An Estimation of the Number of Expected Returned Photons for the HartRAO Lunar Laser Ranger System

S. C. Ndlovu[1, 2], L. Combrinck[1, 2], P. Exertier[3], M. Akombelwa[2] and N. Chetty[2]

1. Space Geodesy Programme, Hartebeesthoek Radio Astronomy Observatory, South Africa
2. College of Agriculture, Engineering and Science, University of KwaZulu-Natal, South Africa
3. French National Centre for Scientific Research, Observatoire de la Côte d'Azur, France

Corresponding Author: sphume@hartrao.ac.za

Abstract: Development of an integrated model and system to enable optimal efficiency and signal path parameter estimation of a Lunar Laser Ranger, is one of the major requirements for the new Hartebeesthoek Radio Astronomy Observatory's Satellite/Lunar Laser Ranger (S/LLR) system; without optimal efficiency of a lunar laser ranger signal path, the number of returned photons could be zero. The mathematical tool under development will be used to evaluate computed and observed photon return efficiency, using as departure point the existing link equation, with the option to add and estimate parameters in the least squares sense. The existing link equation can be used to predict the laser ranging system efficiency and is based on assumed accuracy of all parameters which influences the returned signal, presented as an estimate of expected number of returned photons. However, it does not make provision for model enhancements and parameter optimisations. Optimal efficiency in the S/LLR signal path will yield an improvement in the return-energy of the laser so that ranges to the lunar corner cube retro-reflectors can be measured accurately. This will ensure a high-precision measurement of the Earth-Moon distance, which is highly in demand since determination of the exact Earth-Moon distance is a complex undertaking. The geographic position of the HartRAO station, new state-of-the-art HartRAO S/LLR system under development and the expected number of returned photons will enable HartRAO to play a key role in improving the ranging accuracy to a sub-centimetre level, adding to the current effort to determine highly accurate Earth-Moon distances for various scientific purposes. We estimate the expected photon returns under various scenarios, including variable power levels, lunar distance, atmospheric conditions and system efficiency.

Introduction

The HartRAO Lunar Laser Ranger (LLR) system (Combrinck, 2011) requires a state-of-the-art software tool that enables optimal efficiency and signal path parameter estimation. Such a tool utilizes the existing link budget equation to estimate the number of returned photons for given conditions and LLR system parameters. It calculates the mean number of returned photons recorded by a photon detector as (Degnan 1993),

$$n_p = \eta_q \left(E_T \frac{\lambda}{hc}\right) \eta_t G_t \sigma \left(\frac{1}{4\pi R^2}\right)^2 A_r \eta_r T_a^2 T_c^2 \,, \tag{1}$$

Where η_q is the quantum detector efficiency, E_T is the total energy of the laser pulse, λ is the wavelength, h is Planck's constant, c is the speed of light in a vacuum, η_t is the transmit optics efficiency, G_t is the transmitter gain, σ is the efficiency of the retro-reflector optical cross-section, R is the slant range, A_r is the area of the receiving aperture, η_r is the receive optics efficiency, T_a is the atmospheric transmission and T_c is the cloud cover transmittance.

In this work, we focus on the effects that result from the variable power loss, lunar distance, atmospheric conditions and system efficiency. The simplified link budget equation for the HartRAO LLR system is written as,

$$n_p = C_s G_t \sigma \left(\frac{T_a T_c}{R^2}\right)^2, \;\; C_s = \eta_q \left(E_T \frac{\lambda}{hc}\right) \eta_t \left(\frac{1}{4\pi}\right)^2 A_r \eta_r \tag{2}$$

and

$$G_t = \frac{4\pi}{\lambda^2}\left(\frac{2}{\alpha_t^2}\right)\left(e^{-\alpha_t^2} - e^{-\alpha_t^2 \gamma_t^2}\right)^2 A_t, \tag{3}$$

where C_s is the known constant of the "fixed" system parameters, α_t is the , γ_t is the , A_t is the area of the transmitting aperture and the other parameters are as defined in (Equation 1). The efficiency of the retro-reflector optical cross-section, σ, is still the subject of complex and active studies, which investigate the degradation of returned photons from the reflectors (Murphy et al. 2010). The transmitter gain equation (Equation 3) is modified in such a way that it takes into account the obscuring secondary mirror's support structure (see Figures 1 and 2).

Figure 1: The 0.3 m secondary mirror and spider structure mounted on the 1 m telescope before refurbishment.

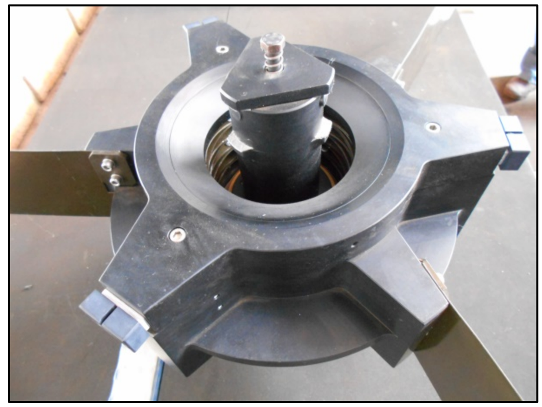


Figure 2: The refurbished spider and secondary mirror support structure after it was removed from the main telescope tube.

The modified transmitter gain equation is then expressed as,

$$G_t = \frac{4\pi}{\lambda^2}\left(\frac{2}{\alpha_t^2}\right)\left(e^{-\alpha_t^2} - e^{-\alpha_t^2\gamma_t^2}\right)^2 (A_t - A_{os}), \tag{4}$$

where A_{os} is the total area of the obscuring structures and the other parameters are as defined in (Equation 3).

Development of an integrated model and system to enable optimal efficiency for HartRAO's LLR signal path will yield an improvement in the return-energy of the laser, hence more data of high quality will be achieved from the only (currently) LLR station in the entire Southern Hemisphere (Combrinck and Botha 2013).

Methodology

An advanced mathematical tool (Figure 3), utilising C++ code, still under development is used to estimate the expected number of returned photons for the HartRAO's LLR system. The tool allows the user to select a targeted

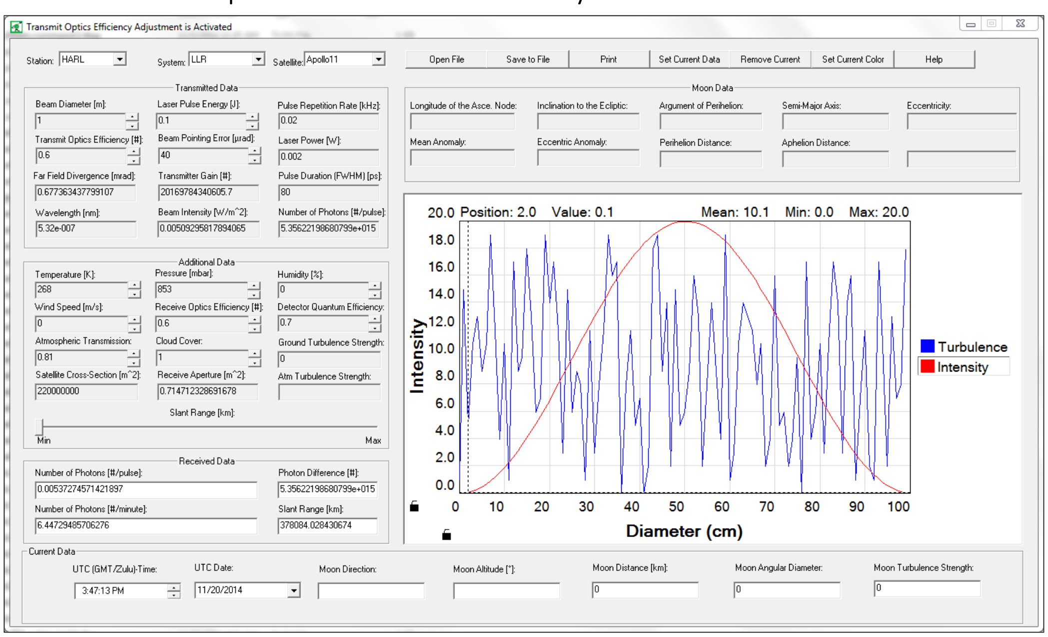
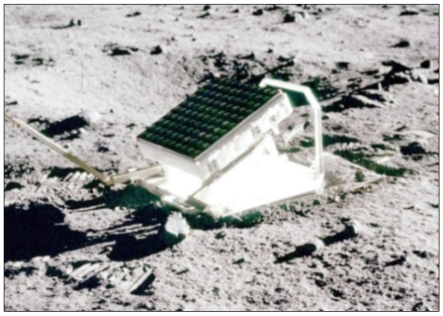


Figure 3: The GUI representing HartRAO's program which estimates the received number of photons per minute.

satellite/reflector (see Figure 4) and adjust the varying parameters.



It will be used to evaluate computed and observed photon return efficiency, using as departure point the existing link equation, with the option to add and estimate parameters in the least squares sense. The values for satellites/reflectors' cross section were calculated from (Degnan 2012) and the atmospheric transmittance and cirrus transmission were obtained from (Degnan 1993). The calculated transmitter gain for HartRAO's system is 2.0×10^{13}.

Figure 4: The Apollo 11 retro-reflector corner cube array placed on a dusty Lunar surface (source: NASA).

Preliminary Results

The preliminary results (Figures 5 and 6) were obtained from the use of equation (1) with a fixed parameter for the retro-reflector's cross-section, σ. This parameter can be varied in order to obtain a more realistic indication of the effect of Lunar's corner cubes mirrors on the number of returned photons.

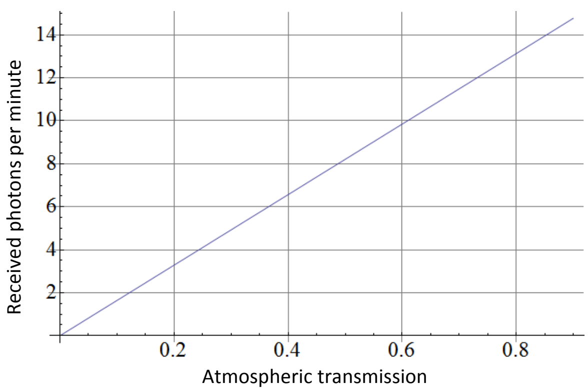


Figure 5: Number of **r**eceived photons per minute vs. varying atmospheric transmission.

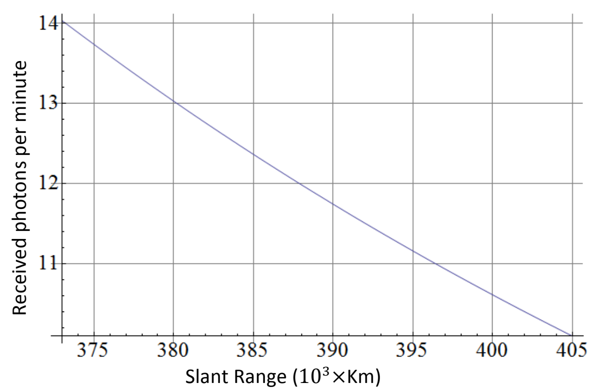


Figure 6: Number of received number of photons per minute vs. the slant range in 10^6 km.

The maximum number of returned photons depends on "better" atmospheric conditions (Table 1) and this is in agreement with the available literature (Degnan 1993).

Table 1: The relationship between varying parameters and number of photons reflected from Apollo 11 Laser Ranging Retro-Reflectors.

Parameter	Worst value	Optimal value
Laser pulse energy (mJ)	100	110
Transmit optics efficiency	0.4	0.9
Slant range (km)	405000	378084
Detector quantum efficiency	0.4	0.7
Receive optics efficiency	0.4	0.9
Atmospheric transmission	0.02	0.81
Cirrus transmission (Cloud cover)	0.1	1
Returned photons/minute	0.003	15

Analysis and Discussion

Preliminary results indicate the influences of atmospheric fluctuations, slant range, retro-reflectors' reflectivity and other degrading parameters on the expected number of returned photons. The slant range, R, is one of the biggest degrading parameters on the number of received photons due to its 4^{th} order of magnitude. Atmospheric transmission and cirrus transmittance also play a huge role in the number of returned photons degradation.

The mathematical tool will include those ranging degrading factors such as the angle of incidence, laser energy variations and efficiency of retro-reflectors, with the aim to add and estimate parameters in the least squares sense. The other restraining effects on the returned laser signal could result from thermal and density fluctuations of the atmosphere; this was recently investigated by our group (Ndlovu and Chetty 2014). Additional research is still necessary to verify the accuracy of the tool. This includes the use of the currently operating LLR stations' system parameters to evaluate computed and observed photon return efficiency.

Conclusions

In conclusion, expansion of the existing link equation is a necessary requirement for more accurate returned photon estimation. This will help in considering parameters (that affect ranging efficiency) and other factors, which can be caused by the detection system, corner cube mirrors and obscuring system structures as the photons are transmitted and received through the LLR signal path. It will also improve the system signal-to-noise ratio, thus ensuring more photon returns for HartRAO's LLR system. Hence, more data can be achieved from the Southern Hemisphere station as the other existing LLR stations are all located in Northern Hemisphere.

Future work will focus on relating the number of transmitted photons with the transmitted laser pulse, laser's radius of curvature on the Moon surface and the number of photons that actually hit and get returned by the retro-reflector mirrors. This will be an additional tool in determining the behavior of retro-reflector mirrors over time.

Acknowledgement

We thank the Department of Science and Technology (South Africa), the Observatoire de la Côte d'Azur and NASA(GSFC) for their support.

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