#### Different approaches to realization the reference frame in SLR-LAGEOS solution



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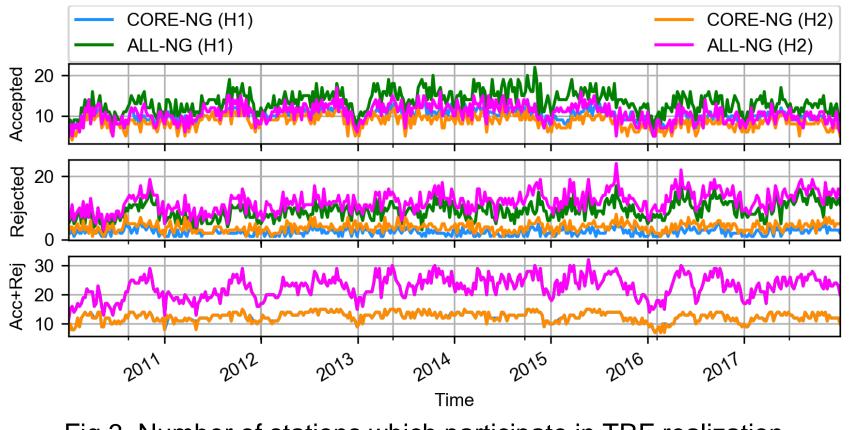
### Motivation

Both, the network configuration and the approach of the geodetic datum definition, affect the quality of the global geodetic products delivered from the SLR solutions. The reference frame in the SLR global solution is realized by applying minimum constraint conditions on the network based on the set of the most stable datum stations. Actually, a No-Net-Rotation condition is imposed because the ERPs station coordinates and orbits are estimated simultaneously. In a global solution the No-Net-Translation condition is related to the geocentre parameters and needs to be handled. This contribution shows differences in station coordinates and geocentre coordinates (GCC) on an example of global geodetic parameters delivered from SLR solution (LAGEOS 1-2; 7-day orbit), which may arise from using different approaches of the datum definition and selection of datum sites.

In this project three main problems were taken into consideration:

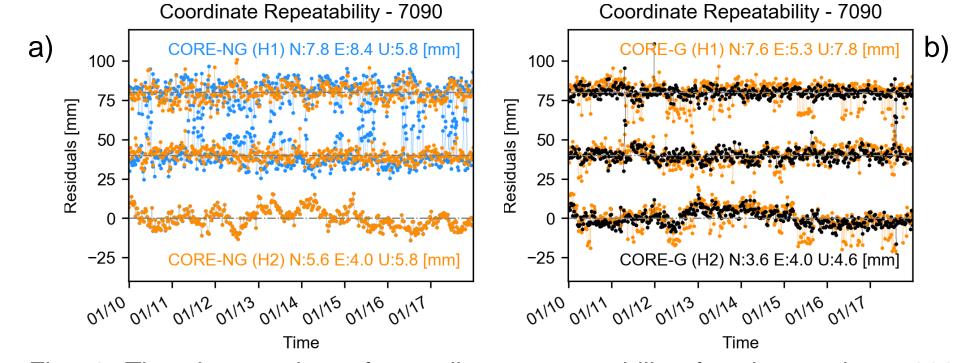
- How do different approaches to datum realization affect the station coordinates?
- Can we use "non-CORE" stations to the datum realization and how

Figure 3 presents the number of stations which participated in TRF realization in each epoch. The sparse and inhomogenous network of SLR stations is one of the major limitation of the SLR technique (see Fig. 2). From 41 stations that deliver SLR observations to LAGEOS 1-2 in the analyzed period, only 14 were considered by ILRS as "core" stations. The median number of stations which participated in the TRF realization increased in H1 approach from 10 to 13 for CORE and ALL, respectively. Applying more rigid conditions (see H2), the median number of accepted stations increased from 9 to 11 for CORE and ALL, respectively. Using a different group of datum-defining stations causes differences in the TRF realization especially in terms of the shift of the center of network in reference to the a priori TRF origin, which determines the quality of the parameters being estimated.



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a)	SOL-NG	N [r	nm]	E [n	nm]	U [r	nm]	SOL-G	N [r	nm]	E [n	nm]	U [r	nm]	b
uj	NNR	CORE	REST	CORE	REST	CORE	REST	NNR+NNT	CORE	REST	CORE	REST	CORE	REST	
	CORE (H1)	13.6	18.8	14.5	23.0	10.5	17.3	CORE (H1)	10.6	19.0	12.6	20.3	12.1	16.9	
	CORE (H2)	10.7	18.6	8.9	18.2	10.4	17.3	CORE (H2)	10.0	18.2	8.6	17.8	9.1	16.4	
	PERC [%]	21.6	0.9	38.5	20.9	0.1	0.0	PERC [%]	5.0	4.0	31.6	12.0	24.8	2.9	
	ALL (H1)	15.8	20.1	14.0	23.8	10.4	17.3	ALL-G (H1)	13.3	19.3	15.6	21.7	14.2	19.0	
	ALL (H2)	10.8	18.9	8.8	17.3	10.4	17.3	ALL-G (H2)	10.0	18.6	8.5	17.5	9.5	16.1	
	PERC [%]	31.8	5.7	37.1	27.0	0.0	0.0	PERC [%]	25.0	3.5	45.3	19.7	33.2	15.2	

Table 6. Median RMS of the station repeatability and its percentage change when using threshold H2 in reference to H1



- does it affects the quality of station coordinates and GCC?
- What threshold should be taken when chosing datum-defining stations in the datum realization?

## Solutions

Basically, a standard 7-day LAGEOS 1-2 SLR solutions have been prepared from the observations delivered by the ILRS network throughout 8-year interval between 2010-2018. The solution follows the standards of the ILRS solutions used for operational products. The following parameters have been estimated: SLR station coordinates, LAGEOS orbit parameters, X and Y pole coordinates, UT1 - UTC, geocenter coordinates and range biases for selected stations (according to the recommendations of the ILRS Analysis Standing Committee). All parameters are estimated with the 7-day interval, except for the pole coordinates and UT1-UTC parameters which are calculated with a 1-day interval. To carry out this experiment we use the modified Bernese GNSS Software ver. 5.2. The analyzed solutions differ mainly in three aspects (see Table 1): (1) the approach for the terrestrial reference frame (TRF) realization, (2) group of stations considered to be used in TRF realization, (3) threshold for the Helmert condition to accept the stations in the TRF realization.

		GCC	NNR	NNT				
NG "No Geod	center"	NO	YES	NO				
G "Geocente	r"	YES	YES	YES				
SLR								
CORE	Only CORE stations included in frame realization							
ALL	All stations included in frame realization							
Helmert conditions								
H1	North: 15 mm, East: 15 mm, Up 15 mm							
H2	North: 10 mm, East: 10 mm, Up 10 mm							

Table 1. Description of the differences between the analyzed solutions

The TRF is realized using minimum constraint conditions imposed on the network with respect to the a priori reference frame. The minimum constraint conditions are based on the 7-parameters Helmert transformation. Using equation (see Figure 1), we can tie any two reference frames, i.e.: (1) a priori reference frame, and (2) a reference frame of the resulted coordinates. Based on minimum constraint conditions we can constrain three translations (NNT), three rotations (NNR) and one scale parameter (NNS) between the two considered frames (see Fig. 1).

Fig 3. Number of stations which participate in TRF realization

## **NNR or NNR+NNT**

First, we checked the differences between solutions which use only the NNR condition in reference to the solutions using both NNR and NNT conditions. When only NNR condition is imposed on the network the realized TRF is shifted by the 7-day mean geocenter motion as seen by SLR. The additional NNT constraints imposed on the network makes the estimated station coordinates consistent in origin with the a priori reference frame by definition. We expect the improvement of the station coordinate repeatability when using NNT because no geocenter motion is included in the signal (see Fig. 5b). However, more important is to choose a proper group of datum-defining stations. When using the H1 threshold, the NNT condition has a negative impact on estimating the Up component by up to 36 % (see Table 5a). Using more rigid threshold (H2) improves the median station repeatability in both CORE and ALL solutions by single millimeters (see Table 5b).

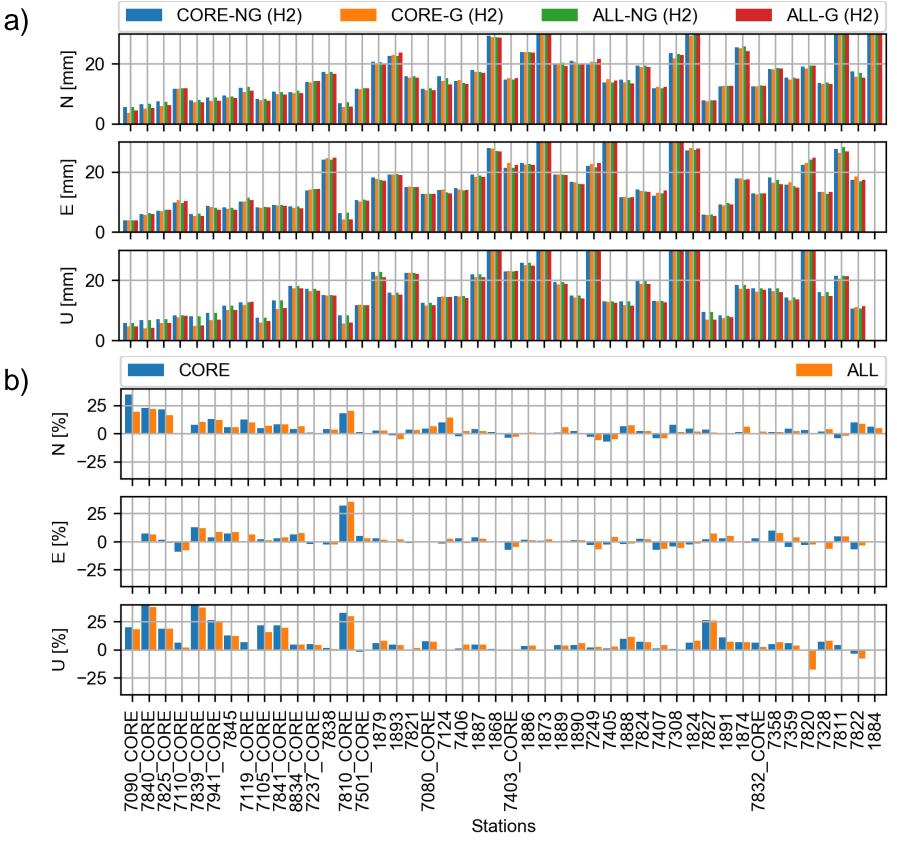


Fig. 6 The time series of coordinate repeatability for the station 7090 decomposed into the North (top), East (middle), Up (bottom) components. The components are shifted on the y-axis by 40 mm.

# Using all available stations

### instead of only core list

We have checked what is the impact of using a different group of datum-defining stations. We tried to consider all possible stations instead of only stations included in the ILRS list of core stations. The effect on the station coordinate repeatability is dependent on the threshold which we use to detect the outlier stations (H1 or H2) and approach to the TRF realization in the solution (see Table 7). The station coordinate repeatability deteriorates for both SOL-NG (H1) and SOL-G (H1) when using ALL stations as compared to CORE only (see Fig. 8a). That means that the 15 mm threshold is insufficient to reduce low-quality stations in datum realization. When using H2 threshold the station coordinate repeatability remains at the comparable level (see Fig. 8b), while the reference frame realization is more robust because of more stations (see Fig. 3) and better geometry.

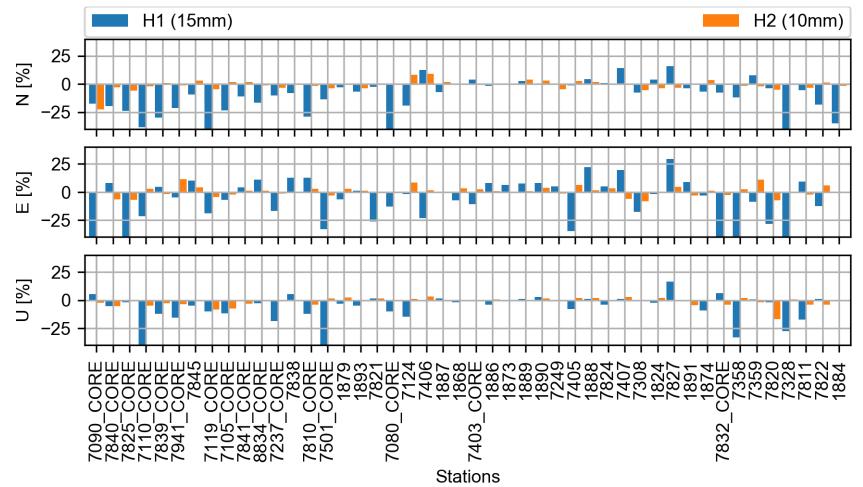
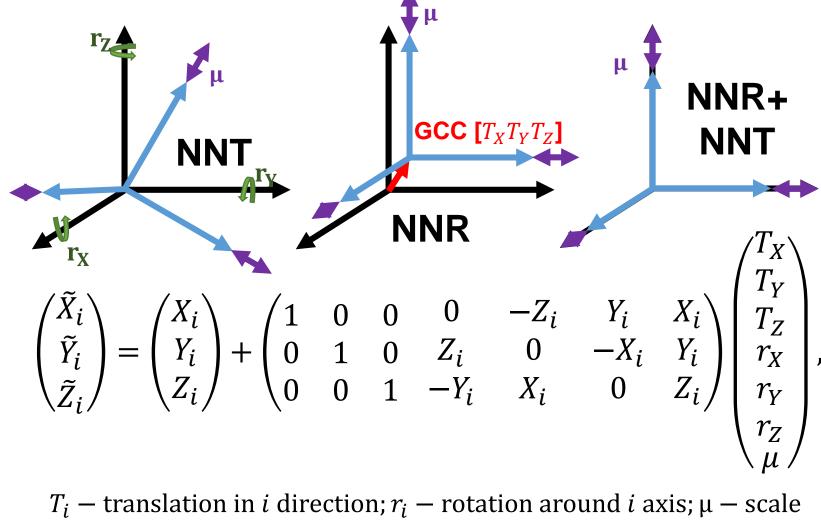


Fig 7. The percentage change of station repeatability when using ALL stations



$$T_i$$
 – translation in *i* direction;  $r_i$  – rotation around *i* axis;  $\mu$  –  
Reference frame  $X_i$  Realized frame  $\tilde{X}_i$ 

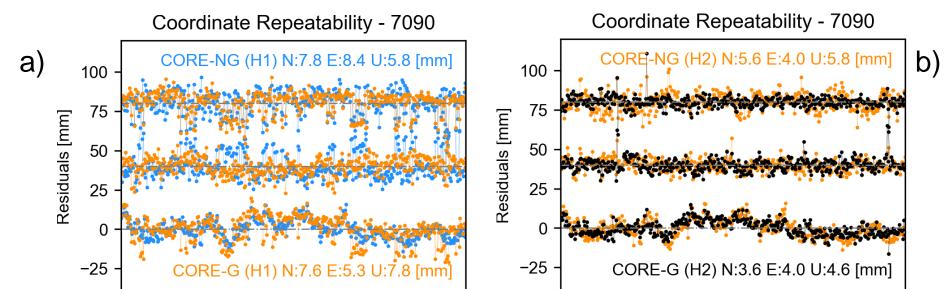
Fig 1. Explanation of using minimum constraint conditions

The TRF should be realized using only selected group of the most stable stations. The estimation process is made in two iterations. After the first iteration the station coordinates are checked using the Helmert transformation in reference to the a priori SLRF2014 coordinates. Only stations whose coordinate residuals do not exceed the assumed threshold are considered as datum-defining in the final parameter estimation. We analyzed two groups of "station-candidates" (see Table 1 and Fig. 2) as well as two thresholds for station-outlier detection (see Table 1).

Fig 4. The difference between solutions NG (NNR) and G (NNR+NNT) when using H2 threshold. RMS of coordinate repeatability and its percentage change for particular SLR stations, decomposed into the North, East and Up component and the percentage change

a)	H1	N [n	nm]	E [n	nm]	U [r	nm]	H2	N [n	nm]	E [n	nm]	U [n	nm]	b)
	(15 mm)	CORE	REST	CORE	REST	CORE	REST	(10 mm)	CORE	REST	CORE	REST	CORE	REST	,
	CORE-NG	13.6	18.8	14.5	23.0	10.5	17.3	CORE-NG	10.7	18.6	8.9	18.2	10.4	17.3	
	CORE-G	10.6	19.0	12.6	20.3	12.1	16.9	CORE-G	10.0	18.2	8.6	17.8	9.1	16.4	
	PERC [%]	22.5	-0.9	13.1	11.9	-16.0	2.1	PERC [%]	6.0	2.3	3.4	1.9	12.7	5.0	
	ALL-NG	15.8	20.1	14.0	23.8	10.4	17.3	ALL-NG	10.8	18.9	8.8	17.3	10.4	17.3	
	ALL-G	13.3	19.3	15.6	21.7	14.2	19.0	ALL-G	10.0	18.6	8.5	17.5	9.5	16.1	
	PERC [%]	15.8	4.0	-11.1	8.5	-35.8	-10.0	PERC [%]	7.4	1.7	3.5	-0.7	9.3	6.7	

Table 5. Median RMS of the station repeatability and its percentage change for SOL-G in reference to SOL-NG



compared to CORE stations in datum realization; Example of SOL-G (NNR+NNT)

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<b>3</b> )	SOL-NG		nm]		nm]	0 [r	nm]	SOL-G	N [n	nmj	E [n	nul	U [r	nmj	b)
~)	NNR	CORE	REST	CORE	REST	CORE	REST	NNR+NNT	CORE	REST	CORE	REST	CORE	REST	,
	CORE (H1)	13.6	18.8	14.5	23.0	10.5	17.3	CORE (H1)	10.6	19.0	12.6	20.3	12.1	16.9	
	ALL (H1)	15.8	20.1	14.0	23.8	10.4	17.3	ALL (H1)	13.3	19.3	15.6	21.7	14.2	19.0	
	PERC [%]	-16.2	-6.9	3.2	-3.4	0.0	0.0	PERC [%]	-26.4	-1.7	-23.8	-7.3	-17.0	-12.5	
	CORE (H2)	10.7	18.6	8.9	18.2	10.4	17.3	CORE (H2)	10.0	18.2	8.6	17.8	9.1	16.4	
	ALL (H2)	10.8	18.9	8.8	17.3	10.4	17.3	ALL (H2)	10.0	18.6	8.5	17.5	9.5	16.1	
	PERC [%]	-1.2	-1.6	1.0	4.6	0.0	0.0	PERC [%]	0.3	-2.2	1.1	2.1	-3.9	1.8	

Table 7. Median RMS of the station repeatability and its percentage change for SOL-ALL in reference to SOL-CORE

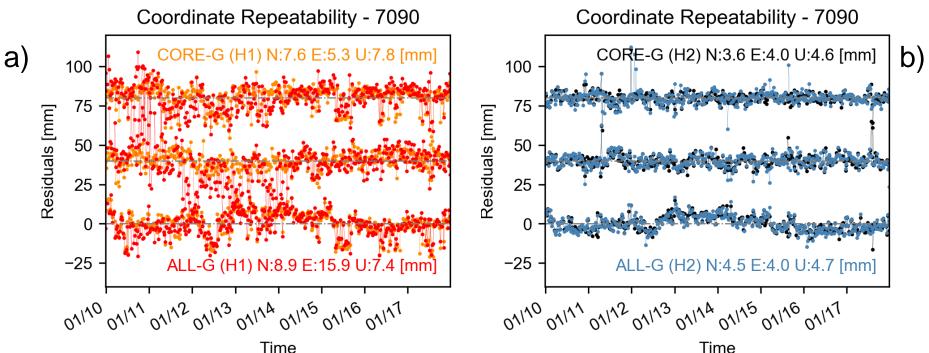


Fig. 8 The time series of coordinate repeatability for the station 7090 decomposed into the North (top), East (middle), Up (bottom) components. The components are shifted on the y-axis by 40 mm.

### **Impact on GCC**

The time series of geocentre coordinates delivered in each solution correspond to each other (see Figure 9a). On the other hand, the differences are visible in the time series of formal errors (see Fig. 9b). There are different epochs with increased values of formal errors of geocentre estimation. That may be caused by the inappropriate geometry of datum-defining stations. For solution ALL\_H2 the number of epochs when formal errors drastically increase is lower than for the other solutions. In that solution, we introduced more stations to maintain a global distribution of stations as well as reduced the number of low-quality stations using more rigid threshold.

ORE H1	- CORE H2	CORE H1	— (

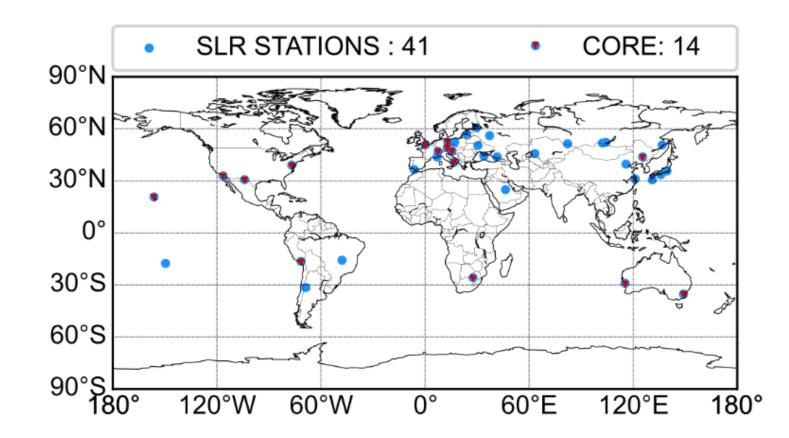


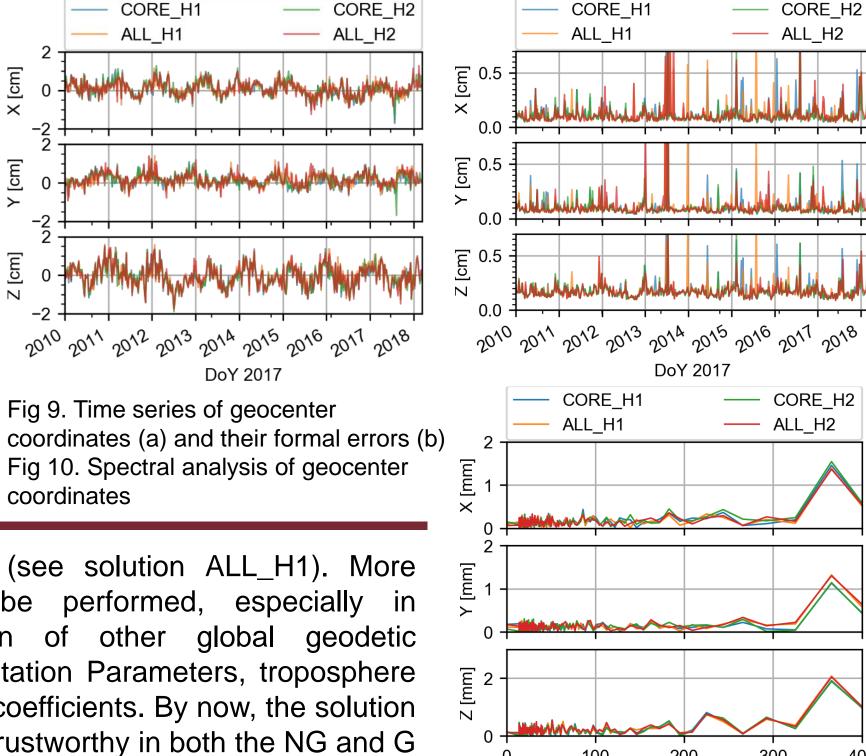
Fig 2. Distribution of SLR stations with the indication of core stations

01/10 01/11 01/12 01/13 01/14 01/15 01/16 01/ 01/10 01/11 01/12 01/13 01/14 01/15

Fig. 5 The time series of coordinate repeatability for the station 7090 decomposed into the North (top), East (middle), Up (bottom) components. The components are shifted on the y-axis by 40 mm.

### **Selection of the correct threshold**

For each solution, we checked what is the impact on station repeatability when changing the threshold in a selection of datum defining stations. We see the clear improvement on all stations when H2 is imposed compared to H1 (see Table 6), especially for the East component up to 45 % (7 mm). When only NNR constraint is imposed on the network, changing the number of datum-defining stations have no impact on the Up component (see Table 6a and Fig. 6a).



Period [days]

# Conclusions

In response to the three main questions raised in the motivation part of this work, a series of analyses have been made. Using NNR+NNT constraints instead of NNR only has a minor effect on station repeatability and is directly dependent on the selection of the group of datum-defining stations.

We can increase the number of the datum-defining stations in parameters being estimated (see solution ALL\_H1). More the TRF realization by 2 or 3 stations using all available SLR complex analyses should be performed, especially in stations instead of only core stations. However, low-quality reference to the estimation of other global geodetic stations have to be eliminated from the TRF realization, for parameters such as Earth Rotation Parameters, troposphere example using more rigid threshold while testing the stations in and time-variable gravity field coefficients. By now, the solution Helmert transformation. Otherwise, using unreliable stations CORE\_H2 seems to be most trustworthy in both the NG and G could worsen the TRF realization and the quality of other approaches depending on the user needs.

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