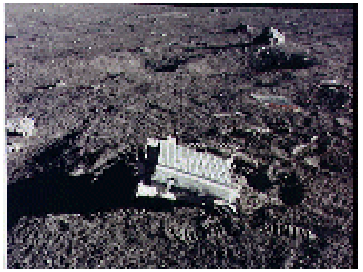
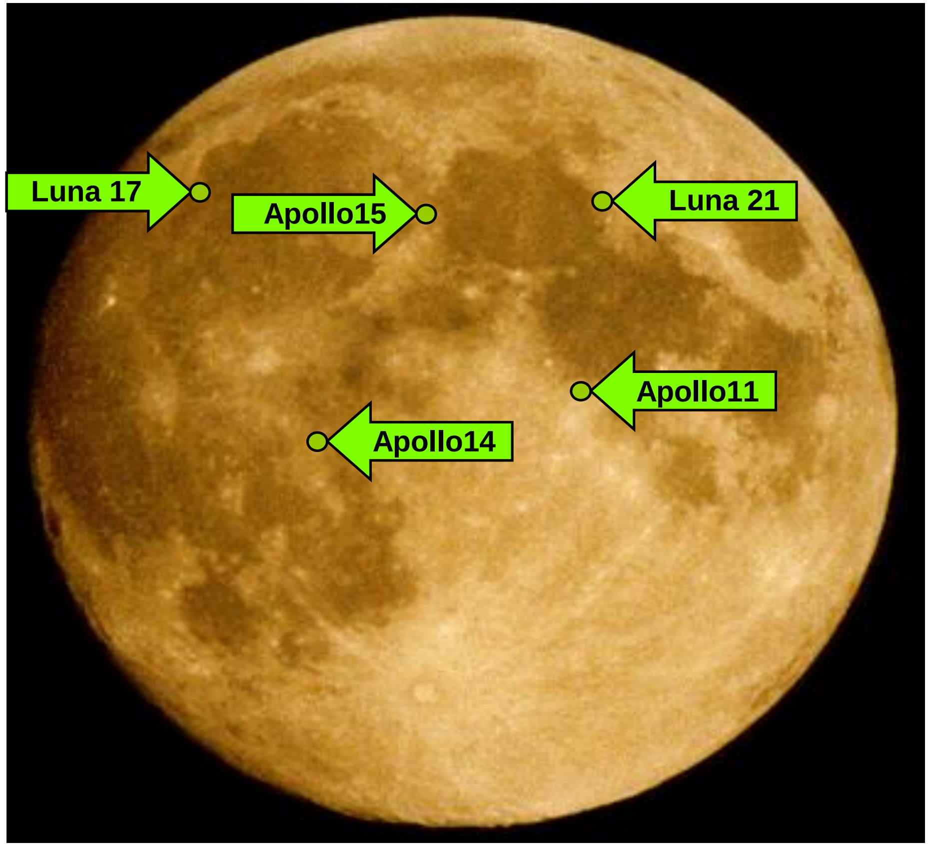
Retro-reflectors on the Moon

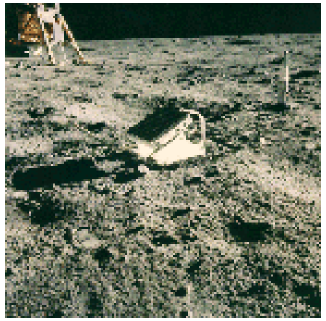

Luna 17

Apollo 15



Luna 21

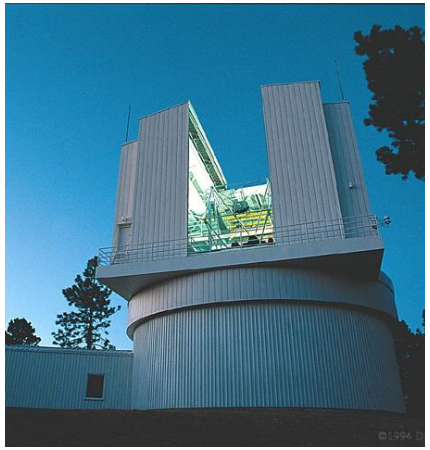

Apollo 14




Apollo 11

deployed 1969-1973 (more to come soon …)

Lunar Laser Ranging Observatories on Earth



APOLLO, 3.5 m

Wettzell

McDonald

Grasse

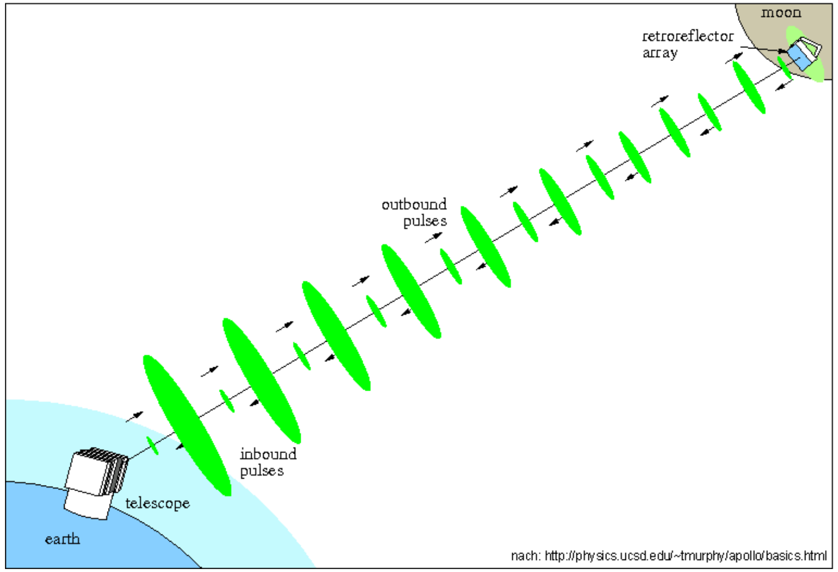
more sites started lunar tracking or may join soon

LLR – Measurement

- telescope, diameter 0.7 m – 3.5 m
- pulsed laser, pulse length 0.07 ns – 0.2 ns
- footprint at the Moon ~20 km² (reflector ¼ - ½ m²)
- 10^{18} - 10^{19} photons/pulse sent, ~1 - 100 photons received
- filtering
  - spatial
  - spectral
  - temporal
- LLR data

  1- 15 min → 1 normal point

  > 31,000 normal points in 53 years



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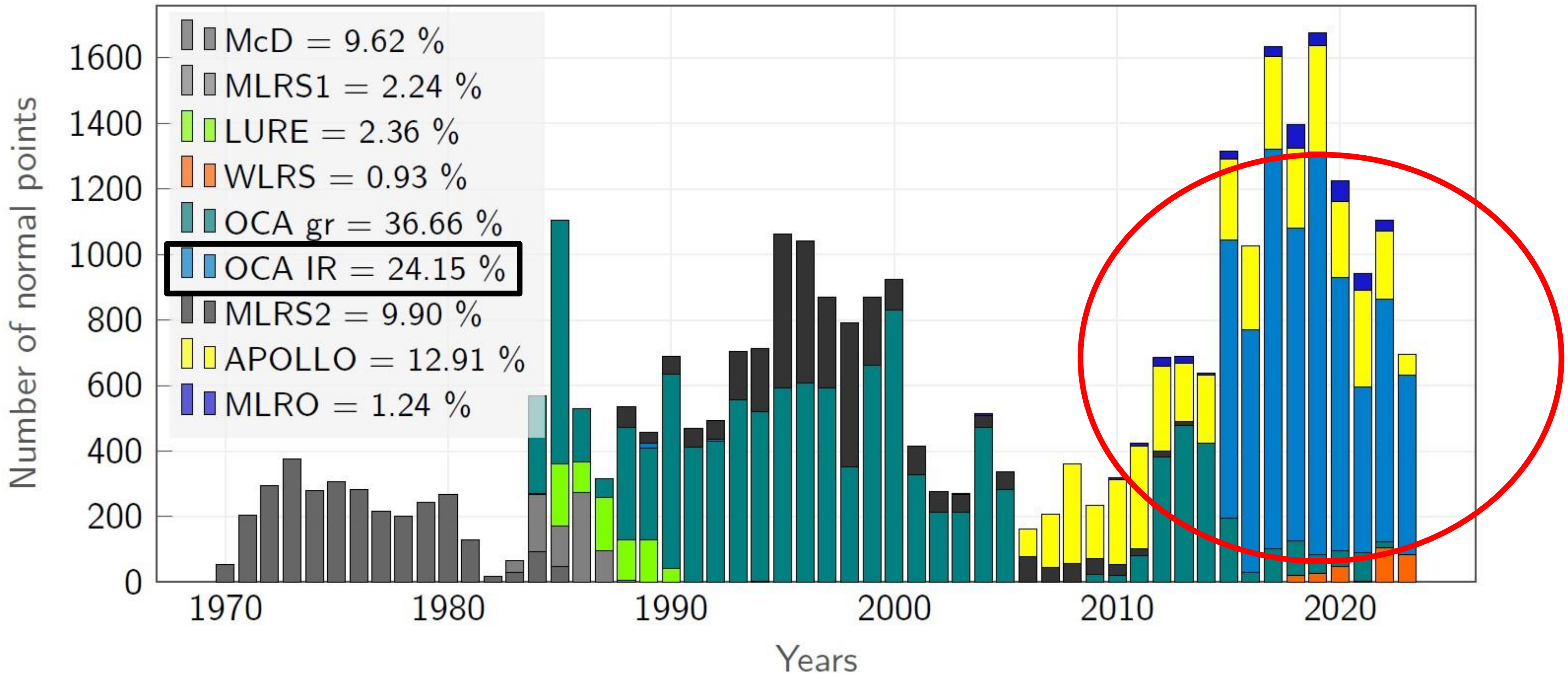
New LLR Normal Points in Infrared

Since 2015 new LLR measurements in **infrared**, especially from Grasse



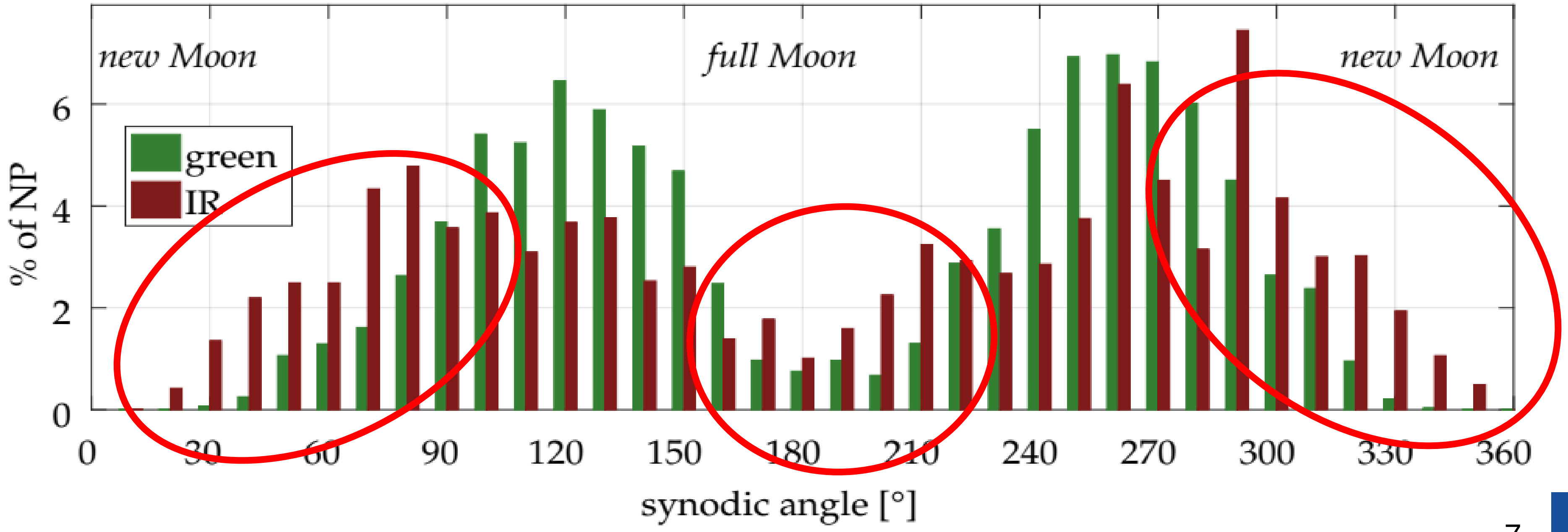
New LLR Normal Points in Infrared

Since 2015 new LLR measurements in **infrared**

- more LLR observations
- higher accuracy
- better orbit and reflector coverage
- potentially more observatories

Very good for testing the equivalence principle (EP)

Distribution of LLR normal points over the synodic month



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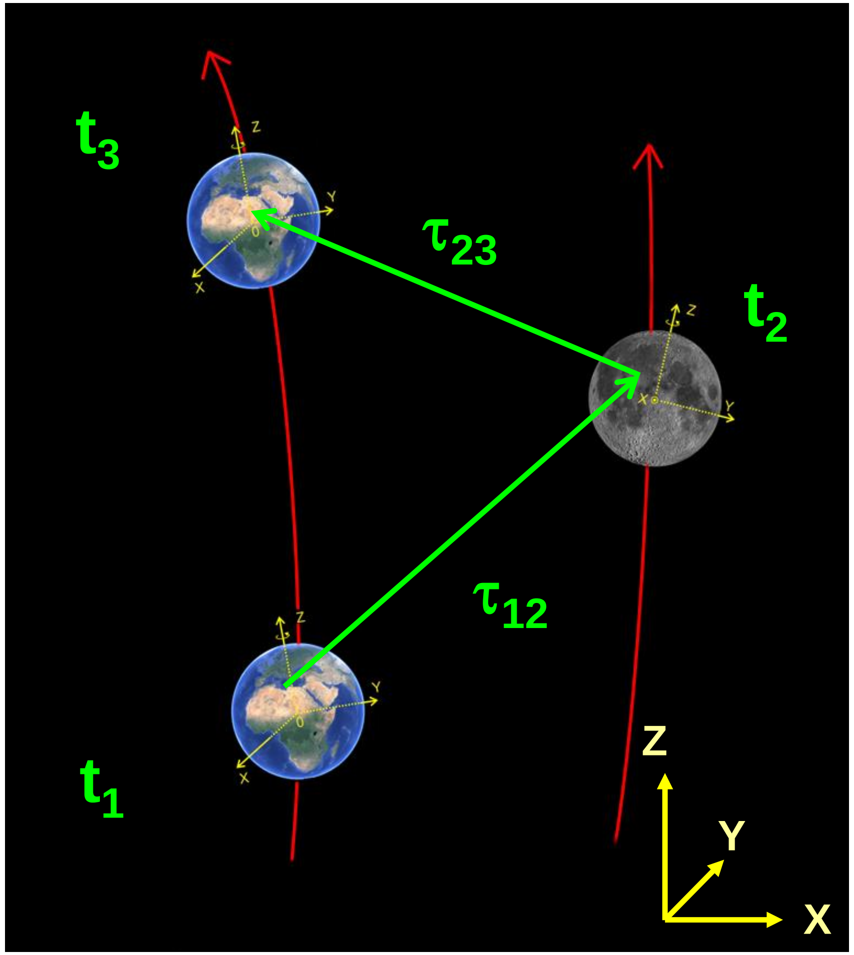
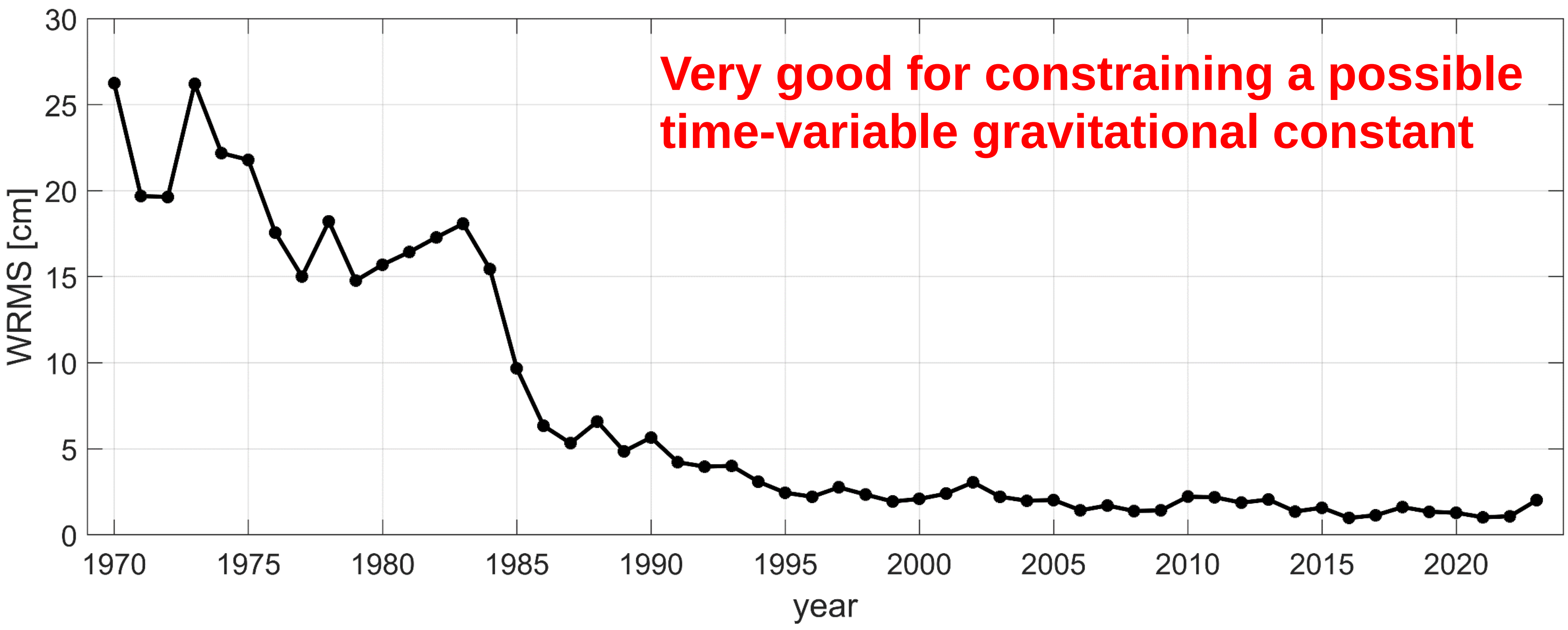
LLR – Analysis Model



- Earth/Moon are moving during measurement
  - Need accurate position/orientation at different times
  - Earth:
    - position from ephemeris
    - orientation from IERS Conv. 2010
  - Moon:
    - position from ephemeris
    - orientation from ephemeris
- Fully (relativistic) model also for
  - Relative change of stations on Earth and reflectors on the Moon
  - Signal propagation
  - …

Weighted Annual LLR Residuals

weighted residuals (observed – computed Earth-Moon distance), annually averaged – for 31,620 NP between April 1970 and July 2023



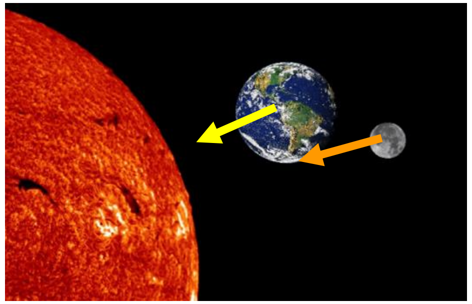
Model Refinement – for Relativistic Tests

Optional extension of the ephemeris model

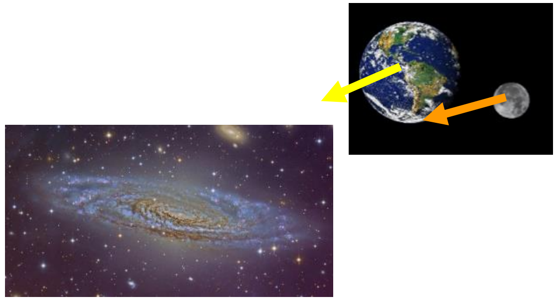
- **time variable gravitational constant** $G = G_0 + \dot{G}\Delta t + \frac{1}{2}\ddot{G}\Delta t^2$
- geodetic precession of the lunar orbit in addition to EIH
- **violation of equivalence principle – 3 versions (see next page)**
- Yukawa term for modifying Newton's $1/r^2$ law of gravity
- preferred-frame effects and metric parameters (Will, 1993)
- gravitomagnetic effects (Soffel et al., 2008)
- optional spin-orbit coupling (Brumberg/Kopeikin)

Three Versions of the Equivalence Principle (EP)

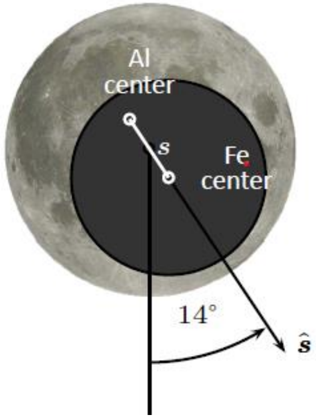
A. Equivalence of **inertial** and **gravitational mass** in the gravitational field of the **Sun**



B. Equivalence of **inertial** and **gravitational mass** with respect to assumed dark matter in the center of the **galaxy**



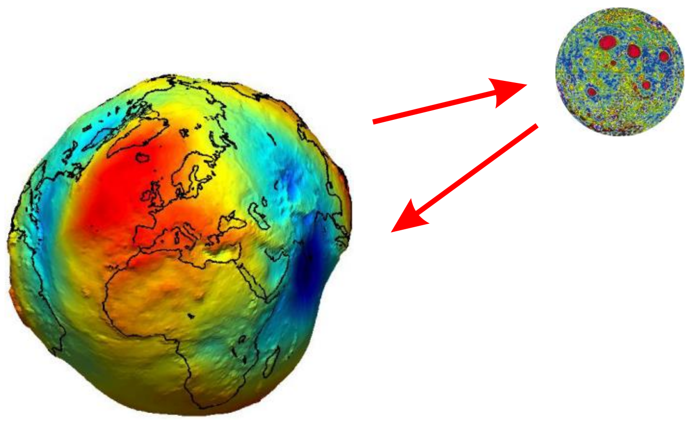
C. Equivalence of **active** and **passive gravitational mass** of the Moon



… using Lunar Laser Ranging

Equation of Motion – EP Tests

$$m_i \ddot{r} \approx -\frac{G m_{pg} m_{ag}}{r_{EM}^2} \ldots$$



EP tests

- Equivalence of inertial mass $\boldsymbol{m_i}$ and gravitational passive mass $\boldsymbol{m_{pg}}$ in the gravitational field of the
  - Sun
  - galactic center
- Equivalence of passive $\boldsymbol{m_{pg}}$ and active gravitational mass $\boldsymbol{m_{ag}}$

A. Test of Strong Equivalence Principle

Earth–Moon system mainly sensitive to **EP test** in gravitational field of the **Sun**

- EP violation would lead to additional acceleration of the Moon w.r.t. Earth depending on

$$\Delta(m_g / m_i)_{EM} = (m_g / m_i)_{Earth} - (m_g / m_i)_{Moon}$$

- resulting in additional range term with synodic period (scaling factor S = -2.9 x 10^{10} m)

$$\Delta r_{\mathrm{EM}} = S\,\Delta(m_g / m_i)_{EM} \cos D$$

- EP test with LLR is a combination of weak and strong EP test, i.e. due to (i) different compositions and (ii) gravitational self energy

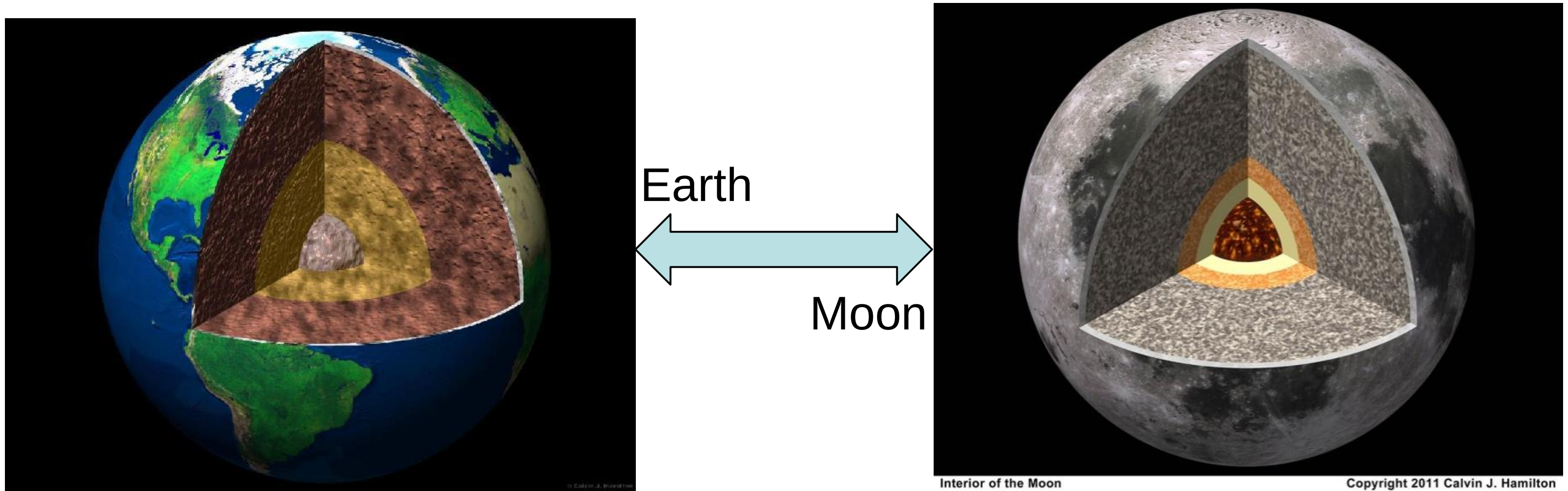
Test of Strong Equivalence Principle (2)

Earth and Moon have non-negligible amount of gravitational self energy

- strong form of EP can be tested
- if strong EP is violated, gravitational self energy itself affects gravitational acceleration
- ratio m_g/m_i can be expressed by

$$\frac{m_g}{m_i} = 1 + \eta \left(\frac{U}{Mc^2} \right)$$

η Nordtvedt parameter
U gravitational self energy
M mass of the body
c speed of light



Test of Strong Equivalence Principle (3)

Introducing Earth/Moon values

$$\left(\frac{U}{Mc^2}\right)_{Earth} = -4.64\times10^{-10}$$

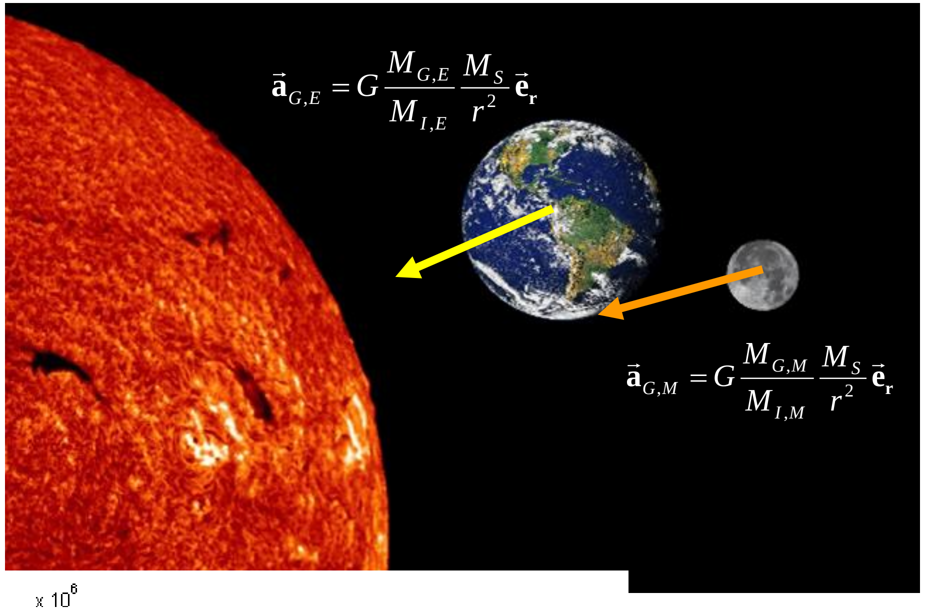
$$\left(\frac{U}{Mc^2}\right)_{Moon} = -1.90\times10^{-11}$$

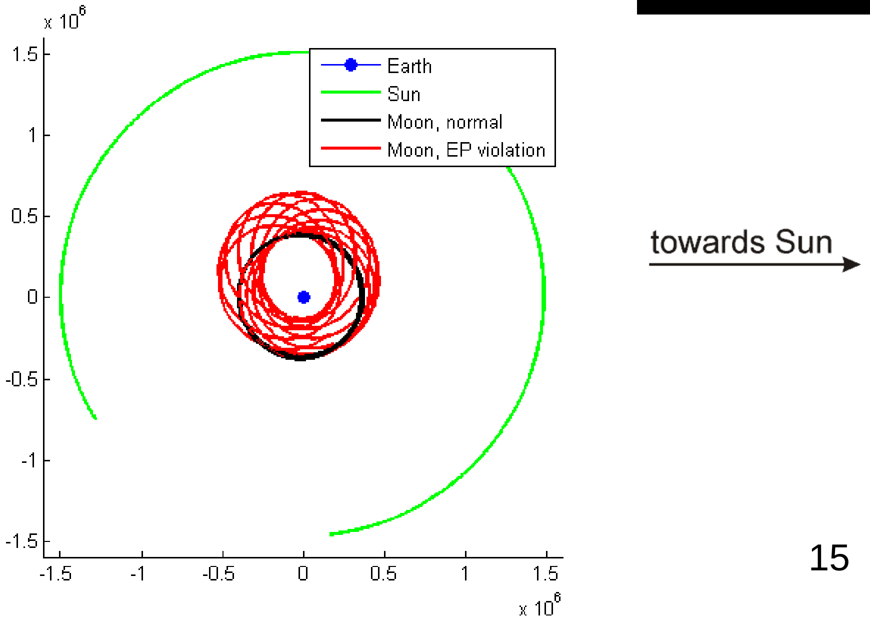
gives $\Delta r_{EM} = 13.1\,\mathrm{m}\,\eta\cos D$

If $\eta \neq 0$

- different accelerations of Earth and Moon
- polarisation of lunar orbit

Result: $\boxed{\eta = \left(-0.2 \pm 1.1\right)\times10^{-4}}$





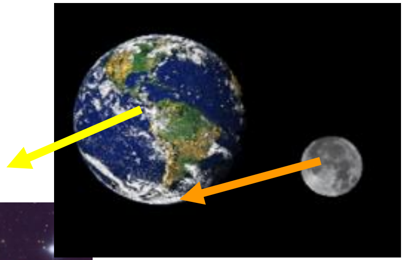
B. „Dark Matter" EP Test – Galactic Center

- part of galactic matter is not observable by electromagnetic radiation
- strong hint of "dark matter" from discrepancy beween observed and calculated orbit velocities of stars close to the center
- test of possible differential acceleration δg between dark matter and different compositions of Earth and Moon (Nordtvedt 1994)

$$\Delta r_{EM} = A\cos(\omega t - \Theta)$$

$$A = -\frac{3}{2}\frac{\delta g}{\omega(\omega - \omega_0)}$$

A *amplitude of in-plane component*
ω siderial frequency
Θ longitude of galactic center



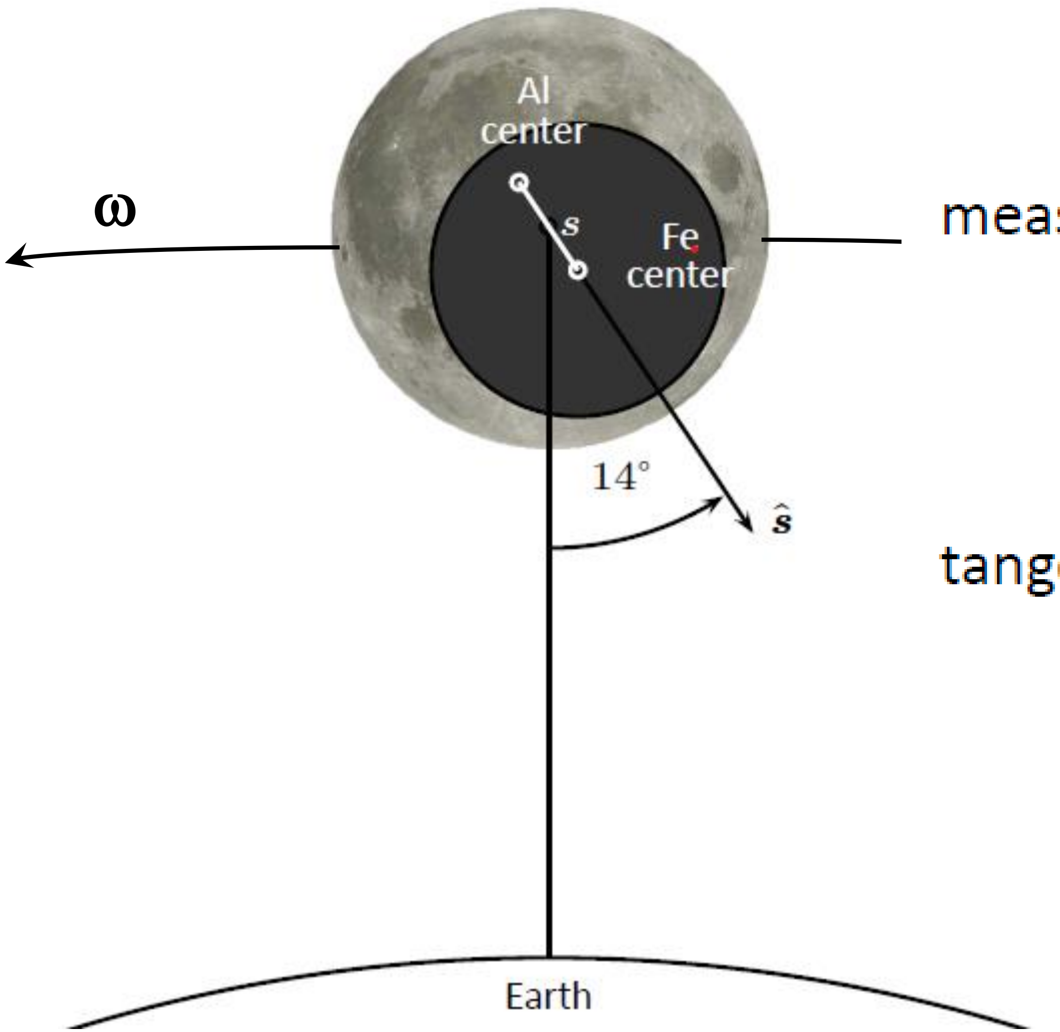
- LLR EP test by estimating *A*,

  δA < 1 mm, corresponding to

$$\boxed{\delta g < 3\times 10^{-15}\ \mathrm{cm/s^2}}$$

Zhang, M., Müller, J., Biskupek, L. (2020): Celestial Mechanics and Dynamical Astronomy, DOI: 10.1007/s10569-020-09964-6

C. Equivalence of Active and Passive Gravitational Mass



$$m_i \ddot{r} \approx -\frac{G\mathbf{m}_{\mathbf{pg}}\mathbf{m}_{\mathbf{ag}}}{r_{EM}^2} \ldots$$

measurement of self-acceleration of center-of-mass

$$\frac{\boldsymbol{F}_{\text{self}}}{F_{\text{EM}}} = C_{\text{Al-Fe}} \frac{M_{\text{M}}}{M_{\oplus}} \frac{r_{\text{EM}}^2}{r_{\text{M}}^2} \frac{s}{r_{\text{M}}} \frac{\rho}{\Delta\rho} \hat{\boldsymbol{s}}$$

Bartlett and van Buren, 1986

tangential component of force increases orbital velocity

$$\frac{\Delta\omega}{\omega} = 6\pi \frac{F_{\text{self}}}{F_{\text{EM}}} \sin 14^\circ \text{ per month}$$

Result: $\mathrm{C}_{\text{Al-Fe}} = 4 \cdot 10^{-14}$

Singh, V., Müller, J., Biskupek, L, Hackmann, E., Lämmerzahl, C. (2023): Phys. Rev. Letters, 131(2), 021401, doi: 10.1103/PhysRevLett.131.021401

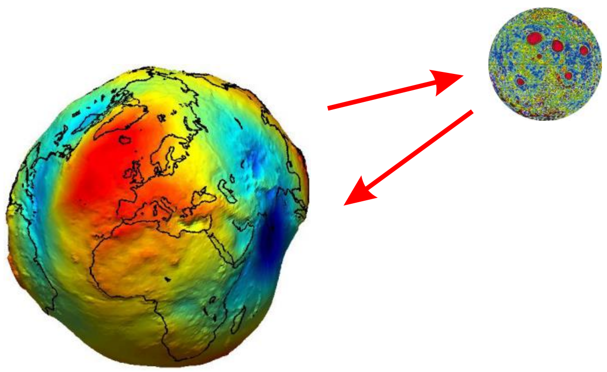
Variation of the Gravitational Constant *G*

Use ansatz

$$G = G_0 + \dot{G}\Delta t + \frac{1}{2}\ddot{G}\Delta t^2$$

in equations of motion

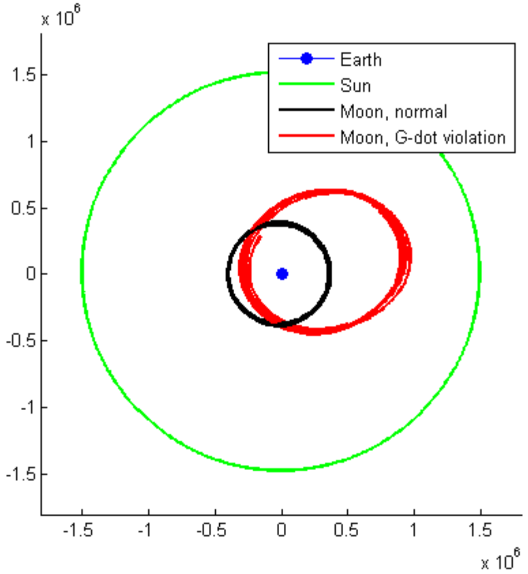
$$\ddot{r}_{EM} \approx -\frac{GM_{E+M}}{r_{EM}^2}\ldots$$



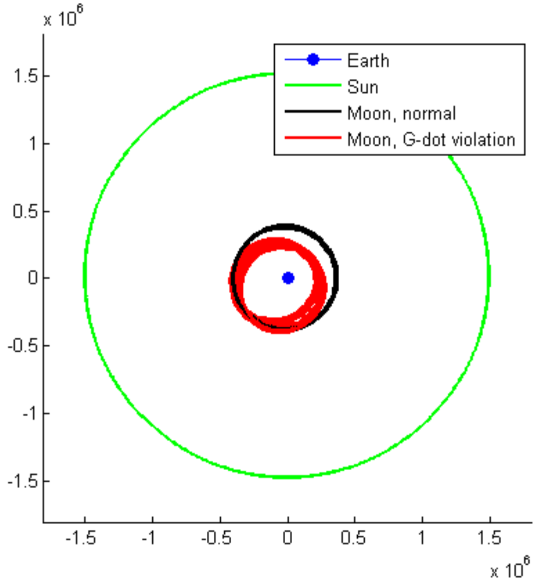
$$\frac{\dot{G}}{G} = (-5.0 \pm 9.6) \times 10^{-15}\,\text{yr}^{-1}$$

$$\frac{\ddot{G}}{G} = (1.6 \pm 2.0) \times 10^{-16}\,\text{yr}^{-2}$$

high correlation with $k_2\delta$



$\dot{G}$ positive



$\dot{G}$ negative

Results – Relativistic Quantitites

Parameter		Results
Nordtvedt parameter η **(test of the strong equivalence principle)**		**$(-0.2 \pm 1.1) \cdot 10^{-4}$**
time variable gravitational constant **(→ unification of the fundamental interactions)**	$\dot{G}/G\,[yr^{-1}]$ $\ddot{G}/G\,[yr^{-2}]$	**$(-5. \pm 9.6) \cdot 10^{-15}$** **$(1.6 \pm 2.0) \cdot 10^{-16}$**
difference of geodetic precession Ω_{GP} - Ω_{deSit} [″/cy] (1.92 ″/cy predicted by Einstein's theory of gravitation)		$(-5.6 \pm 8.5) \cdot 10^{-4}$
metric parameter γ - 1 (space curvature; γ = 1 in Einstein)		$(1.7 \pm 1.6) \cdot 10^{-4}$
metric parameter β - 1 (non-linearity; β = 1) or using $\eta = 4\beta - \gamma_{Cassini} - 3$ with $\gamma_{Cassini}-1$ (~10^{-5})		$(6.2 \pm 7.2) \cdot 10^{-5}$ $(0.5 \pm 2.5) \cdot 10^{-5}$

Results – Relativistic Quantitites (2)

Parameter	Results
Yukawa coupling constant $\alpha_{\lambda=400\,000\ \mathrm{km}}$ (test of Newton's inverse square law for the Earth-Moon distance)	$(-3.7 \pm 4.5)\cdot 10^{-12}$
equivalence of passive and active gravitational mass $C_{Al\text{-}Fe}$	**$4 \cdot 10^{-14}$**
influence of dark matter δ_{gc} [cm/s²] **(in the center of the galaxy; test of strong equivalence principle)**	**$(2 \pm 3)\cdot 10^{-15}$**
preferred frame effects α_1 α_2 (coupled with velocity of the solar system)	$(-1.1 \pm 1.5)\cdot 10^{-5}$ $(-6.0 \pm 9.0)\cdot 10^{-6}$
preferred frame effect α_1 (coupled with dynamics within the solar system)	$(6 \pm 6)\cdot 10^{-4}$

Biskupek, L., Müller, J., Torre, J.-M. (2021): Universe, 7, 34
https://doi.org/10.3390/universe7020034

Conclusions

- LLR is a unique tool for studying the Earth-Moon system and testing general relativity
- Test of the **equivalence principle**, with respect to
  - **the Sun** $\eta = \left(-0.2 \pm 1.1\right) \times 10^{-4}$
  - **the galactic center** $\delta g \leq 3 \times 10^{-15}\ \mathrm{cm/s^2}$
  - **active/passive mass** $C_{Al-Fe} = 4 \cdot 10^{-14}$
- **Gravitational constant** $\dot{G}/G = (-5.0 \pm 9.6) \cdot 10^{-15}\ 1/\mathrm{yr}$
- Only possible thanks to fantastic long-term lunar tracking by observatories (> 53 years of data)

Biskupek, L., Müller, J., Torre, J.-M. (2021): Universe, 7, 34
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