REDUCING THE SATELLITE CONTRIBUTION TO RANGE ERROR

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MM ACCURACY GEODETIC SATELLITES

- **Technical Challenges**
  - Geodetic satellites (e.g., LAGEOS, Starlette, etc.) are typically in low to medium altitude orbits, have large velocity aberrations, rotate freely in space, and are therefore spherical in shape to permit simultaneous and unbiased ranging from multiple SLR stations.
  - The satellite must present a high enough cross-section, consistent with its altitude, to support ranging by the entire ILRS network. Since velocity aberration limits the size of the individual retroreflectors, cross-section must be achieved by increasing the number of retroreflectors contributing to the station return. Furthermore, the strength of an individual retro return diminishes as one gets farther from normal incidence
  - To simultaneously achieve maximum range accuracy, the satellite impulse response must be made as short as possible, but the large number of retros, combined with the spherical shape of the surface, causes a variable range between the retros and the station, thereby spreading the pulse.

- **Possible Solutions**
  - Building supporting spheres with larger radii.
  - Improving the packing density to increase the number of retros per unit surface area and hence the effective array cross-section.
  - For spheres, limit target pulse spreading by restricting returns over a smaller range of incidence angles through the use of hollow or recessed solid retros .

Starlette and LAGEOS

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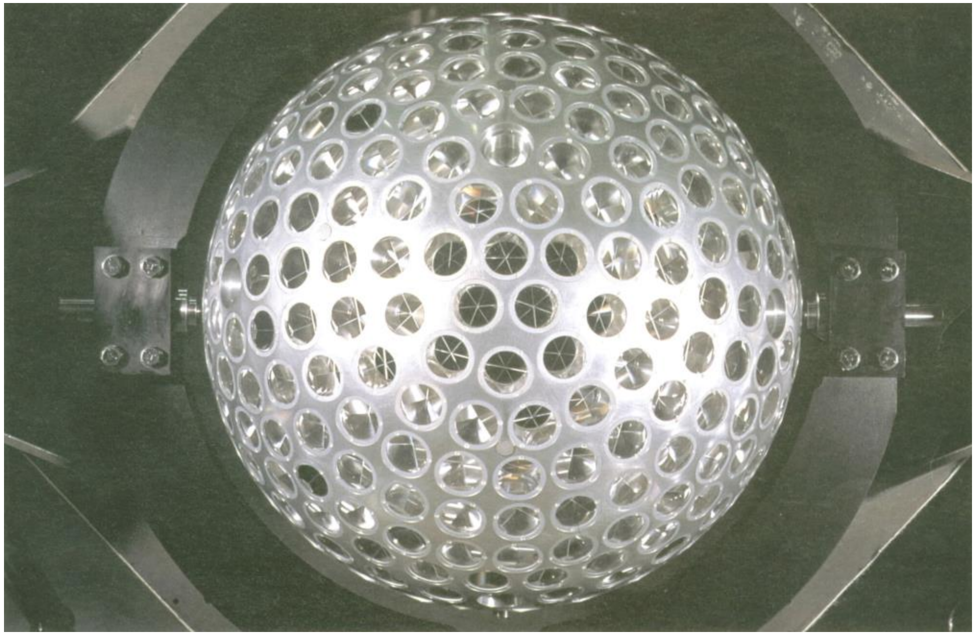


Starlette
CNES, France
Launch: 1975
Diameter: 24 cm
Number of Retros: 60

LAGEOS
NASA, USA
Launch: 1976
Diameter: 60 cm
Number of Retros: 426 (4 Ge for NIR)

* LAGEOS-2 (Italy) was launched from the NASA Space Shuttle in 1992.

MM ACCURACY GEODETIC SATELLITES

J. Degnan, Contributions of Space Geodesy to Geodynamics: Technology, Geodynamics 25, pp. 133- 162 (1993)

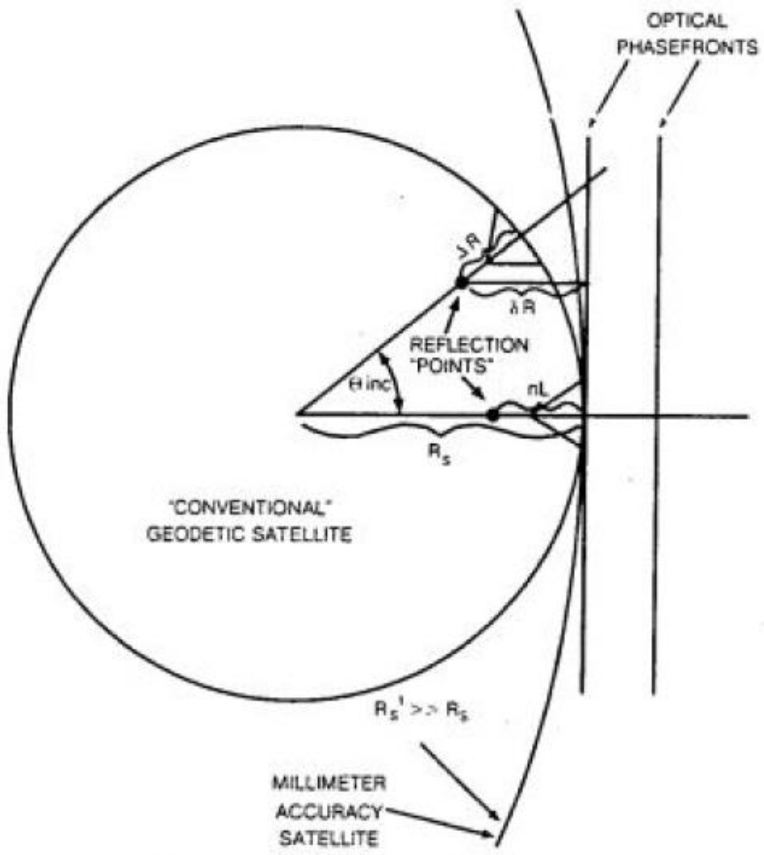


Fig. 25. Definitions of quantities used in the discussion of satellite impulse response. Satellites capable of supporting millimeter accuracy two color measurements are characterized by larger radii, high retroreflector densities, and limited angular field of view. The large satellite radius provides a better "match" to the incoming planar phasefront.

Basic Design Guidelines

- Increase radius of sphere to better approximate a flat surface
- Recess the individual cubes in their holders to restrict the response at large incidence angles and minimize the target signature
- Increase packing density of the cube corners on the satellite surface (ratio of cube aperture within a given surface area). (LAGEOS has a packing density of 0.435.)
- Hexagonal arrays have the greatest packing density but restrict "clocking" to 60 degree increments. Choose retro diameter and clocking scheme to optimally fill the annular FOV, Ω, between α_{max} and α_{min}

Impulse Response in the Large Satellite Limit

J. Degnan, Contributions of Space Geodesy to Geodynamics: Technology, Geodynamics 25, pp. 133- 162 (1993)

In the following graph, τ is a time normalized to the time it takes a light pulse to travel the diameter of the satellite, i.e. $2R_s/c$. Increasing the radius of the satellite to make it appear flatter will increase the impulse response. However, narrowing the incidence angle range by using hollow cubes or recessing the solid cubes reduces the impulse response of the satellite and improves range accuracy.

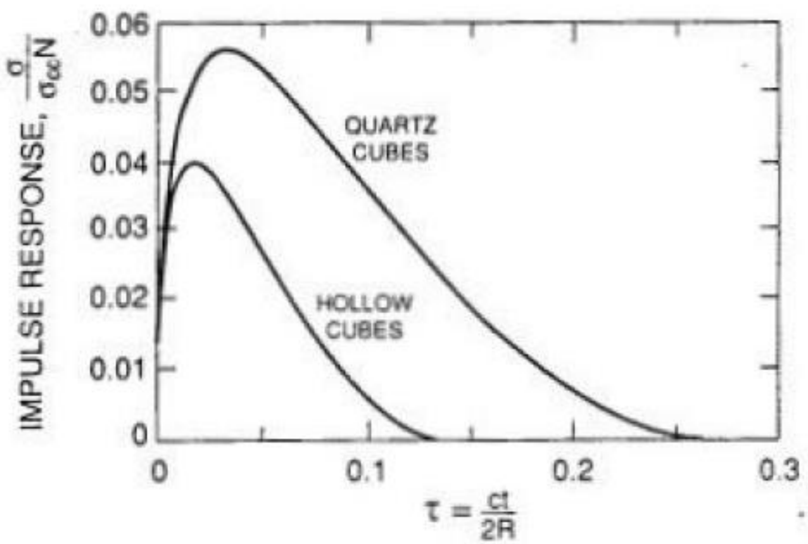
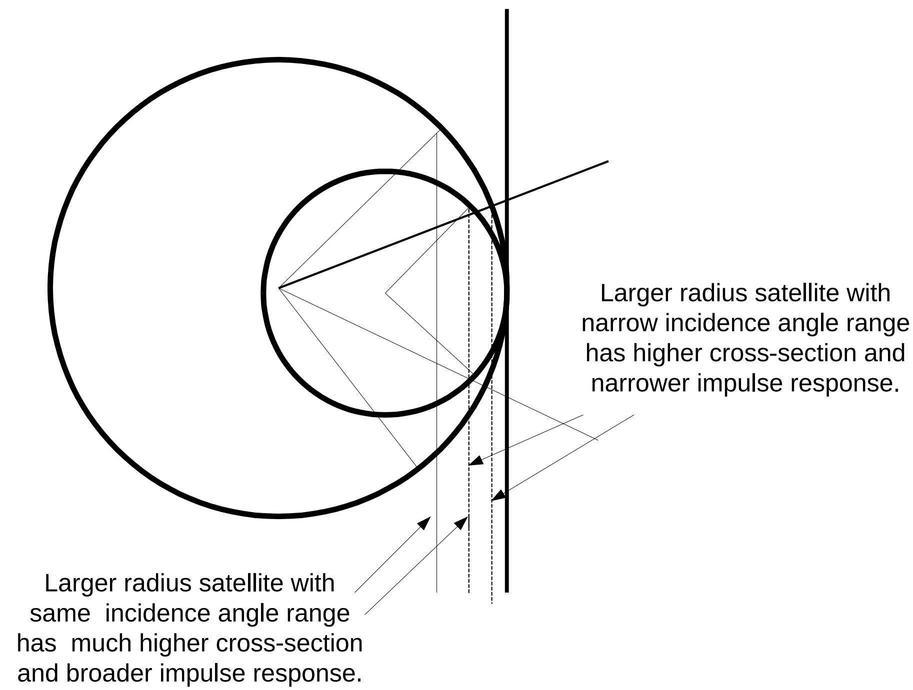


Fig. 27. Impulse response in the large satellite limit ($R_s \gg nL$) for both hollow and solid quartz cube corners.



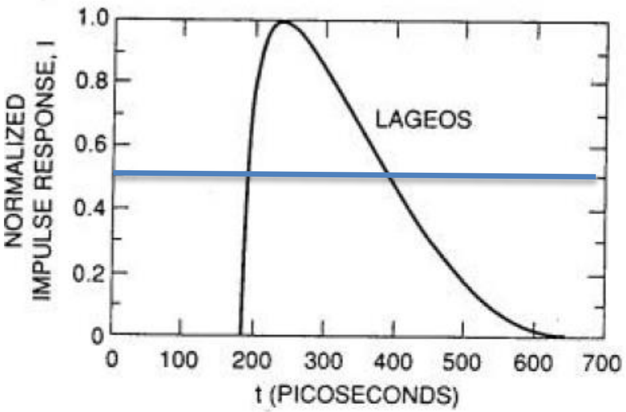
Time Spread of LAGEOS

J. Degnan, Contributions of Space Geodesy to Geodynamics: Technology, Geodynamics 25, pp. 133- 162 (1993)

The total time spread (0 to 0) introduced by the satellite is given by the equation

$$\Delta t = \frac{2R_s}{c}\left\{1-\cos\theta_{\max}\left[1-\frac{nL}{R_s}\sqrt{1-\frac{1}{n^2}+\left(\frac{\cos\theta_{\max}}{n}\right)^2}\right]\right\} \cong \frac{R_s}{c}\theta_{\max}^2\left[1-\frac{nL}{R_s}\left(1+\frac{1}{n^2}\right)\right]$$

where the approximation holds for spherical satellites with large radius (R_s) and small maximum incidence angles (θ_{max}) as shown in the graph below. The following values apply to the LAGEOS satellite.



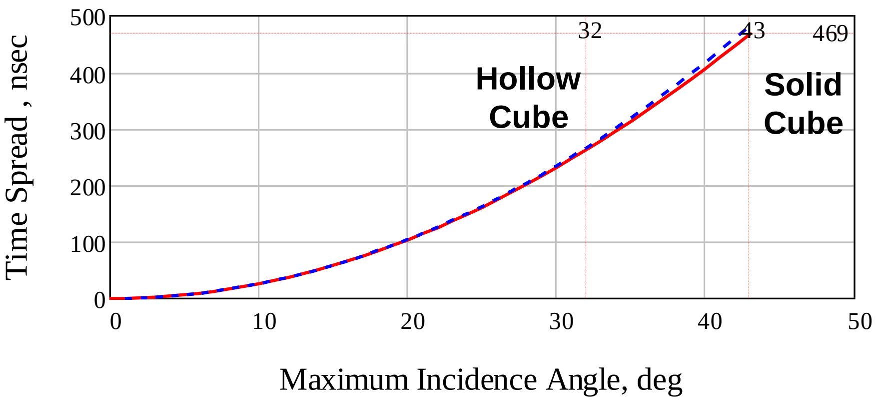
c = speed of light

n =retro index of refraction = 1.455 for fused quartz

L =retro face to vertex distance = 1.095 cm for a 38 mm diameter retro

R_s = LAGEOS satellite radius = 30 cm

Exact = solid red; Approximation = dashed blue



Satellite Cross-Section

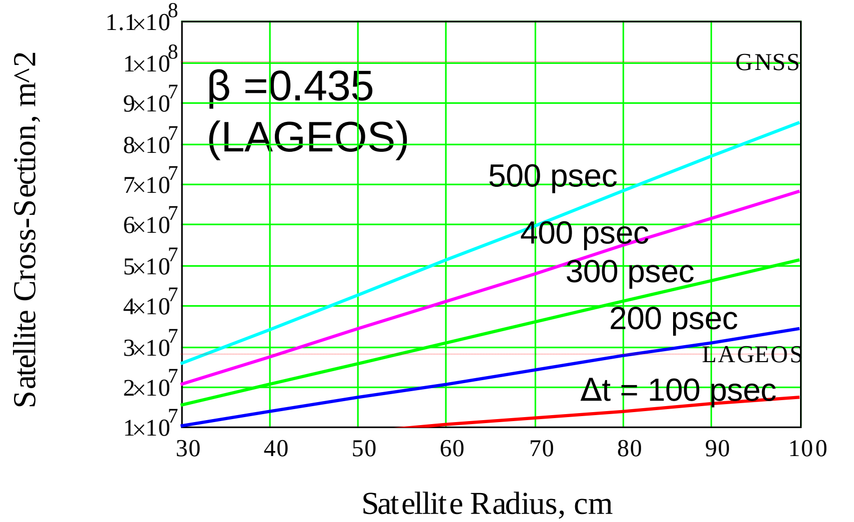
For a spherical satellite, the peak array cross-section is given by [Degnan, 1993]

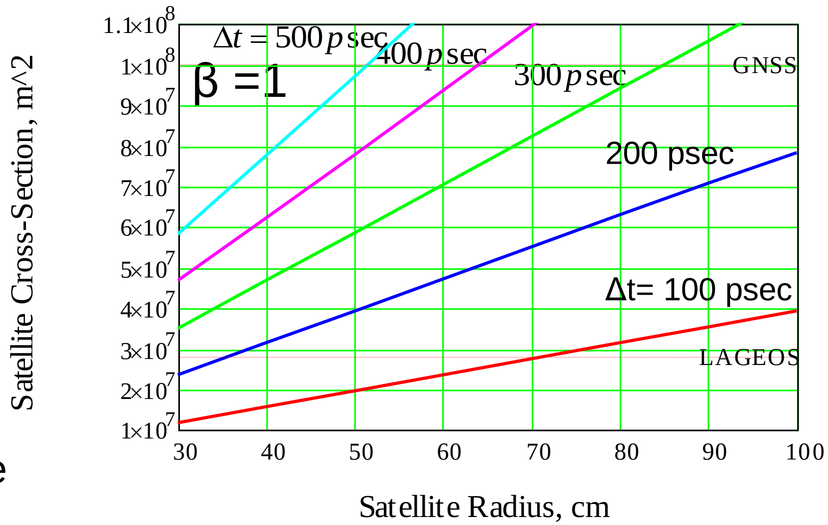
$$\sigma = \frac{\sigma_{cc} N}{2}\left[1 - \frac{\sin^2\left(\frac{\theta_{\max}}{2}\right)}{\left(\frac{\theta_{\max}}{2}\right)^2}\right] = \frac{\sigma_{cc}}{2}\frac{\beta 4\pi R_s^2}{A_{cc}}\left[1 - \frac{\sin^2\left(\frac{\theta_{\max}}{2}\right)}{\left(\frac{\theta_{\max}}{2}\right)^2}\right]$$

where θ_{max} is the maximum acceptance angle of the cube corner, σ_{cc} is the optical cross-section of a single cube corner at normal incidence, N is the total number of retroreflectors on the spherical satellite, A_{cc} is the surface area occupied by a cube corner, R_s is the satellite radius, and β is the "packing density". From the previous vugraph, θ_{max} can be expressed as a function of the time spread Δt and the satellite radius, i.e.

$$\theta_{\max} \cong \sqrt{\frac{c\Delta t}{R_s - nL\left(1 + 1/n^2\right)}}$$

From the bottom graph, a 70 cm radius Super-LAGEOS with the same cross-section and a 100 psec total spread is feasible. Retros would be recessed such that θ_{max} = 11.9°, and the satellite would be heavier to counteract the increased drag.





GNSS and Geostationary Satellites

GNSS and Geosynchronous Satellites have the following characteristics:

1. Their orbital altitudes correspond to several Earth radii
2. They generally perform a utilitarian function (Earth observation, communications, navigation, etc.) which keeps the nadir side of the satellite approximately facing the Earth CoM.
3. The velocity aberration α is typically 20 to 25 μrad and the variation is very small.
4. For a maximum zenith tracking angle of 70°, beam Incidence angles can vary from 0 to θ_{lim} where

$$\theta_{\lim} = a\sin\left[\frac{R_E}{R_E + h}\sin(110^0)\right]$$

=13.1 deg for GNSS satellites at 20,000 km
= 8.2 deg for GEO satellites at 36,000 km

The smaller range of incidence angles implies limited pulse spreading from a flat array, especially if the array is compact in size and the retros are densely packed together to achieve the necessary cross-section. Nevertheless, the maximum flat panel induced spreading due to zenith tracking angle is still 474 psec and 292 psec per linear foot of array for GNSS and GEO satellites respectively. This spreading can increase further if satellite attitude deviations from true nadir extend the range of incidence angles. However, flat panels can be replaced by a segment of a large sphere that simultaneously provides the desired cross-section (10^8 m^2) and the same amount of pulse spreading from all view angles. (Not true of existing GNSS satellites.)

GNSS Array Characteristics

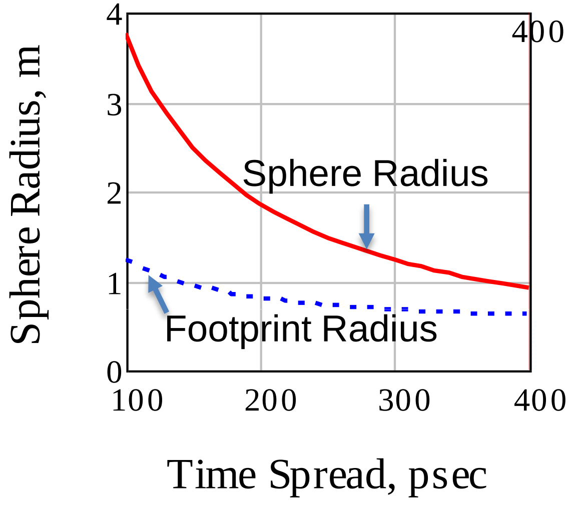
If we replaced the current flat panels on GNSS satellites with a segment of a sphere with radius R_s, the minimum radius of the array footprint on the nadir-viewing side of the satellite would be given by

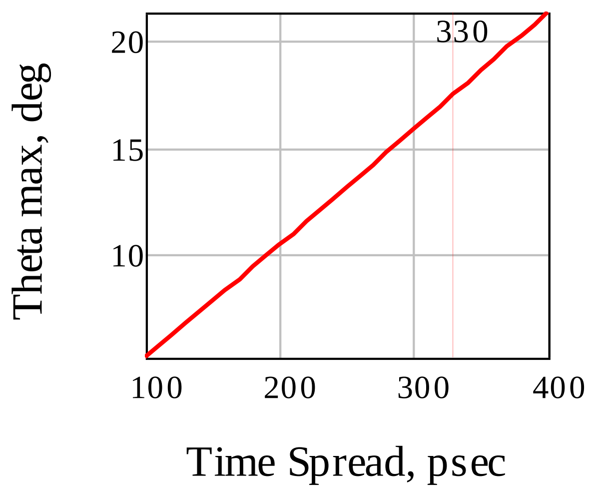
$$R_A = R_S \tan(\theta_{\max} + \theta_{\lim})$$

where we have assumed β ~0.8, θ_{lim} = 13.1° and used the following equation

$$\theta_{\max} \cong \sqrt{\frac{c\Delta t}{R_s - nL\left(1 + 1/n^2\right)}}$$

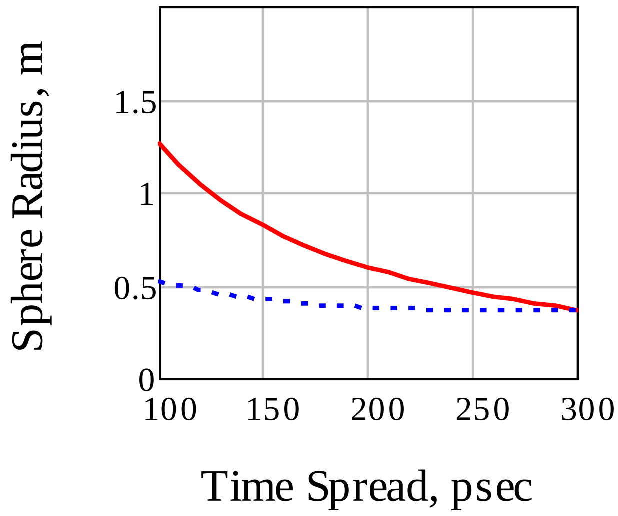
and assumed the ILRS recommended GNSS cross-section of 10^8 m^2 to generate the following graphs

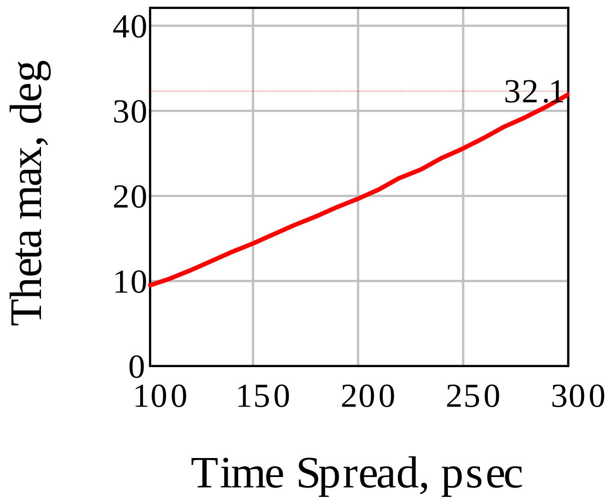




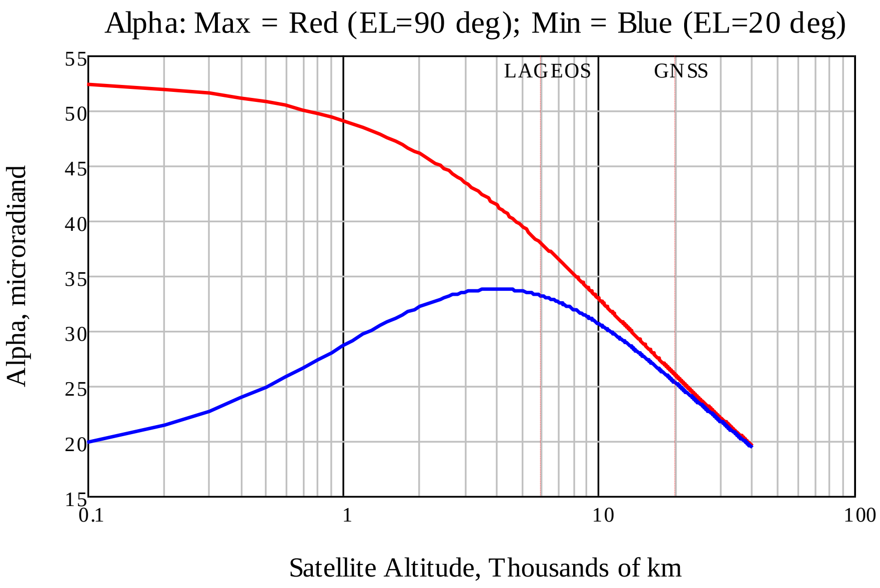
GNSS Retroreflector Size

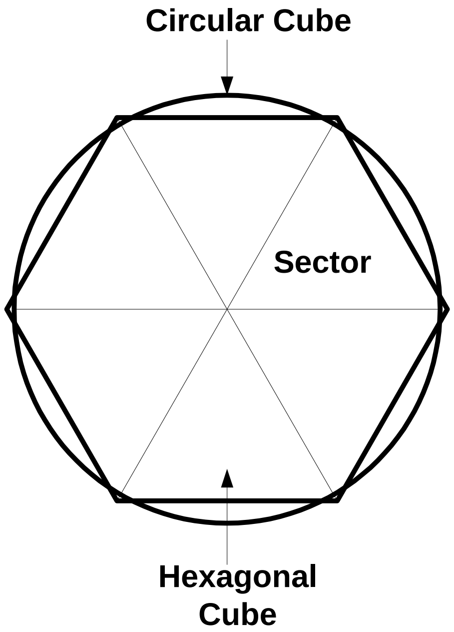
The results on the last slide assumed a standard 38 mm diameter retroreflector and resulted in a rather large footprint (1 to 2 m) on the nadir-viewing face of the spacecraft. However, because the range of velocity aberration is rather narrow for GNSS and higher satellites, one has the option of using a larger retroreflector. Since the optical cross-section increases as D^4 power but the occupied area increases only as D^2, one can envision a smaller sphere with fewer retros embedded into it and therefore a smaller footprint. A cursory analysis suggests that retro diameters up to 64 mm can be used, yielding the following results for a fill factor of β =0.8.





Velocity Aberration



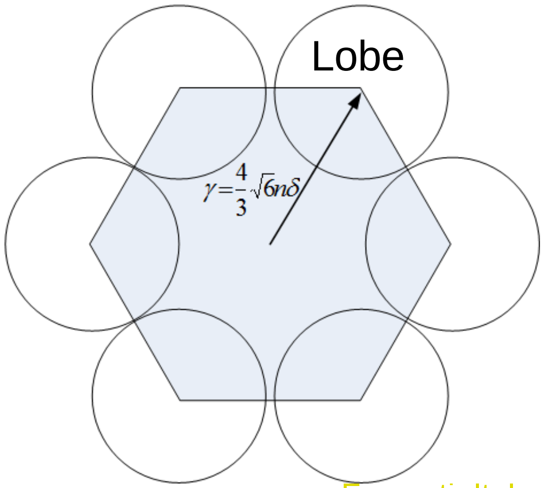


$$\gamma = \frac{4}{3}\sqrt{6}n\delta = 26\mu rad$$

$$\delta = \frac{3(26\mu rad)}{4\sqrt{6}(1.455)} = 5.47\mu rad$$

$$C = 2\pi\gamma = 163\mu rad$$

$$N_C = \frac{C}{6\delta} = \frac{163}{6(5.47)} = 5$$



Summary

mm Accuracy LEO to MEO Geodetic Satellites

- Large radius satellites to better match the incoming plane wave
- Greater packing density allows more reflectors in the active area to increase cross-section
- Reduce range of accepted incidence angles to minimize satellite impulse response through the use of hollow retros or recessed hollow or solid retros
- Also incidence angles < 17^o do not leak light in solid TIR reflectors
- Selection of cube diameters and clocking to best match the "α annulus" while favoring high zenith (low elevation) angles is key to efficient array design

GNSS and GEO Satellites

- Typically have nadir face pointed near Earth center due to other functions (Earth observation, communications, navigation, etc.)
- Flat panels exhibit several hundred psecs of temporal spread per linear foot of array at the lower elevation angles and do not present the same cross-section or temporal spread at all viewing angles.
- Range accuracy and signal uniformity would benefit from using a segment of a large sphere.