Atmospheric range correction for two-frequency SLR measurements

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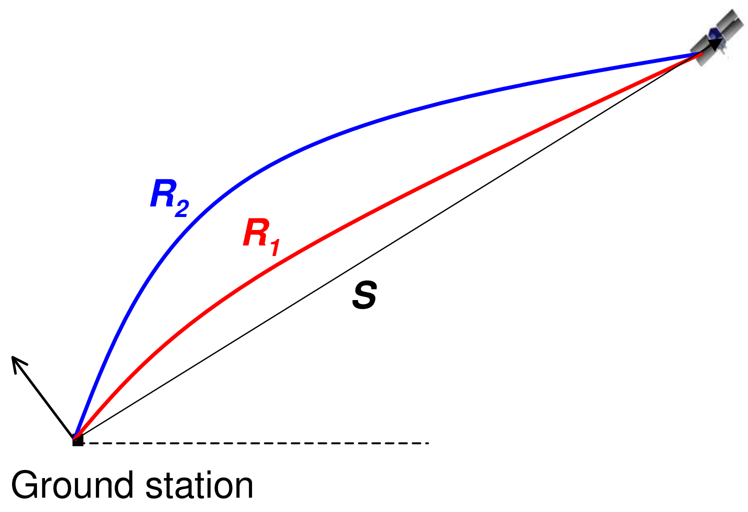
1. Previous atmospheric correction formula for 2F-SLR



$$S = R_1 + \nu (R_1 - R_2)$$

$$\nu = \frac{k_d(f_1)}{k_d(f_2) - k_d(f_1)}$$

R_1 & R_2 : SLR measurements

k_d : Dispersive constant

Limitations:

- Neglecting **water vapor** and **curvature/bending** effects
- A millimeter level of accuracy for the range correction is hard to achieve
- Within the framework of GGOS, the standard model needs to be improved

(Abshire & Gardner, 1985)

2. New formula: Extension of the previous formula

$$S = R_1 + \nu(R_1 - R_2) + (\nu P_{21} - K_1) + H_{21}\, SIWV$$

T_1: dispersion (observed)

T_2: curvature (modeled)

T_3: water vapor (observed)

previous formula

- Geometrical optics approximation
- The new formula considers all of the atmospheric propagation effects, except turbulence

dispersive power

$$\nu = \frac{k_d(f_1)}{k_d(f_2) - k_d(f_1)}$$

water vapor factor

$$H_{21} = 10^{-6}\, k_v(f_1)\, \nu \left(\frac{k_v(f_2)}{k_v(f_1)} - \frac{k_d(f_2)}{k_d(f_1)} \right)$$

slant integrated water vapor

$$\mathrm{SIWV} = \int_{p_1} \rho_v \, ds_1$$

Propagation correction

$$P_{21} = \int_{p_2} n(r_2, f_2)\, ds_2 - \int_{p_1} n(r_1, f_2)\, ds_1$$

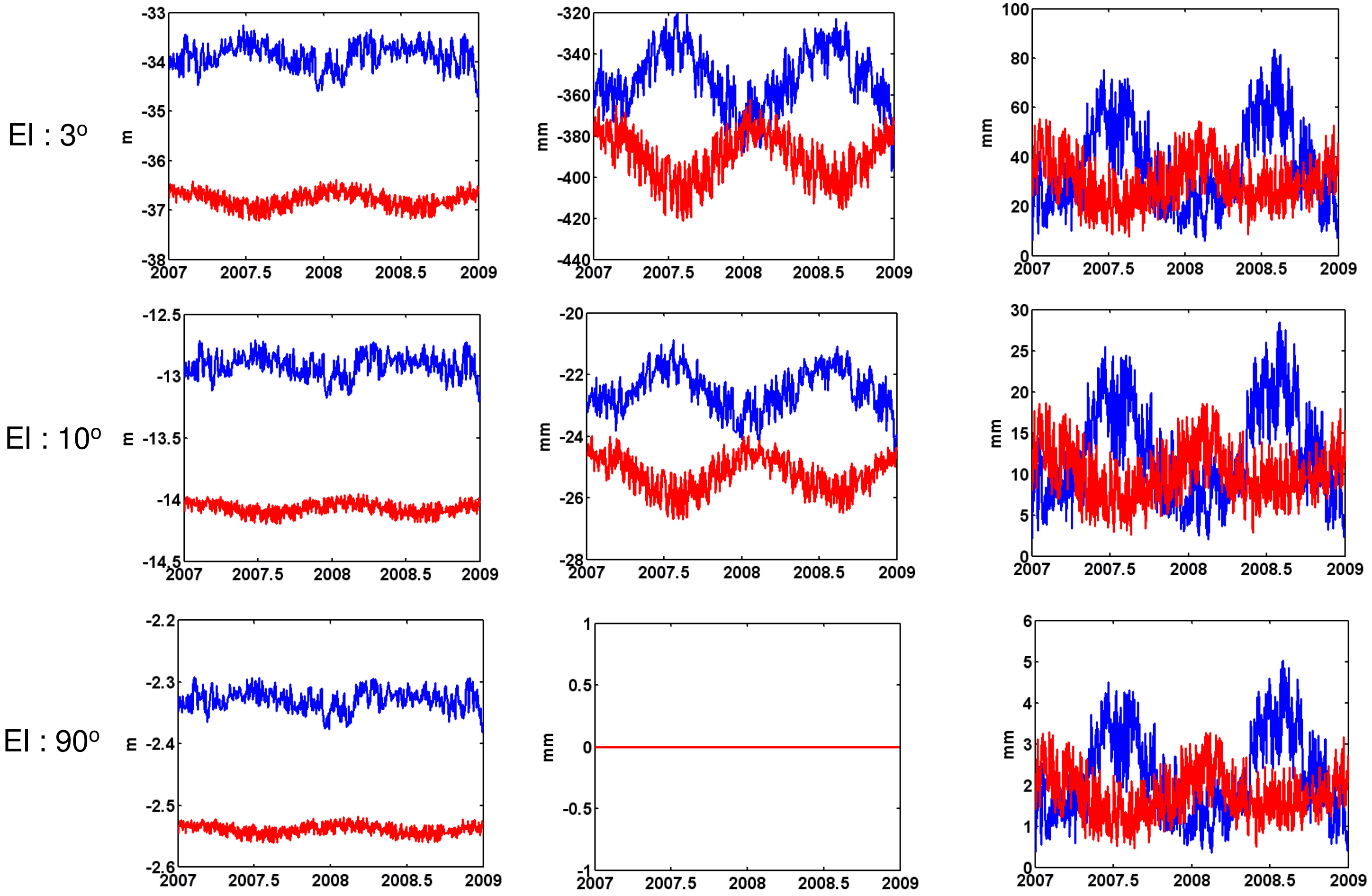
arc-to-chord correction

$$K_1 = \int_{p_1} ds_1 - S$$

(Wijaya & Brunner, 2011)

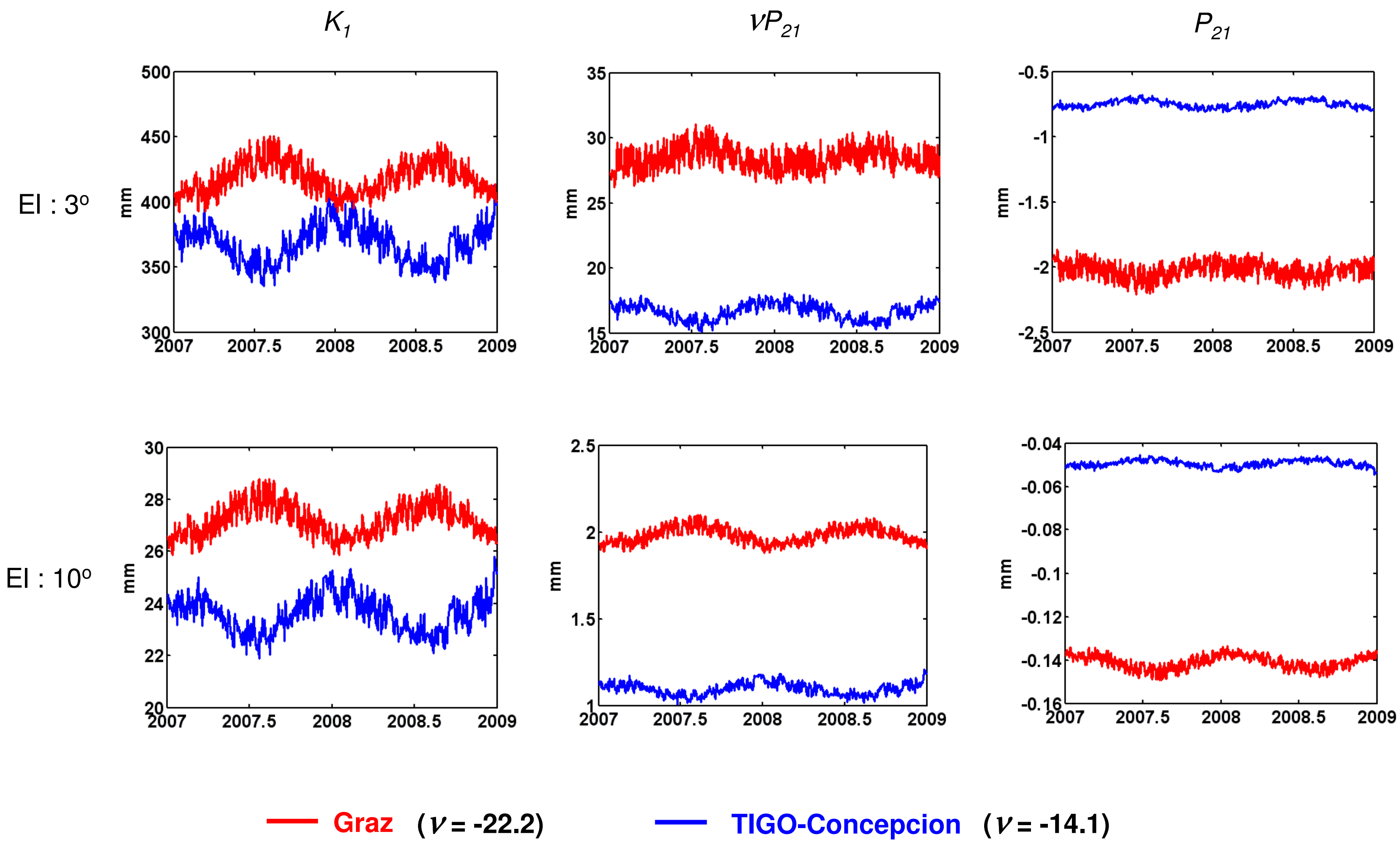
2. New formula: Magnitude of the corrections simulated by ray-tracing

$\nu\,(R_1 - R_2)$ dispersion	$(\nu P_{21} - K_1)$ curvature	$H_{21}\,SIWV$ water vapor



Graz (ν = -22.2) **TIGO-Concepcion** (ν = -14.1)

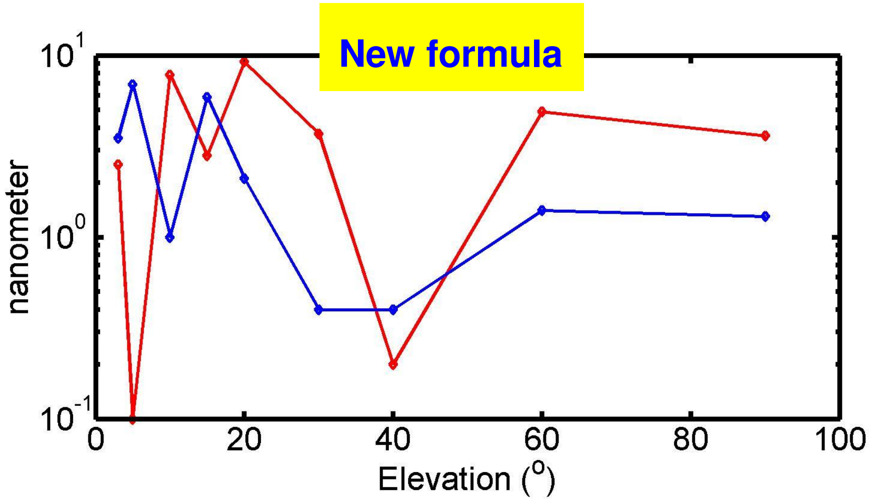
2. New formula: Magnitude of the corrections simulated by ray-tracing

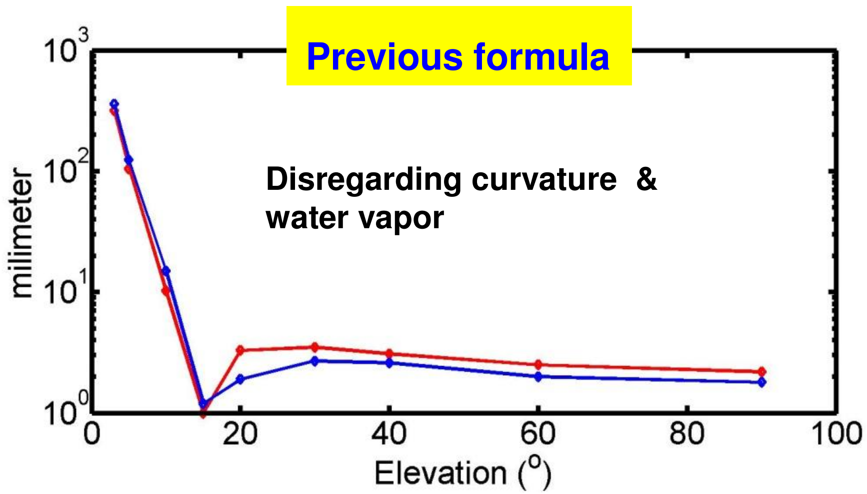


2. New formula: Comparison with the previous formula

- **Residual range error (RRE)**

$$RRE = | S_{formula} - S_{ray\text{-}tracing} |$$





Graz   TIGO-Concepcion

2. New formula: Precision requirements

$$S = R_1 + \nu (R_1 - R_2) + (\nu P_{21} - K_1) + H_{21}\, SIWV$$

Parameter	Precision	Method/Model
$R_1 - R_2$	8 μm	Observable ??
R_1	1 mm	Observable
P_{21}, K_1	1 mm	Accurate model
SIWV	6 kg/m²	Accurate model, GNSS

The values of ν and H_{21} for various SLR frequencies

$$\nu = \frac{k_d(f_1)}{k_d(f_2) - k_d(f_1)}$$

$$= 8 - 30$$

$$H_{21} = 10^{-6}\, k_v(f_1)\, \nu \left(\frac{k_v(f_2)}{k_v(f_1)} - \frac{k_d(f_2)}{k_d(f_1)} \right)$$

$$= 1.3 \times 10^{-4} \sim 1.7 \times 10^{-4} \ \mathrm{m^3/kg}$$

3. Calculation for elevation above 10°: Curvature effect K_1

$$S = R_1 + \nu(R_1 - R_2) + \underline{(\nu P_{21} - K_1)} + H_{21}\,SIWV$$

$$\nu P_{21} - K_1 \approx -\left(\nu\left[1 - \frac{k_d(f_2)}{k_d(f_1)}\right]^2 + 1\right) K_1$$

…depends only on K_1 (bending effect)…

Two-year mean & std of K_1

El (°)	K_1 (mm) Graz	TIGO
10	23.6 ± **0.7**	27.2 ± **0.6**
15	7.5 ± **0.2**	8.6 ± **0.2**

- K_1 does not vary significantly during a year
- A simple model relating K_1 with elevation angle can be easily developed
- The model is independent of time

3. Calculation for elevation above 10°: Integrated water vapor

$$S = R_1 + \nu(R_1 - R_2) + (\nu P_{21} - K_1) + H_{21}\,\underline{SIWV}$$

$$SIWV = \int \rho_v \, ds$$

…accuracy 6 kg/m²…

Results from optical delay modeling

- Zenith wet delay model and Mendes-Pavlis mapping functions (Mendes et al., 2002; Mendes & Pavlis, 2004)
- Ray-tracing through numerical weather model (Hulley & Pavlis, 2007)

Results from microwave measurements

$$SIWV \approx \frac{1}{10^{-6} k_3 R_v \left(\frac{1}{\overline{T}} + k_5 \right)} SWD_{microwave}$$

Techniques for obtaining $SWD_{microwave}$:

- Microwave measurements: GNSS, Water Vapor radiometer, VLBI
- ECMWF provided with VMF1 (Boehm et al., 2006)

4. The VMF1 for correcting optical delays of SLR measurements

Vienna Mapping Function (Boehm et al., 2006):

- The VMF1 was originally developed for correcting atmospheric propagation effects in microwave measurements
- The coefficients of the VMF1 as well as the (microwave) zenith hydrostatic and wet delays are determined 6-hourly

The VMF1 for correcting optical delays of two-frequency SLR measurements

$$SWD_{microwave} = ZWD_{microwave} \times VMF1$$

The VMF1 for correcting optical delays of single-frequency SLR measurements

$$SWD_{optic} = ZWD_{microwave} \times \boxed{C_v(f)}\ VMF1$$

$$SHD_{optic} = ZHD_{microwave} \times \boxed{C_h(f)}\ VMF1$$

- Scaling factors $C_v(f)$ and $C_h(f)$ depend on the dispersion constants of optical frequency
- The scaling factors can be determined empirically by ray-tracing
- Rigorous determination can be accomplished by reformulating the mapping functionformula as a function of the density and dispersion constants

5. Summary and conclusions

• A new atmospheric correction formula for two-frequency SLR measurements has been developed

•The new formula improves the previous formula by adding two terms for calculating curvature effects and water vapor distribution

• The curvature effect can be determined by a simple model

• The water vapor effect can be determined by results from optical delay modeling or from microwave measurements

• Requirement accuracy for two-frequency SLR measurements exceeds by far the capability of current state of the art SLR system

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