Relativistic corrections in the European Laser Timing (ELT) experiment

Stefan Marz *, Anja Schlicht, Christoph Bamann

Forschungseinrichtung Satellitengeodäsie, Technical University of Munich
* s.marz@tum.de

Abstract. *The European Laser Timing (ELT) experiment, which is part of the ESA mission ACES (Atomic Clock Ensemble in Space), aims at enabling picosecond time transfer between ground based clocks and the ultra-stable time scale of the ACES module aboard the International Space Station. To this end, both a classical two-way and an additional one-way optical link shall be established, both of which are based on timing via ultra-short laser pulses. For maximum timing precision, the space based ELT hardware will be equipped with a novel single photon avalanche diode (SPAD), which needs to be gated to reduce the signal-to-noise ratio to an acceptable level. To synchronize pulse transmission dates with the gating of the ELT detector, space and ground clocks shall be referred to a common time scale like UTC. Hence, the ELT data center is required to compute the relativistic drift of the ACES clock with respect to UTC, as the payload has no access to a sufficiently accurate approximation of UTC, for example through GNSS. We briefly present the underlying relativistic effects and our approach to compute the associated clock correction products. In doing so, we discuss the impact of the resolution of the employed gravity model as well as the effects of the tidal potentials of Sun, Moon, and planets. In addition, we suggest how these clock correction products could efficiently be delivered to the SLR stations that contribute to ELT. Finally, we further address relativistic pulse delay corrections, namely the standard Shapiro time delay and first and second-order Sagnac corrections.*

1 Introduction

The objective of the European Laser Timing (ELT) experiment as part of the ESA mission ACES (Atomic Clock Ensemble in Space), is the establishing of an optical time transfer between SRL station clocks and the clock aboard the International Space Station with picosecond accuracy. The on-board hardware of ELT consists of a corner cube retro-reflector (CCR), a single-photon avalanche diode (SPAD), and an event timer connected to the ACES time scale. The SPAD detects laser pulses fired by a SLR station. While the detection dates are recorded in the ACES time scale, a corner cube retro-reflector (CCR) reflects the laser pulses back to the ground station. Hence, with the two-way time of flight precise ranging is provided. The principle of the ELT experiment is illustrated in figure 1. The offset between the ground and ACES time scale can be determined with the information of the laser fire time ($\mathrm{t_{start}}$), the detection time at the SPAD ($\mathrm{t_{stop1}}$) and the detection time of the reflected pulses again at the SLR station ($\mathrm{t_{stop2}}$). Since the ISS has no access to a sufficiently accurate approximation of UTC for opening the detector gate, it is required to calculate the offset of the ACES time scale to UTC. To meet the requirement of a picosecond accuracy in time transfer the observance of relativistic corrections is also essential. Thereby, relativistic effects due to the movement of the clocks, potentials and pulse delay need to be considered. Such relativistic effects do not only affect the spacecraft, but also the ground station. Hence, it is a requirement for the SLR stations to determine their own offset to UTC before the pass and relativistic corrections of the ground clocks have to be taken into account. In the following we analyze the corrections according to the movement of the clock and gravitational potentials for space and ground clocks as well as the relativistic corrections to time transfer.

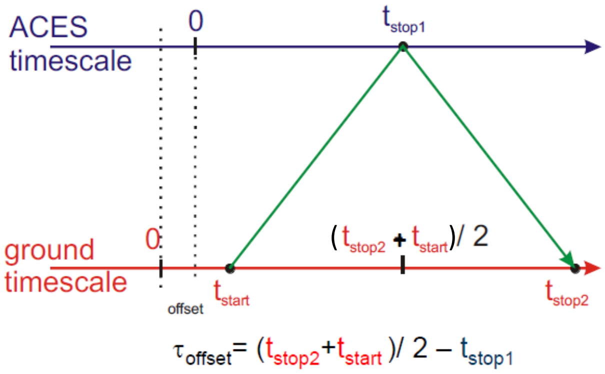


Figure 1. Principle of the ELT experiment

2 Relativistic correction due to the movement of clocks in a gravitational potential

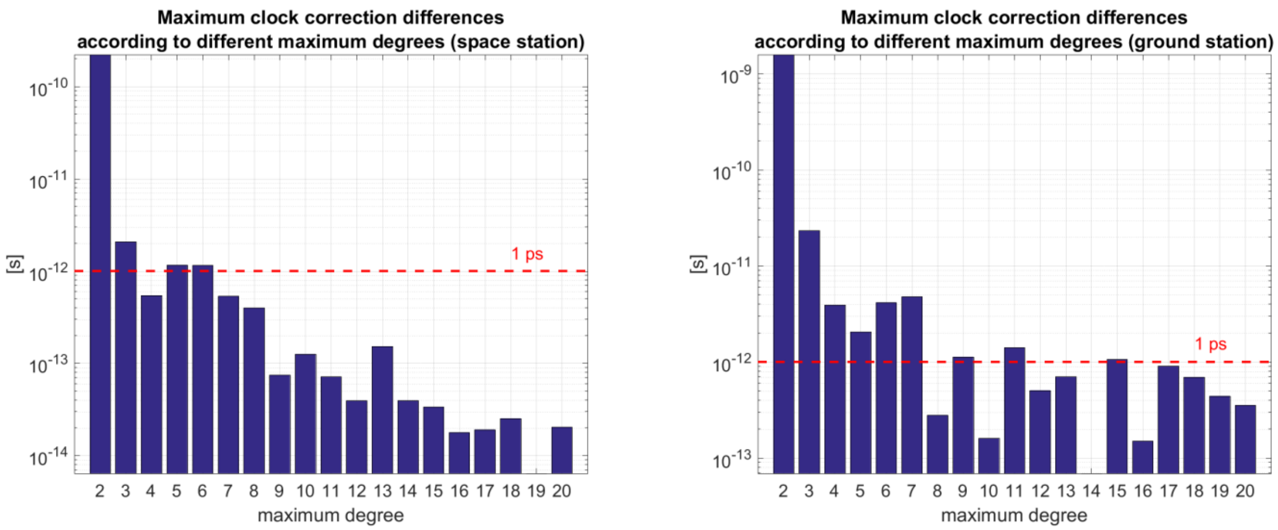
The relativistic behavior of a clock due to the movement of the clock as well as the gravitational potentials along a selected path is calculated according to Ashby (2003) as follows:

$$\int_{path} dt = \int_{path} \Delta\tau \left[1 - \left(\frac{V - \phi_0}{c^2}\right) + \frac{v^2}{c^2} \right] \tag{2.1}$$

where $\Delta\tau$ is the coordinate time (TT), v the space or ground station velocity in ECI, c the speed of light, ϕ_0 the potential of the geoid and V the sum of the gravitational potential of the earth and the tidal potentials of Moon, Sun and planets. Below required considerations and calculations relating to the potentials are briefly explained.

Gravitational potential of the Earth

ELT is expected to compare ground clocks with the ACES time scale in picosecond resolution. To find out to which degree and order the gravitational potential of the Earth has to be expended, we calculate the clock corrections each with different maximum degrees. Subsequently, the maximum increase of accuracy for clock correction calculated with a potential to a certain degree ensures by subtraction of the clock correction calculated with a potential of one degree less and determining each the maximum absolute value. Figure 2 illustrates these maximum clock correction differences according to calculations with different maximum degrees, for the space (Fig. 2(a)) and ground (Fig. 2(b)) station clock. For the analysis we used predicted TLE orbits as well as a time span of two hours.
As can be seen in figure 2, it is necessary to calculate the gravitational potential at least until degree 15.



(a) (b)

Figure 2. Maximum clock correction differences according to different maximum degrees; (a) space, (b) ground station. The maximum differences ensure by subtraction of the clock corrections calculated with a gravitational potential to a certain degree with the clock correction calculated with a potential of one degree less and determining each the maximum absolute value. For the analysis predicted TLE orbits as well as a time span of two hours were used.

Tidal potentials

Next to the gravitational potential of the earth also the tidal potentials of Moon, Sun and planets need to be considered. Figure 4 illustrates the increase of accuracy for the clock correction calculation in consideration of the tidal potential of each listed celestial object (Fig. 3). Hence, for a picosecond time transfer the effects of Moon and Sun have to be incorporated, whereas the effects of the planets can be neglected.

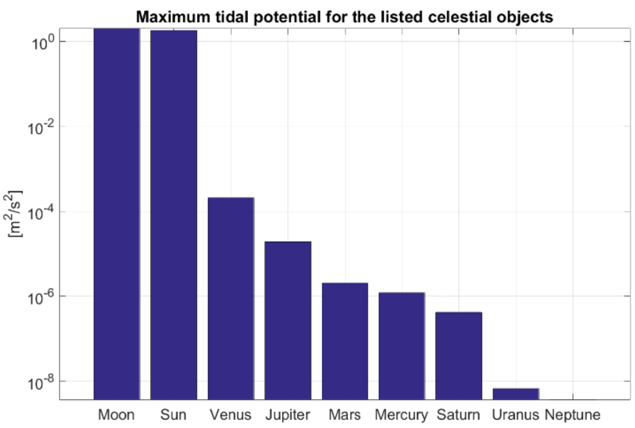


Figure 3. Maximum tidal potentials for the listed celestial objects.

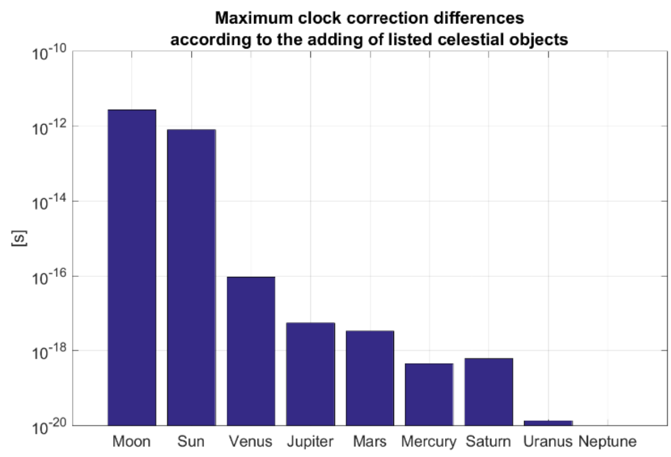
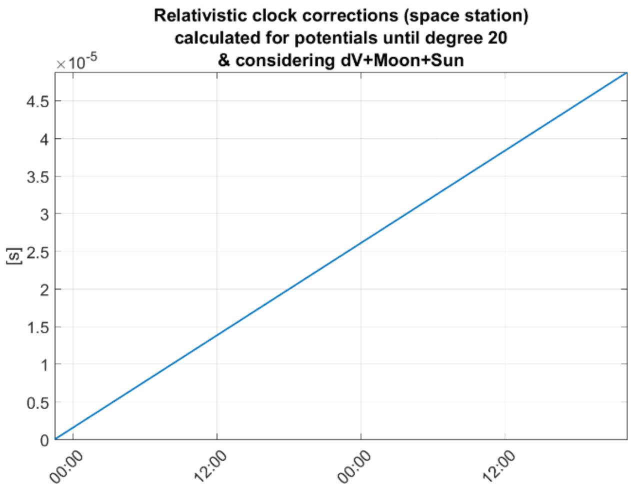


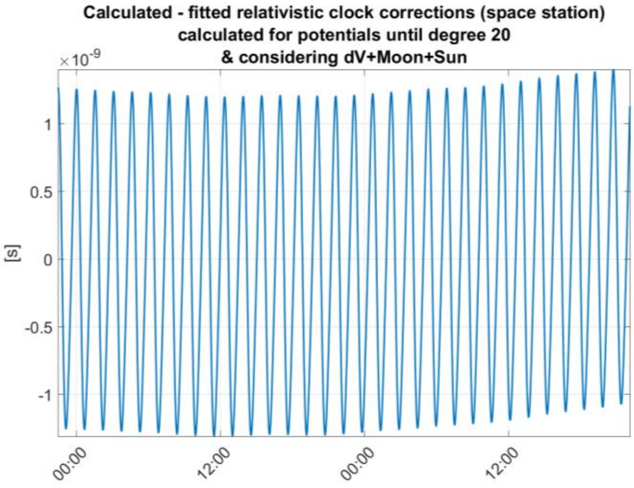
Figure 4. Maximum clock correction differences according to the adding of celestial objects. The maximum differences ensure by adding the tidal potential of a celestial object to the calculation of the clock corrections, followed by a subtraction of clock correction calculated with the potentials of the preceded objects according to the listing in fig. 3 and determining each the maximum absolute value.

2.1 Relativistic clock corrections

According to formula 2.1 the relativistic clock corrections for the space station (Fig. 5(a)) and ground station are calculated for a gravitational potential until degree 20 and considering the tidal potentials of Moon and Sun. To offer SLR stations a less intricate calculation of the space and ground station clock offsets to UTC though the relativistic effects of the clocks movements and the potentials, a linear coefficient as well as the corresponding offset coefficient will be integrated in the consolidated prediction file. Fig. 5(b) illustrates the difference between the calculated and the linear fitted clock correction for the space station. With a maximum difference of about 1,5 ns for the space station clock we are well below 10 ns, which is the UTC calculation precision of the SLR stations.



(a)



(b)

Figure 5. Clock correction for the space station; (a) shows the calculated clock correction, (b) shows the difference between the calculated and the linear fitted clock correction.

According to formula 2.1 we calculate clock corrections each with a predicted and a precise orbit.
The subtraction of the clock corrections finally results in the difference between the true and predicted clock behavior (Fig. 6). As can be seen in figure 6, for a period of two days the difference is in the order of about 1,5 ns.

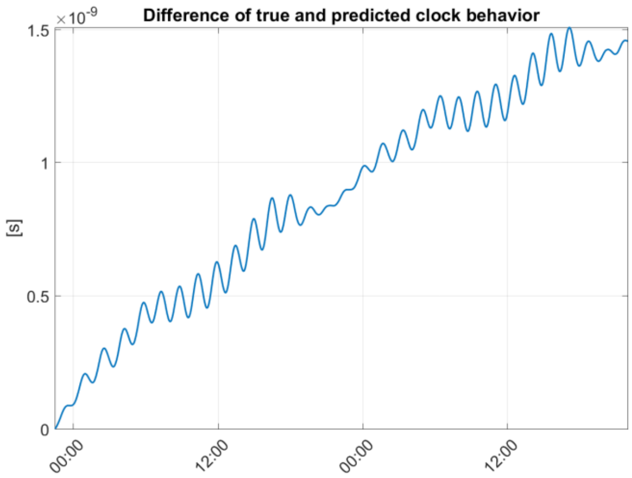


Figure 6. Difference between true and predicted clock behavior.

With the error of the linear clock prediction less than 10 ns, the general prediction accuracy of 5 ns and the accuracy of the ACES clock of about 50 ns we are within the accuracy limit of 100 ns. Comparing the prediction accuracies with the accuracy of the ACES clock these are more or less subordinated.

3 Relativistic effects due to pulse delay

3.1 Shapiro delay

The Shapiro delay (Fig. 7) refers to relativistic pulse delay corrections and describes the curvature of the signal way. It is necessary for a one- and two-way link. According to Blanchet et al. (2000) the effect is defined as follows:

$$\frac{2GM_E}{c^3}\ln\left(\frac{r_A+r_B+D_{AB}}{r_A+r_B-D_{AB}}\right) \tag{3.1}$$

where D_{AB} is the absolute value of coordinate distance between the receiving and emitting station in ECI, r_A the absolute value of coordinates of station A in ECI at the emitting time, r_B the absolute value of coordinates of station B in ECI at the emitting time, GM_E the geocentric gravitational constant and c the speed of light.

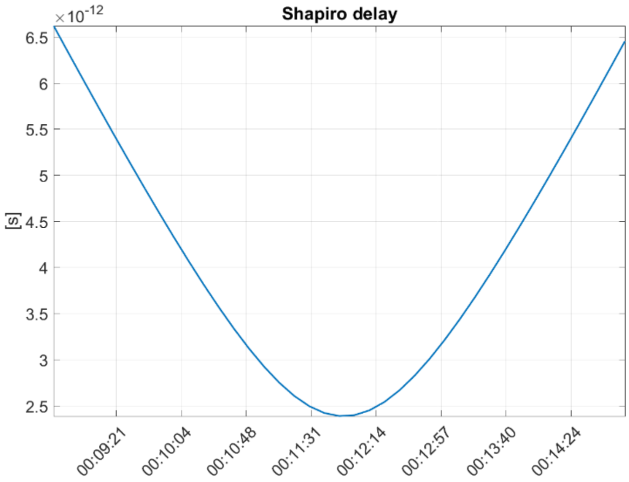


Figure 7. Shapiro delay for a simulated pass

3.2 Sagnac delay

If one-way measurements are evaluated at only receive or transmit time, a correction due to the satellite movement has to be taken into account. This correction is called Sagnac delay. The Sagnac delay, which is not a real relativistic effect but often added to them, refers to pulse delay corrections. The first order term (formula 3.2), as defined according to Blanchet et al. (2000), corrects the movement of the clock during the time of flight of the signal. The second order term (formula 3.3, according to Blanchet et al. (2000)) corrects the Shapiro delay. The effect of the first order Sagnac delay for a simulated pass is shown in figure 8, for the second order delay in figure 9.

First order delay: $$\frac{\mathbf{D}_{AB} * v_B(t_A)}{c^2} \tag{3.2}$$

Second order delay: $$\frac{D_{AB}}{2c^3}\left(v_B^2 + \frac{(\mathbf{D}_{AB} * v_B)^2}{D_{AB}^2} + \mathbf{D}_{AB} * a_B\right) \tag{3.3}$$

where $\mathbf{D}_{AB}$ is the coordinate distance between the receiving and emitting station in ECI, $D_{AB} = |\mathbf{D}_{AB}|$, v_B the velocity of the receiver station in ECI, a_B the acceleration of the receiver station in ECI and c the speed of light.

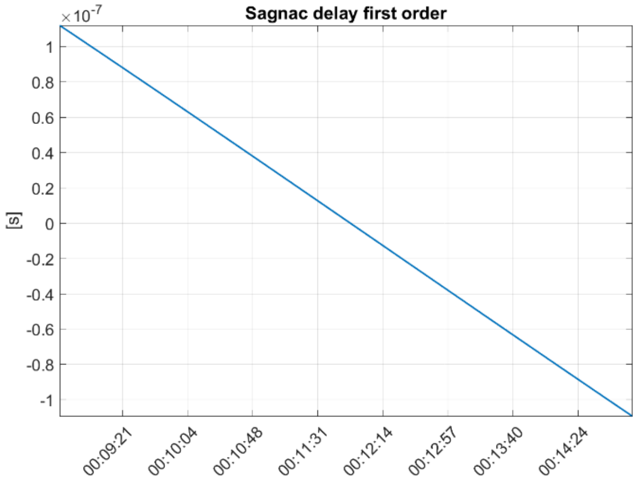


Figure 8. Sagnac delay first order for a simulated pass

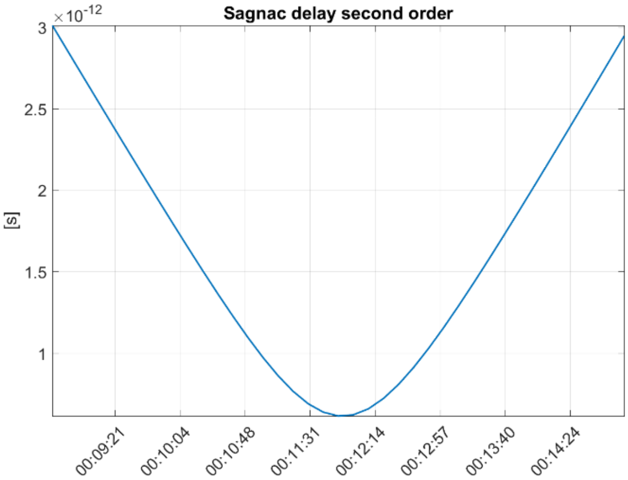


Figure 9. Sagnac delay second order for a simulated pass

4 Summary

Relativistic clock corrections need to be considered on the one hand due to the movement of the clocks and gravitational potentials and on the other hand due to the pulse delay. The corrections due to clocks movements and potentials will be given to SLR stations as a linear coefficient and an offset coefficient. The error of a linear clock prediction amounts to about 1,5 ns for the space station is thus well below the UTC calculation precision of the SLR station of about 10 ns. With the general prediction accuracy of about 5 ns and the accuracy of the ACES clock of about 50 ns we are within the accuracy limit of 100 ns in which the prediction accuracies have more or less an underpart in comparison. Relating to the simulation the corrections for the pulse delay are for the Shapiro delay around 6,5 ps, the Sagnac delay of first order 110 ns and second order 3 ps. To the proper pulse delay all relativistic delays as well as required non relativistic corrections for instance due to the troposphere and geometry will be added up. Hence, the SLR stations can calculate the offset of the ACES time scale and determine their own offset to UTC.

References

Ashby, N. (2003): Relativity in the Global Positioning System, Living Rev. Relativity 6.

Blanchet, L.; Salomon, C.; Teyssandier, P.; Wolf, P. (2000): Relativistic Theory for Time and Frequency Transfer to Order 1/c3, Astron. Astrophys. 370, 320.