Global Glacial Isostatic Adjustment: Target Fields for Space Geodesy

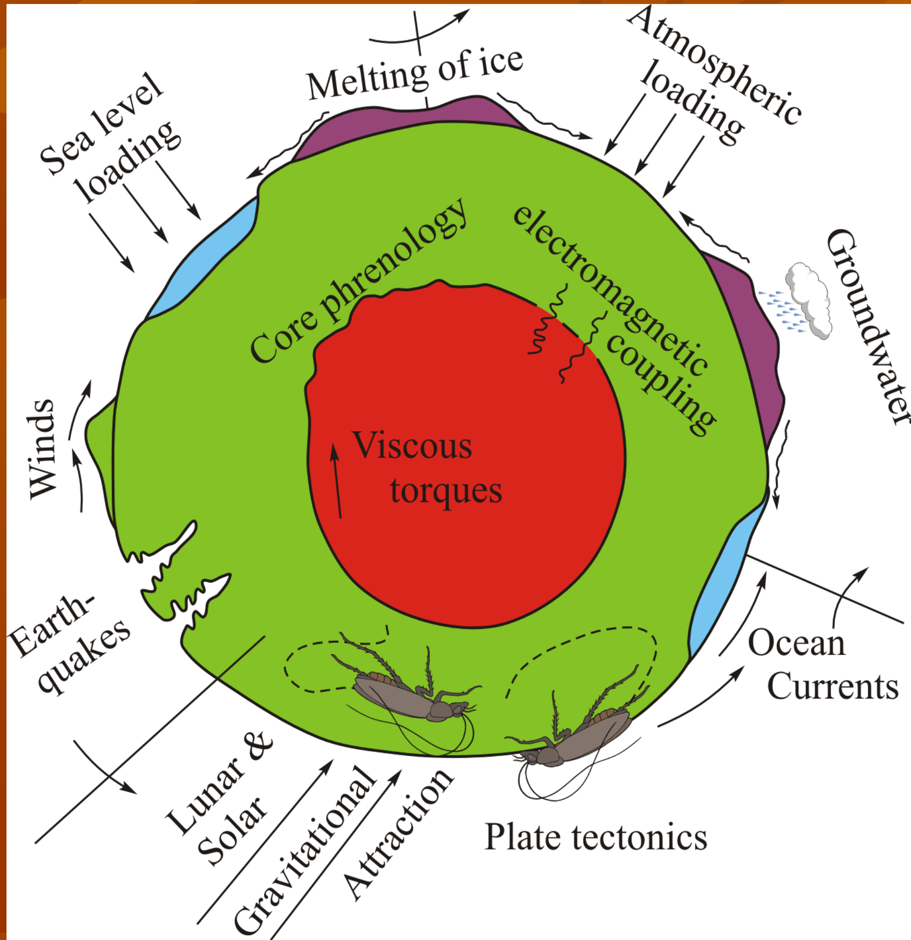
W.R. Peltier

Department of Physics

University of Toronto

Outline

- Global glacial isostasy and space geodesy: a fundamental geodynamic process
- Theoretical basis: the Sea Level Equation+models of glaciation
- The unique SLR contribution: time dependent Stokes coeff's of low degree—the non-tidal acceleration
- The VLBI+exILS++contribution: The process of true polar wander
- A challenge to the GIA explanation of the modern day rate and direction of true polar wander: A new test based upon "old" data



Theoretical Basis of the GIA Process

ROTATIONAL FEEDBACK IN THE SEALEVEL EQUATION

Because a change in rotational state is accompanied by a change in centrifugal potential and because sea level (msl) is constrained to lie on an equipotential, a change in rotational state will clearly induce a change in sea level.

∴ A Modified Sea Level Equation

$$S(\theta, \lambda, t) = C(\theta, \lambda, t)\Big[\int_{-\infty}^{t} dt' \int_{\Omega e}\int d\Omega' \Big\{ L(\theta', \lambda', t')\, G_\phi^L(\gamma, t-t') + \psi^R(\theta', \lambda', t')\, G_\phi^T(\gamma, t-t') \Big\} + \frac{\Delta\Phi(t)}{g}\Big]$$

Where, to first order in perturbation theory

$$\psi^R = \psi^{00} + \sum_{m=-1}^{+1} \psi_{2m}\, Y_{2m}(\theta, \lambda)$$

$$\psi_{00} = +\frac{2}{3}\,\omega_3\,\Omega_0\, a^2$$

$$\psi_{20} = -\frac{1}{3}\,\omega_3\,\Omega_0\, a^2\sqrt{4/5}$$

$$\psi_{21} = +(\omega_1 - i\omega_2)(\Omega_0 a^2/2)\sqrt{2/15}$$

$$\psi_{2-1} = -(\omega_1 + i\omega_2)(\Omega_0 a^2/2)\sqrt{2/15}$$

Geoid Height Time Dependence and the $\dot{J}_{\ell m}$

(1) Geoid Height Time Dependence (take G (θ, λ, t) = geoid height)

$\dot{G}(\theta, \lambda, t) = \dot{R}(\theta, \lambda, t) + \dot{R}SL(\theta, \lambda, t)$, R = local radius wrt com

RSL = sea level wrt solid surface

(2) $$\dot{G}(\theta, \lambda, t) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{+\ell} \dot{G}_{\ell m}\, Y_{\ell m}(\theta, \lambda)$$

(3) Definition of the $J_{\ell m}$

$$\dot{G}(\theta, \lambda, t) = -a \sum_{\ell=2}^{\infty} \sum_{m=0}^{\ell} (\dot{J}^1_{\ell m} \cos m\lambda + \dot{J}^2_{\ell m} \sin m\lambda)\, P_\ell^m(\cos\theta)$$

With the P_ℓ^m normalized such that

$$\int\!\!\int_{\text{unit sphere}} \left(P_\ell^m(\cos\theta) \begin{matrix}\cos m\lambda \\ \sin m\lambda\end{matrix}\right)^2 \sin\theta\, d\theta\, d\lambda = \frac{4\pi\,(\ell+m)\,!}{(2\ell+1)\,(\ell-m)\,!\,(2-\delta_{om})}$$

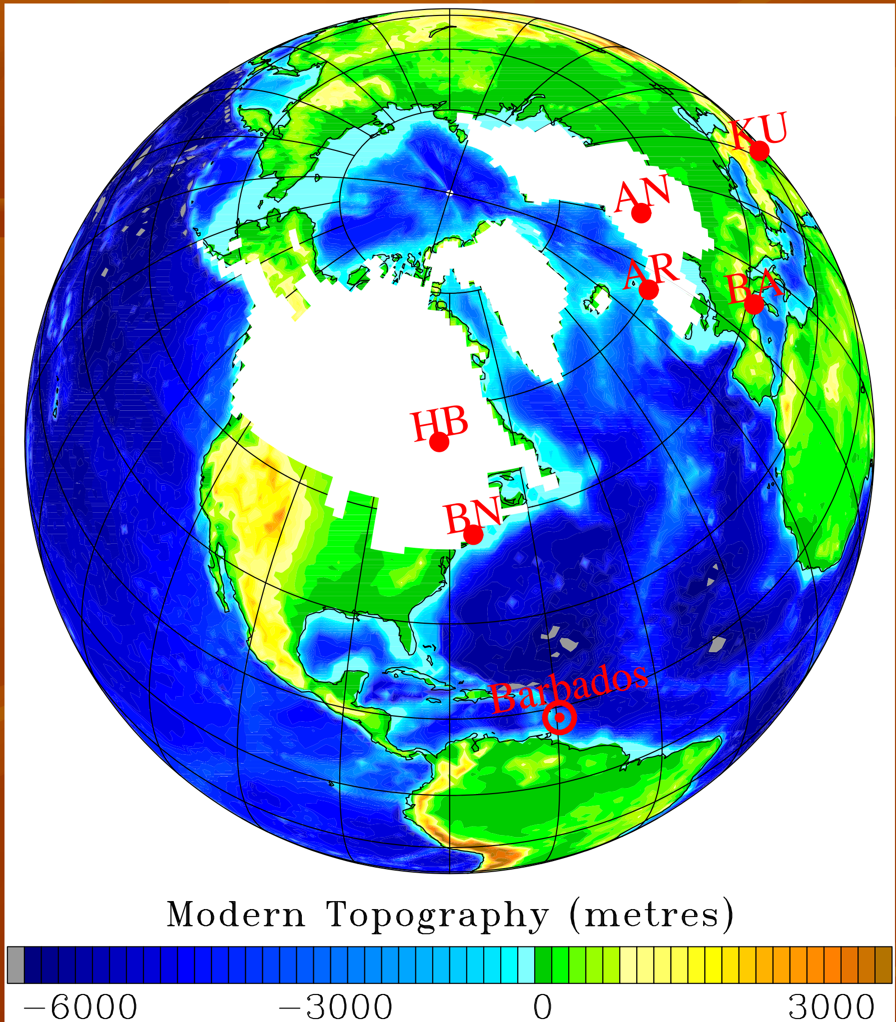
(4) Thus $$\dot{J}^1_{\ell m} = \frac{-(2-\delta_{om})}{a}\sqrt{\frac{(2\ell+1)\,(\ell-m)\,!}{(\ell+m)}}\ \mathrm{Re}\,[\dot{G}_{\ell m}]$$

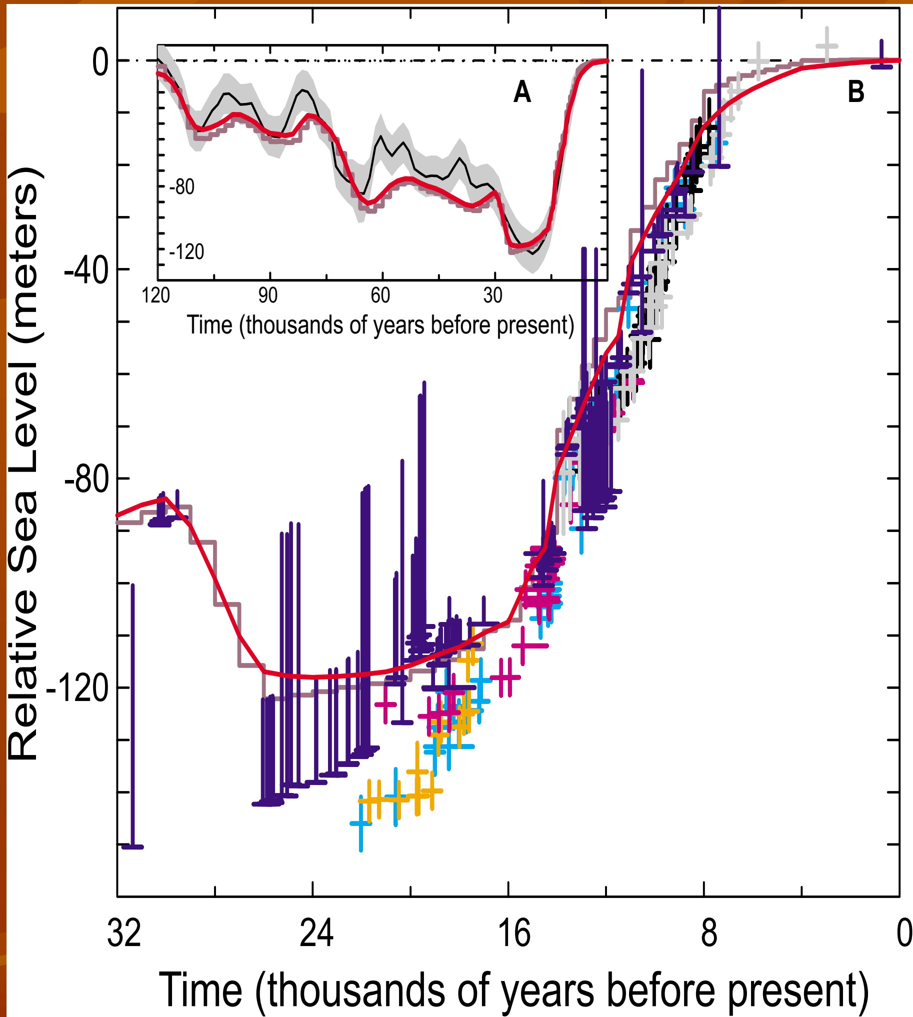
$$\dot{J}^2_{\ell m} = \frac{2}{a}\sqrt{\frac{(2\ell+1)\,(\ell-m)\,!}{(\ell+m)}}\ \mathrm{Im}\,[\dot{G}_{\ell m}]$$

(5) Variance Spectra

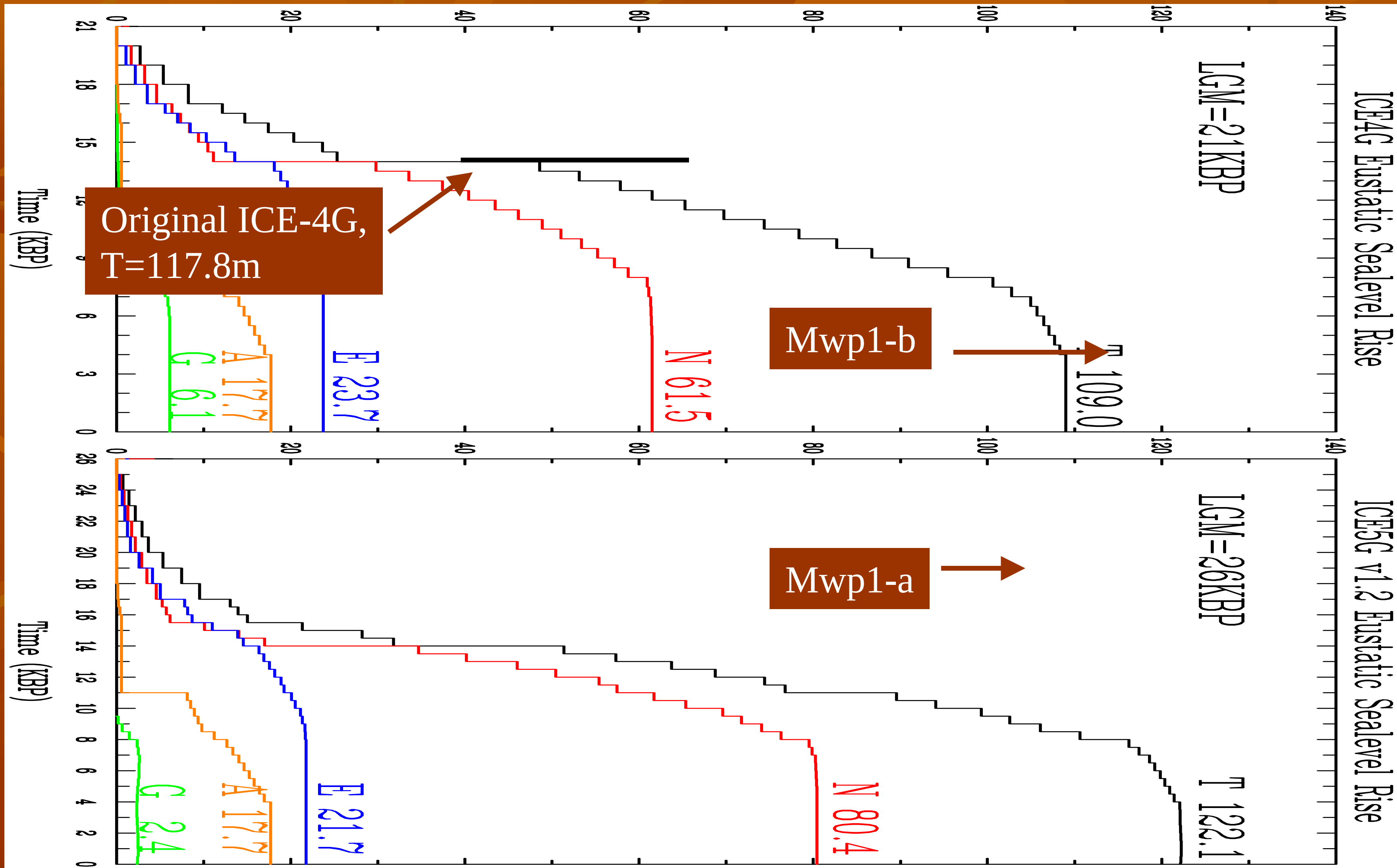
$$\sigma_{\dot{J}}^2 = \sum_{m=0}^{\ell} [(\dot{J}^1_{\ell m})^2 + (\dot{J}^2_{\ell m})^2]$$

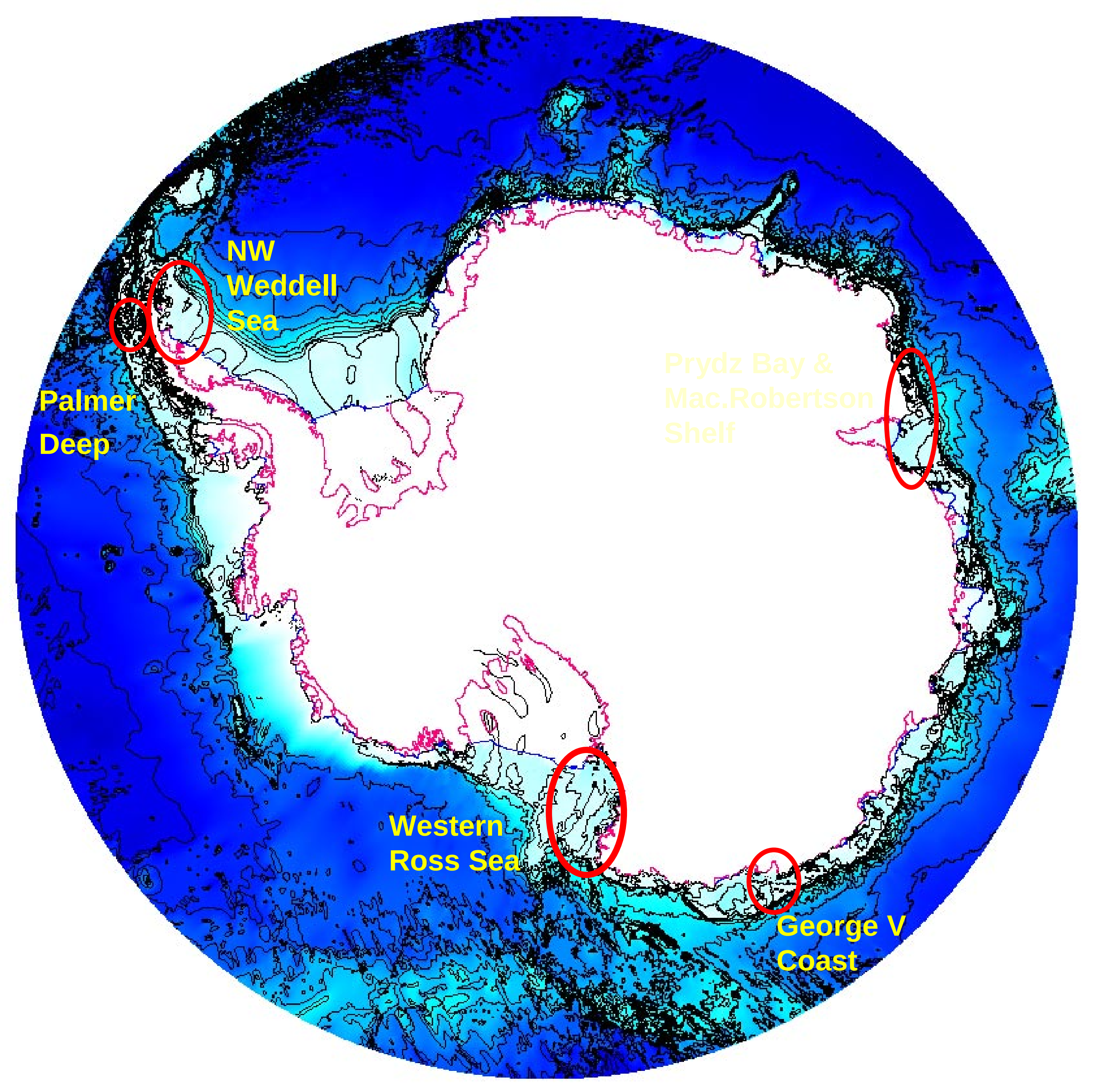
Models of Glaciation History---ICE-5G(VM2)





ICE-4G/5G Eustatic Intercomparison





NW Weddell Sea
Palmer Deep
Prydz Bay & Mac.Robertson Shelf
Western Ross Sea
George V Coast

Leventer et al., in prep.

Domack et al., 1998 (Ant. Sci.)

Leventer et al., in prep.

Domack et al., 1999 (GSA Bulletin)

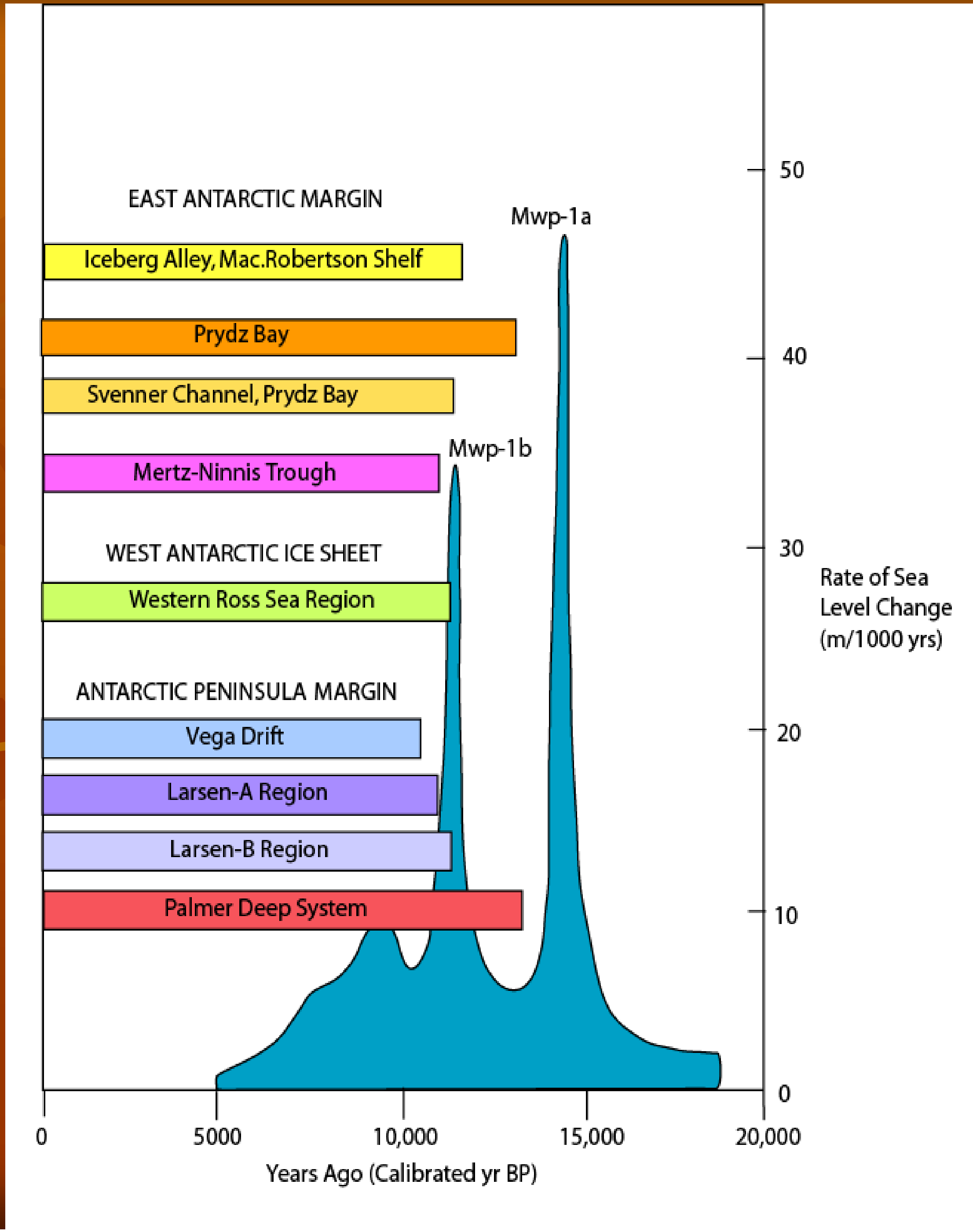
Domack et al., in prep.

Brachfeld et al., 2003 (Geology)

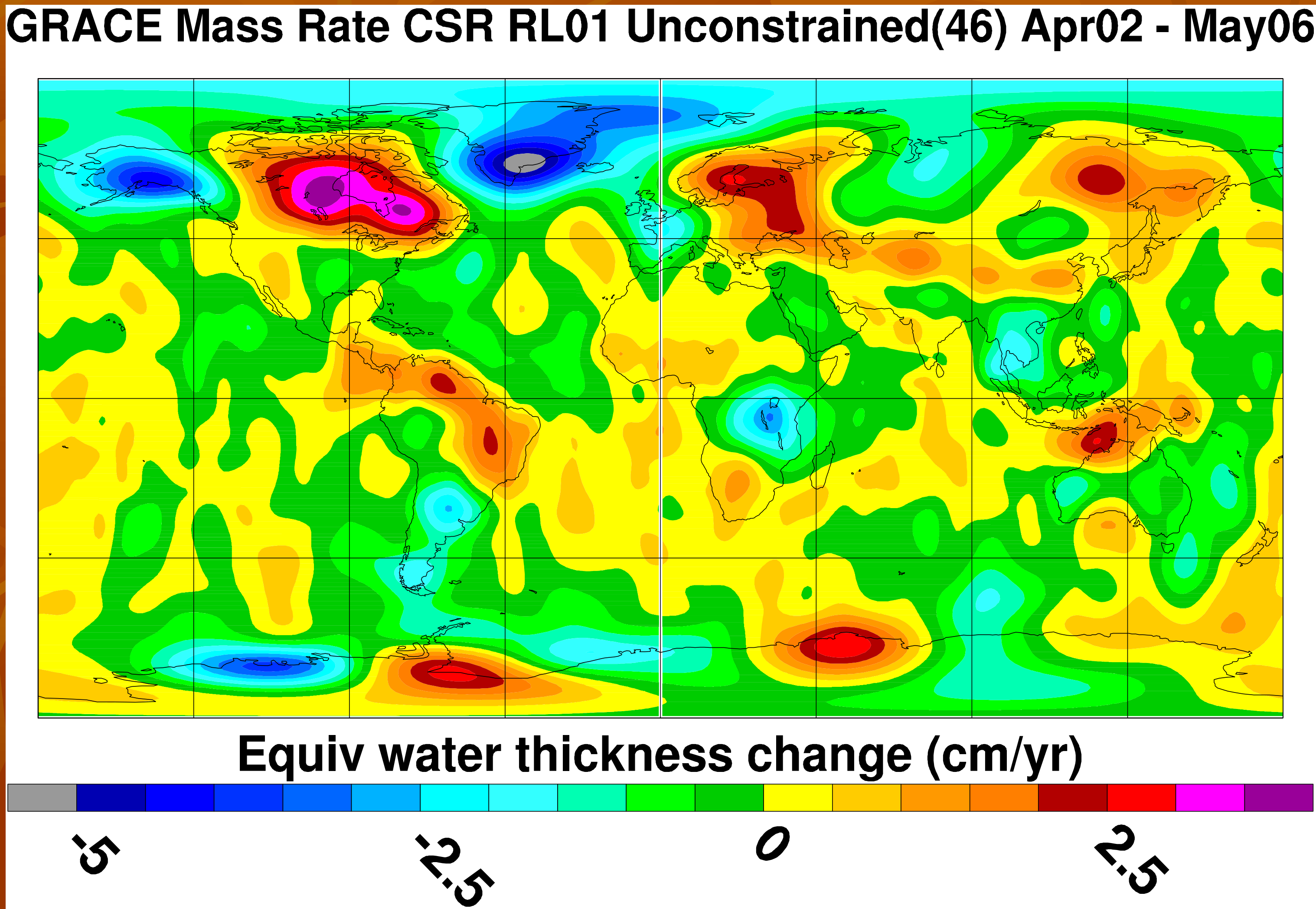
Domack et al., 2005 (Nature)

Domack et al., 2001 (Holocene)

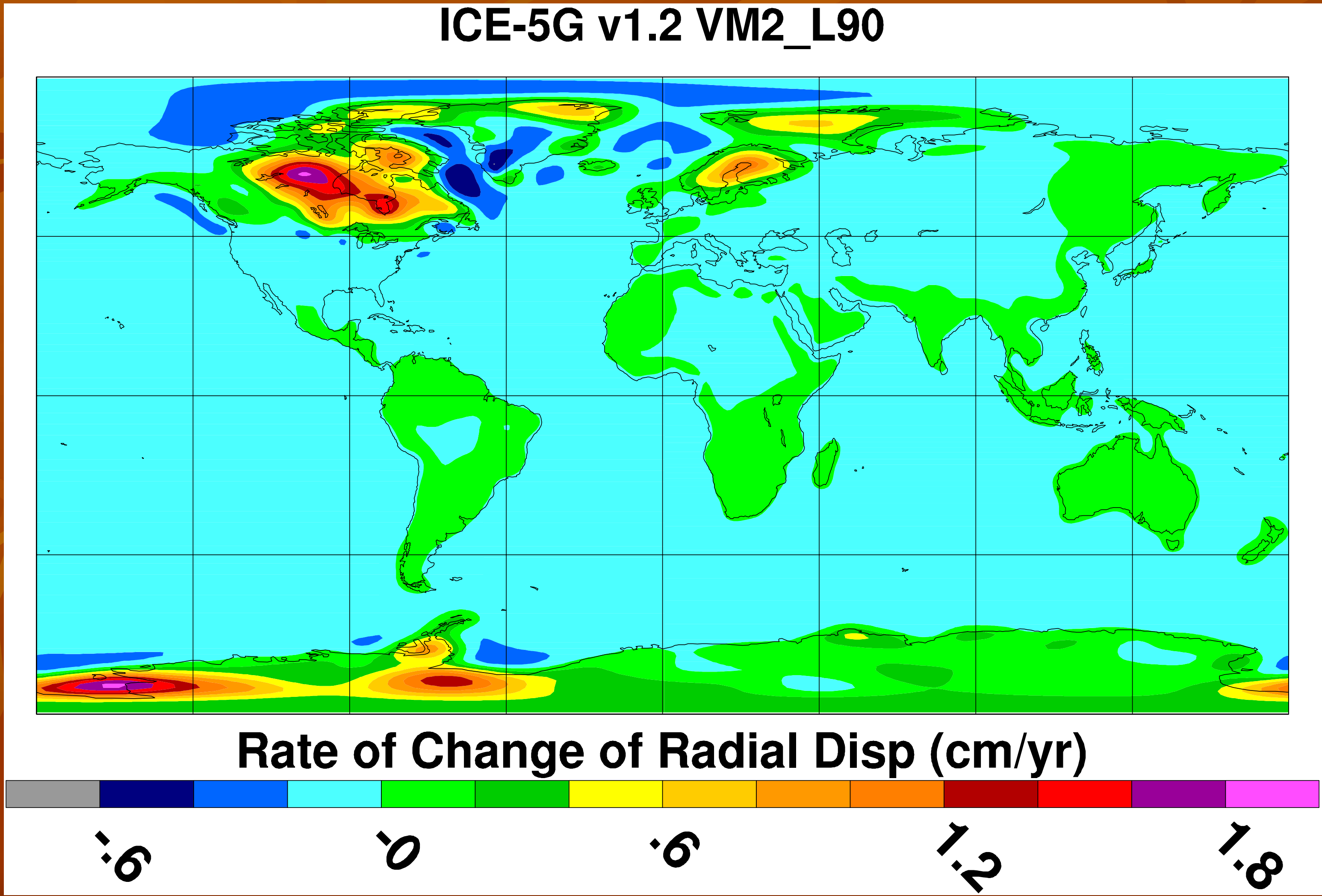
Domack et al. 2005 (Geomorphology)



A GRACE test of the Validity of ICE-5G(VM2)



Rate of radial displacement predicted by ICE-5G(VM2)



Computation of the Rotational Response of the Earth to the GIA Process

(1) The angular velocity vector with components ω_j is determined by a soln. to the classical Euler equation in which J_{ij} is the moment of inertia tensor

$$\frac{d}{dt}\left(J_{ij}\omega_j\right)+\varepsilon_{ijk}\,\omega_j J_{kl}\,\omega_l = 0$$

(2) Because we are interested in small departures from the modern state of steady rotation with angular velocity Ω ,we may employ first order perturbation theory to construct a solution for the GIA forced modification to the steady background solution, by employing the expansion:

$$\begin{aligned}
\omega_i &= \Omega\,(\delta_{i3} + m_i) \\
J_{11} &= A + I_{11} \\
J_{22} &= B + I_{22} \\
J_{33} &= C + I_{33}
\end{aligned}$$

$$J_{ij} = I_{ij}, i \neq j$$

(3) Substitution into the Euler equation and linearization leads to the three governing equations for the m_j , namely

$$\left.\begin{aligned} &\frac{dm_1}{dt} + \frac{(C-B)}{A}\Omega m_2 = \Psi_1 \\ &\frac{dm_2}{dt} - \frac{(C-A)}{B}\Omega m_1 = \Psi_2 \end{aligned}\right\} \text{Polar wander}$$

$$\left.\frac{dm_3}{dt} = \Psi_3 \right\} \text{Length of day}$$

(4) Where the so-called "excitation functions" are given by:

$$\Psi_1 = \left(\frac{\Omega}{A}\right)I_{23} - \left(\frac{dI_{13}/dt}{A}\right)$$

$$\Psi_2 = -\left(\frac{\Omega}{B}\right)I_{13} - \left(\frac{dI_{23}/dt}{B}\right)$$

$$\Psi_3 = -\left(\frac{I_{33}}{C}\right)$$

(5) It is critical to recognize that there exist perturbations to the inertia tensor due to 2 distinct causes, namely due to the direct influence of the GIA process AND due to the deformation induced by the changing rotation. These may be expressed as:

(a) Due to the direct influence of GIA

$$I_{ij} = \left(1 + k_2^L\right) * I_{ij}^R$$

, in which $*$ indicates convolution in time

(b) Due to the rotation induced deformation, from MacCullagh's formula

$$I_{13}^{Rot} = \left(\frac{k_2^T * a^5 \omega_1 \omega_3}{3G}\right) = \left(\frac{k_2^T}{k_f}\right) * m_1 (C - A)$$

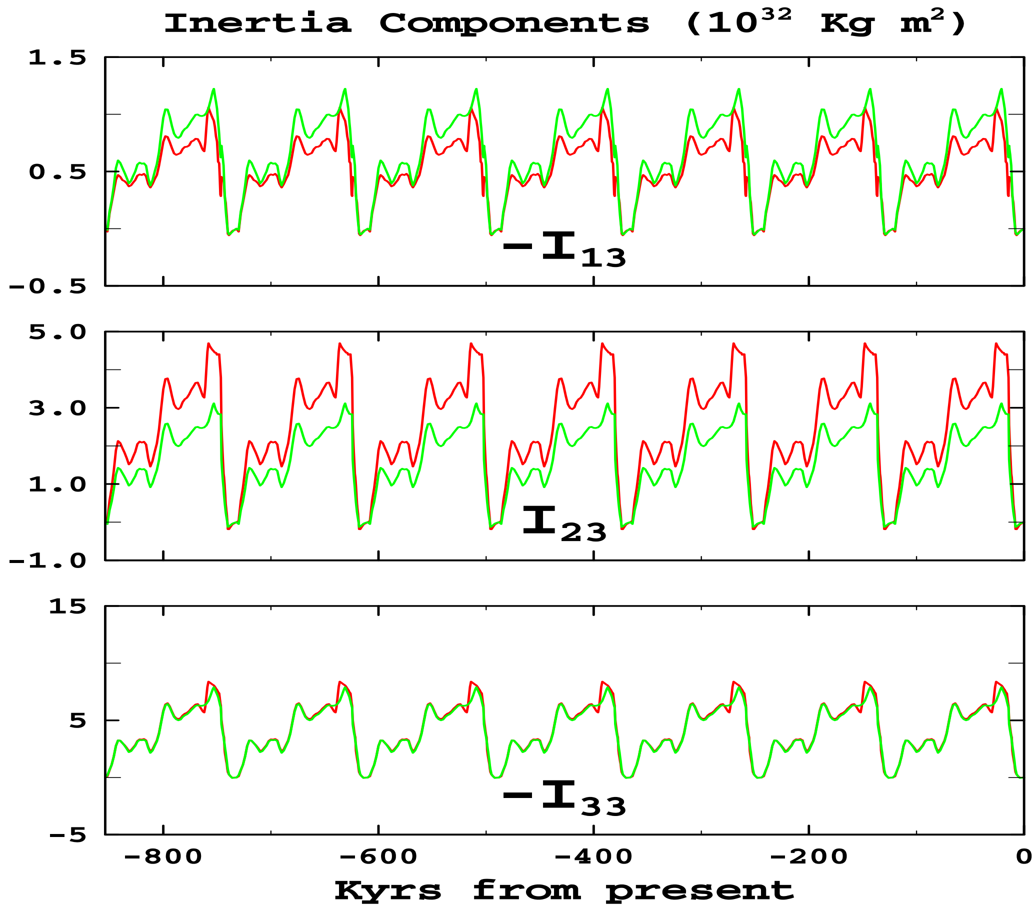
$$I_{23}^{Rot} = \left(\frac{k_2^T * a^5 \omega_2 \omega_3}{3G}\right) = \left(\frac{k_2^T}{k_f}\right) * m_2 (C - A)$$

Assuming A=B

$$k_f = \left(\frac{3G}{a^5 \Omega^2}\right) (C - A)$$

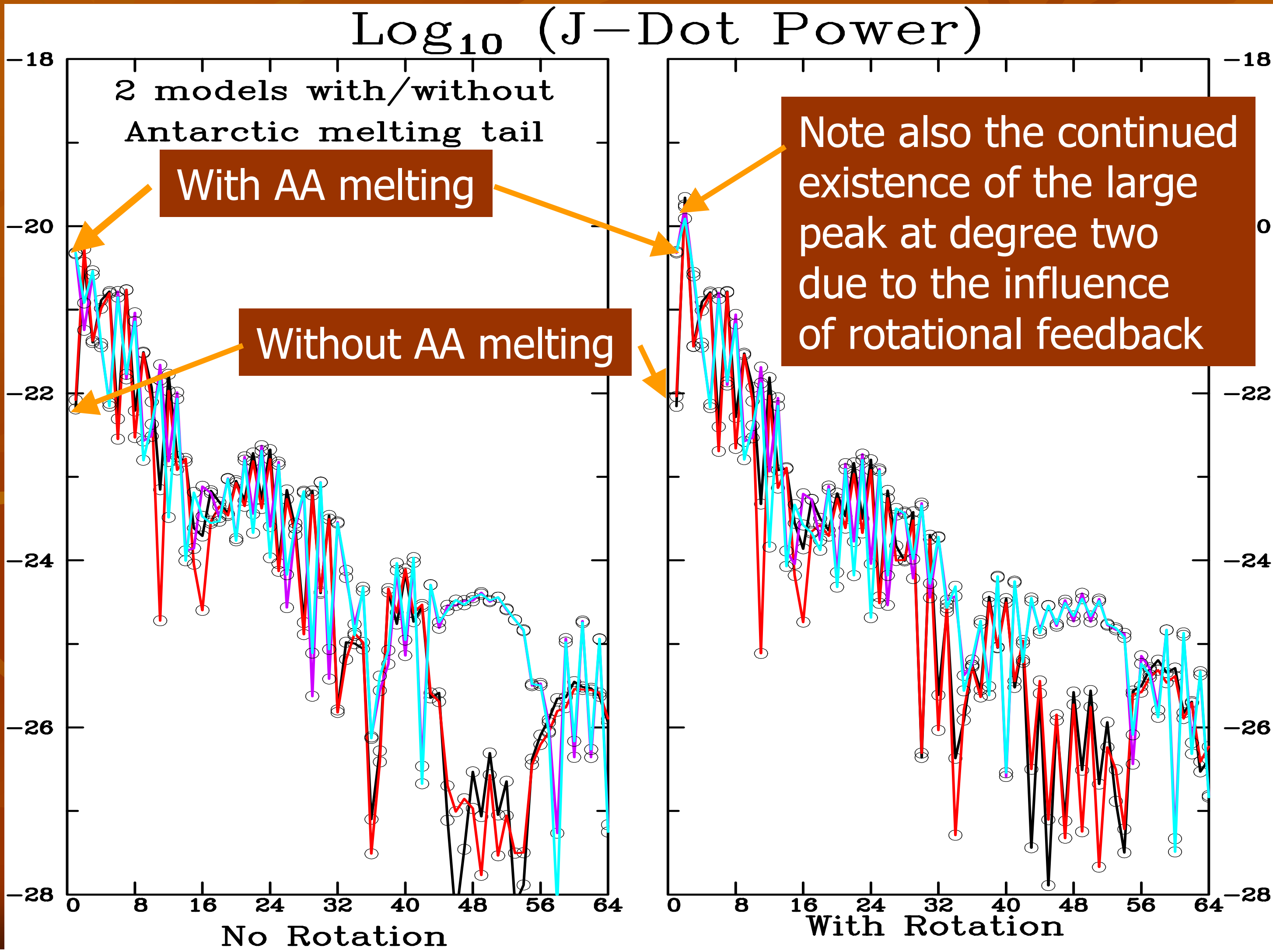
This is the traditional fluid Love number based upon the observed oblateness!! It it ~0.9382

Inertia perturbations for the ICE-4G (green) and ICE-5G(red) models of the global process of glacial isostatic adjustment. The calculations to be discussed are based upon the assumption that 7, 100,000 year cycles of glaciation have occurred during Late Pleistocene time.

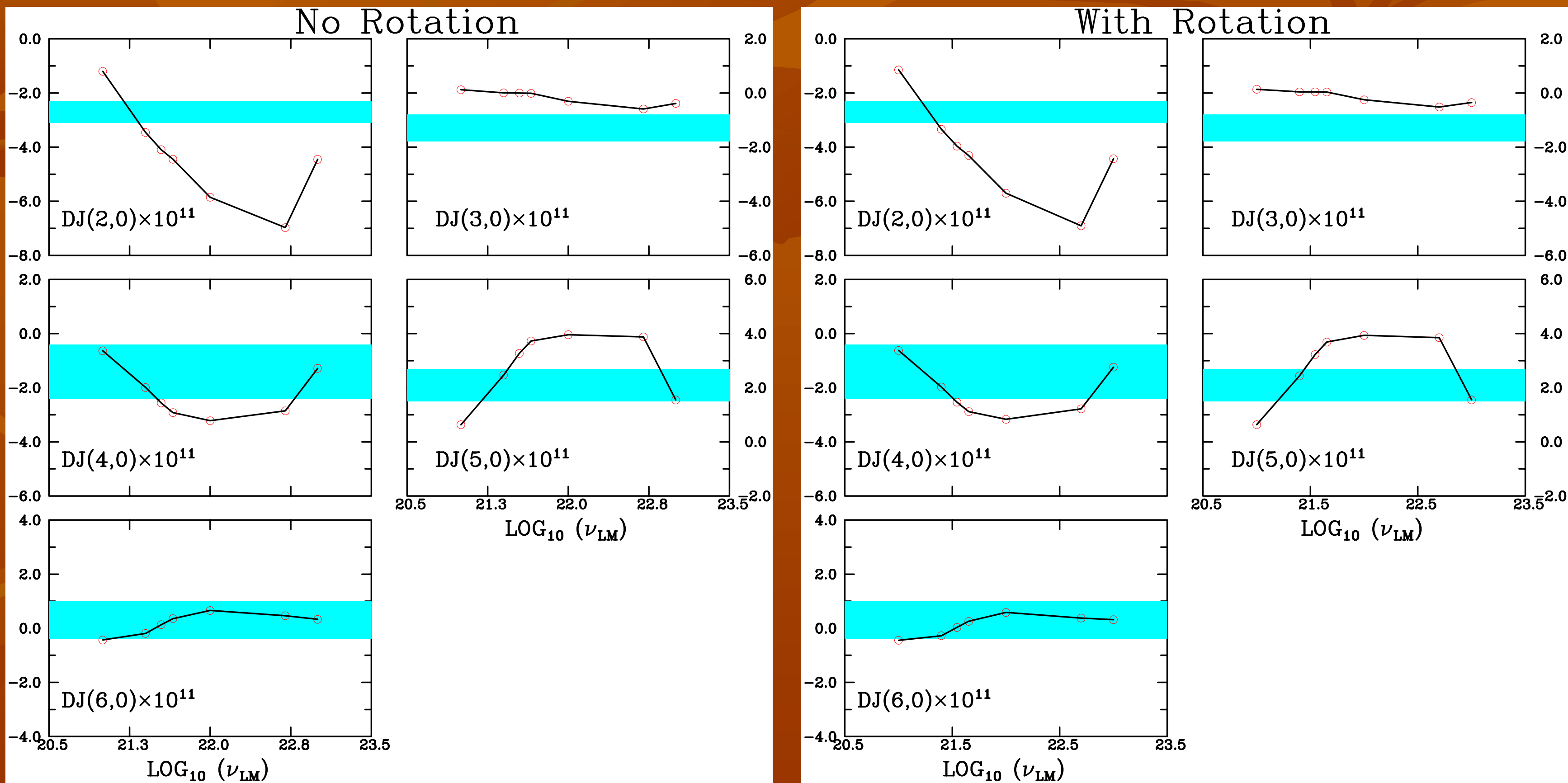


Power spectra of geoid height time dependence-with and without an Antarctic melting tail of global strength 1 mm per year

Note the very Large amplitude Of the peak in the Spectrum at Spherical Harmonic degree One that an Expected Consequence of AA melting at a Rate sufficient to Cause global sea Level to rise at a Rate of 1 mm per Year.



It is useful to compare the complete set of low order zonal Jn-dots to observations based upon Satellite Laser Ranging



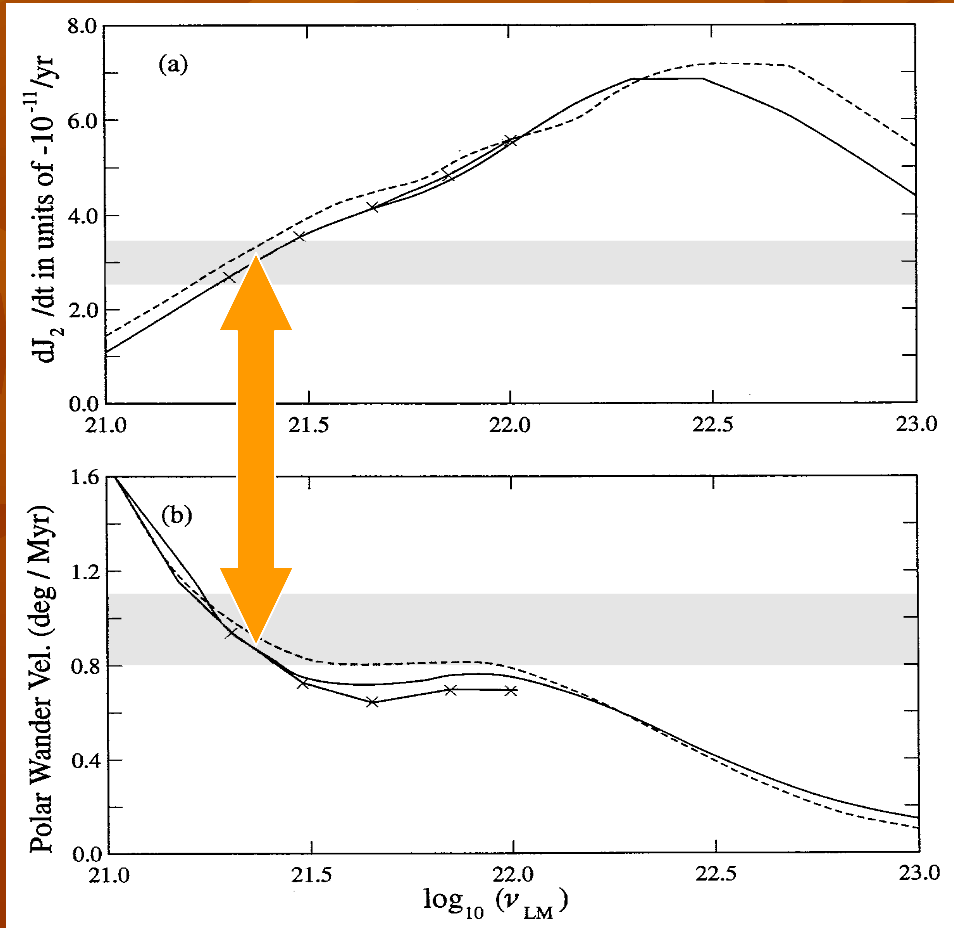
Theory employs a 1 cycle decription of the loading history. The data is from the paper by Cheng,Shum and Tapley, JGR,vol.102,no. B10, 22377-22390, 1997.Note that there is essentially no influence of rotational feedback on the low order zonal coefficients.

All of these results serve to reinforce the validity of the earlier analyses of the planet's rotational response to GIA forcing

In this earliest work based upon the assumption

$$k_f = k_2^T(s=0)$$

It was argued that the assumptions on the basis of which the theory was developed, specifically the above, was correct on the basis of the fact that BOTH rotational observables were fit by the same model of the radial viscoelastic structure.



(8) The polar wander component of the solution in the Laplace transform domain is then simply:

$$m_1(s) = \left[\frac{1+k_2^L(s)}{s^2+\sigma^2(1+k_2^T(s)/k_f)^2}\right]\left[(\frac{\Omega\sigma}{A})(1-k_2^T(s)/k_f)-s^2/A\right]I_{13}^{Rigid}(s)$$

$$m_2(s) = \left[\frac{1+k_2^L(s)}{s^2+\sigma^2(1+k_2^T(s)/k_f)^2}\right]\left[(\frac{\Omega\sigma}{A})(1-k_2^T(s)/k_f)-s^2/A\right]I_{23}^{Rigid}(s)$$

Neglecting terms of order s^2/σ^2 the solutions become

$$m_1(s) = (\frac{\Omega}{A\sigma})\left[\frac{1+k_2^L(s)}{1-k_2^T(s)/k_f}\right]I_{13}^{Rigid}(s) = H(s)I_{13}^{Rigid}(s)$$

$$m_2(s) = H(s)I_{23}^{Rigid}(s)$$

The Equivalent Earth Model Approach of Munk and MacDonald

In the denominator of the expression for the frequency domain impulse Response H(s) there appears the factor:

$$1 - k_2^T(s)\big/k_f = 1 - k_2^T(0)\big/k_f + \left(\frac{s}{k_f}\right)\sum_{j=1}^{N}\frac{t_j/s_j}{s+s_j}$$

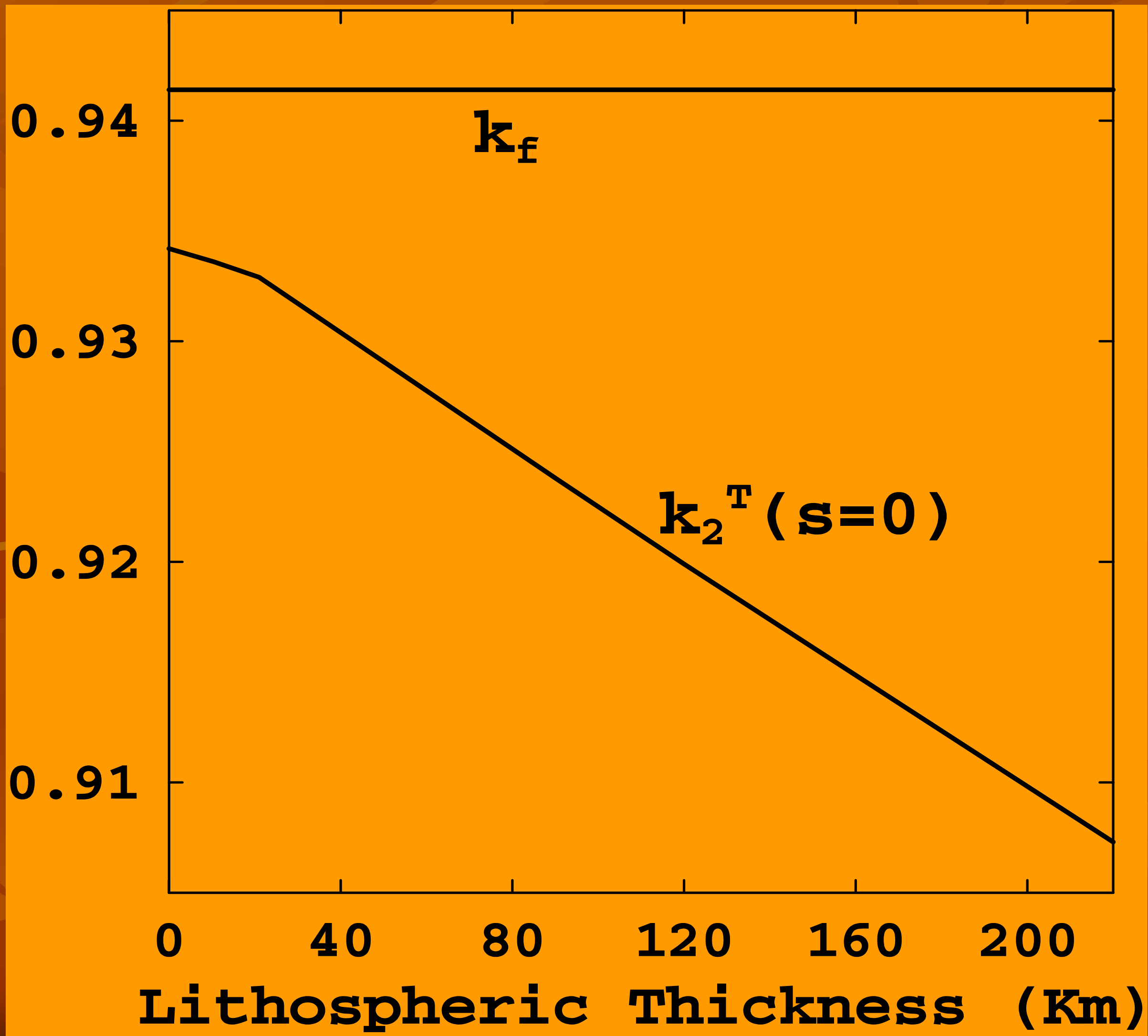
In the EqEM approach we adjust the properties of the visco-elastic model by Assuming:

$$1 - \frac{k_2^T(0)}{k_f} \approx 0$$

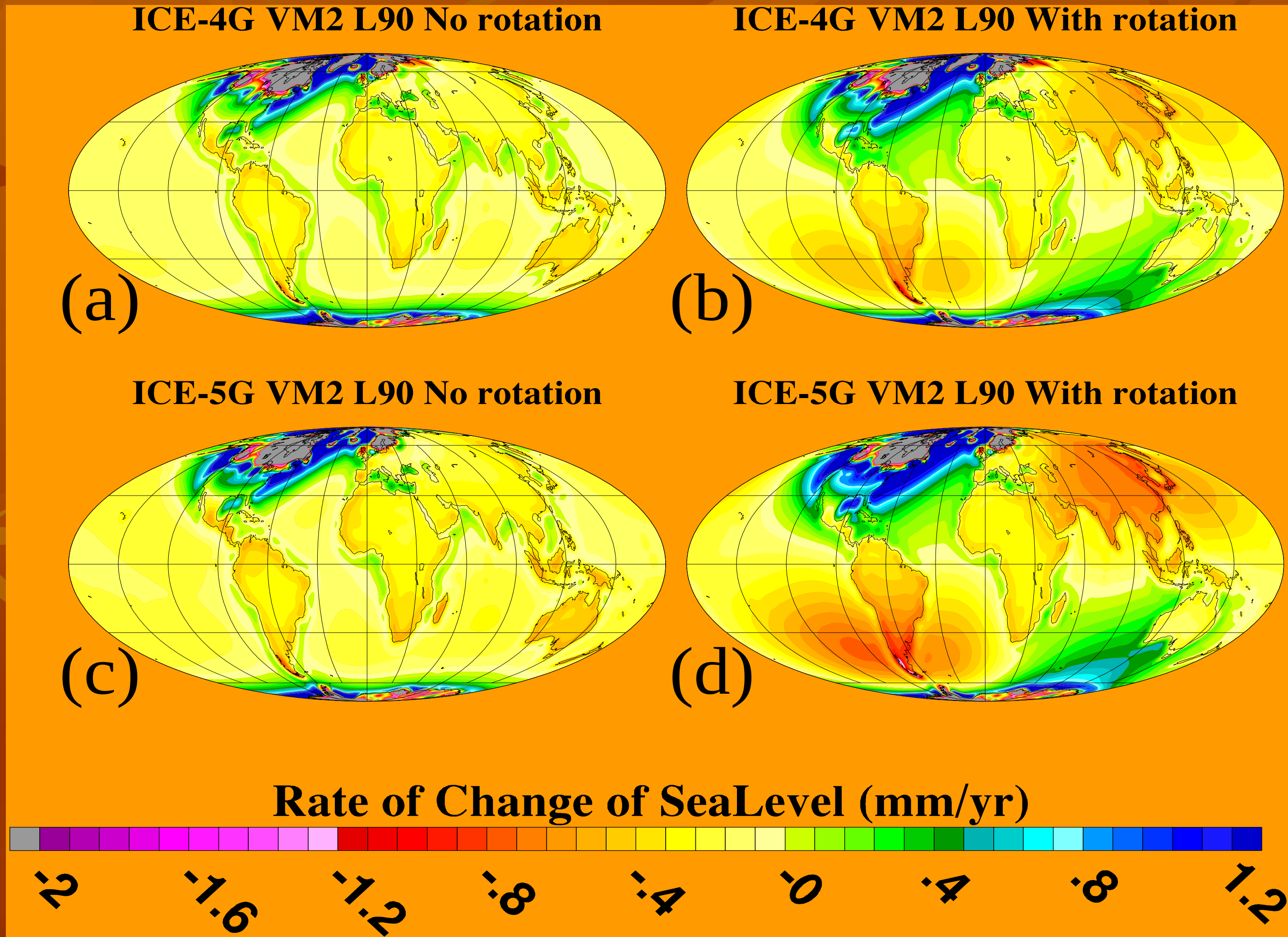
In general this quantity will be a parameter of the model. For the ICE-5G(VM2) model the quantity differs from zero by <1% if it is (reasonably) assumed that the effective thickness of the lithosphere may be assumed to vanish in the limit of long time insofar as the degree 2 tidal deformation is concerned.

The "fluid Love number" is determined by the observed flattening of Earth's figure

Note that in the infinite time limit the value of the tidal Love number of degree 2 approaches the value of the fluid Love number as the lithospheric thickness approaches zero. In this limit the error is less than 1%!



The Present Day Rate of relative Sea Level Rise for 4 Models of the GIA Process



It is important to note the following properties of the ICE-4G(VM2) and the ICE-5G(VM2) models insofar as their rotational response is concerned to the forcing associated with the GIA process.

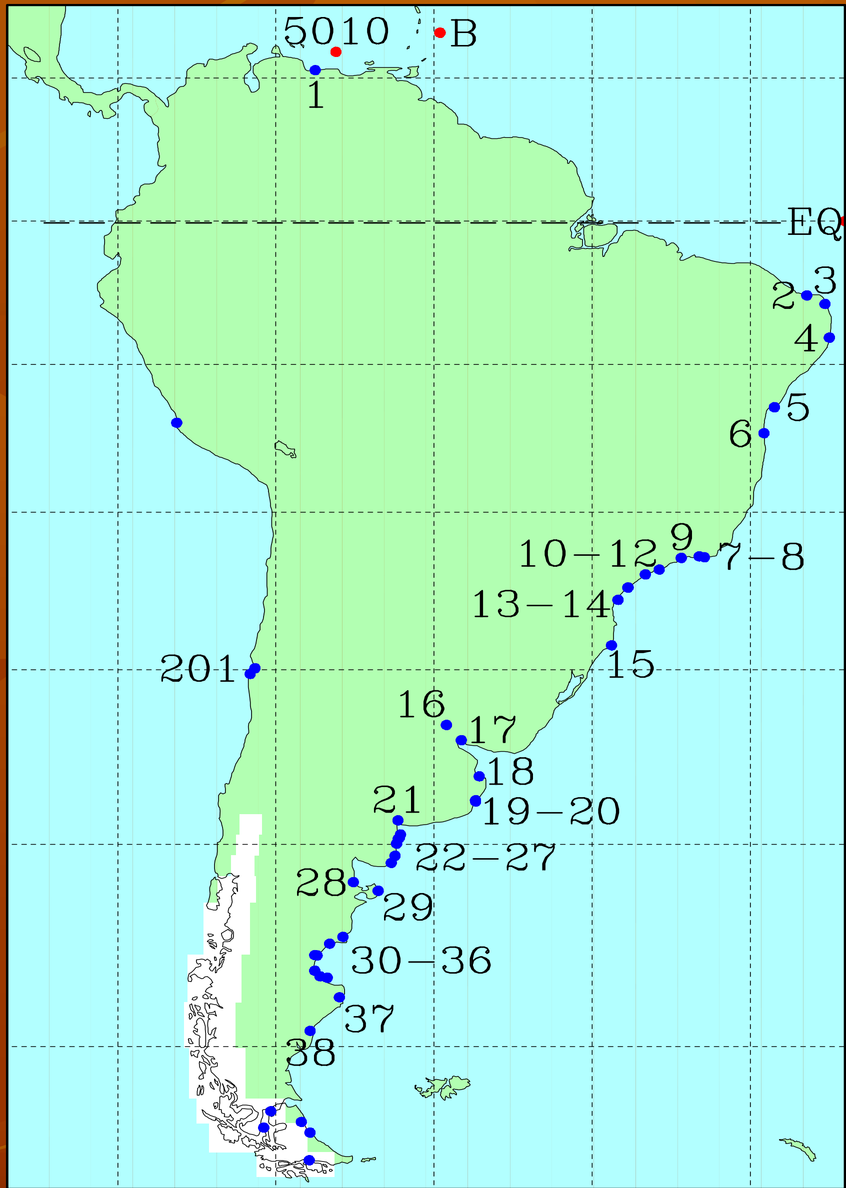
	Ice-4G(VM2)	ICE-5G(VM2)	Observed
TPW speed-no rot.	.810 deg/Ma	1.06 deg/Ma	0.8-1.1 deg/Ma
TPW direction-no rot.	69.9 deg WLong	83.1 deg WLong	~76 deg WLong
TPW speed-rot.	.57 deg/Ma	.72 deg/Ma	0.8-1.1 deg/Ma
TPW direction-rot.	73.4 deg WLong	88.8 deg WLong	~76 deg WLong

Note: the TPW speed delivered by the ICE-4G(VM2) model is somewhat too slow and the direction is too far to the east.

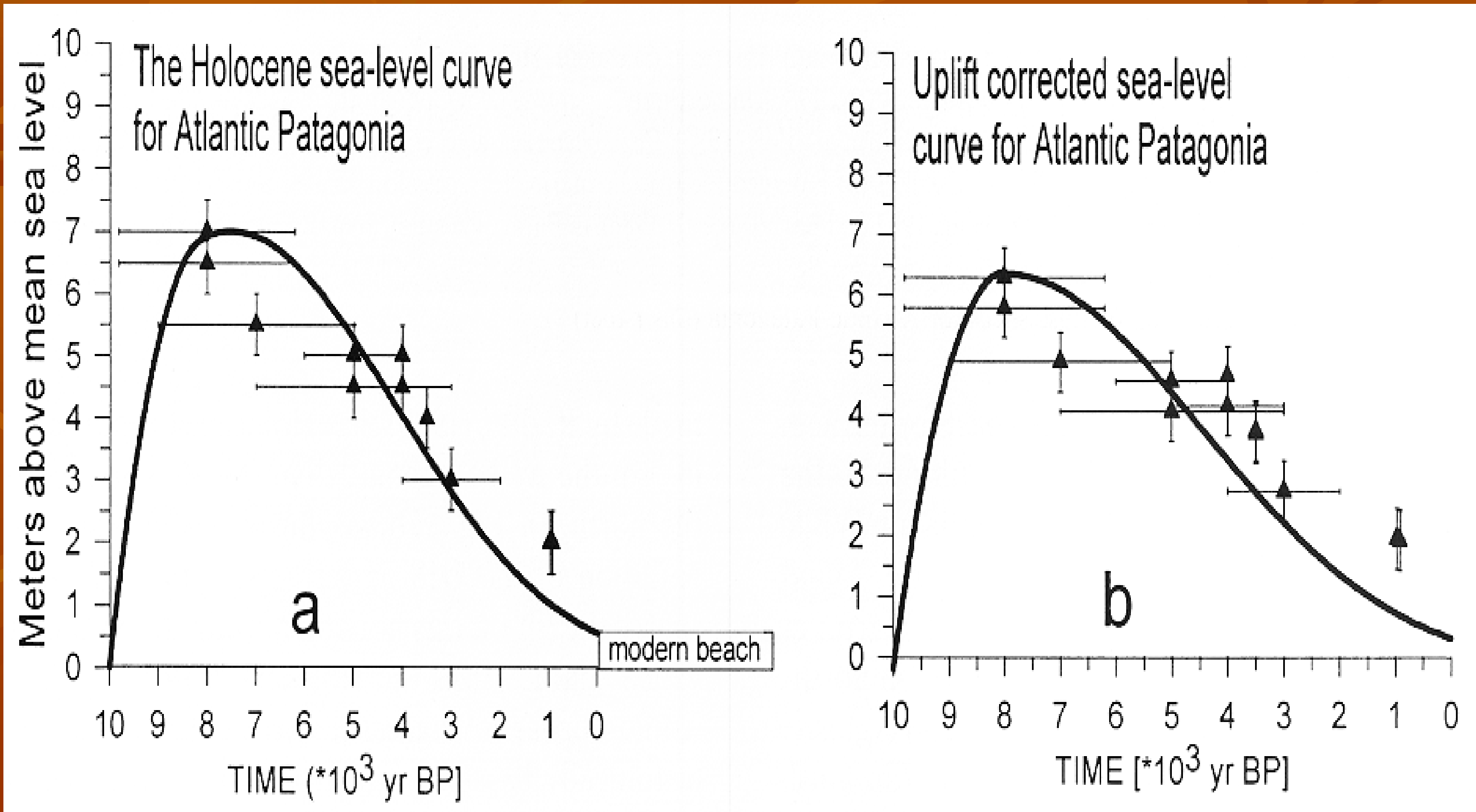
- Note also that the TPW direction delivered by the ICE-5G(VM2) model is shifted too far to the west! although the speed is about right.

The use of rsl data to confirm the validity of the rotational response theory

It is fortunate that we have available a detailed set of relative sea level data, from the entire east coast of the South American continent that provides a traverse from north to south through one of the 4 “bulls-eyes” in the degree two and order one pattern produced by the influence of earth rotation upon relative sea level history. These data were published by Rostami, Peltier and Mangini (QSR 19, 1459-1525, 2000)

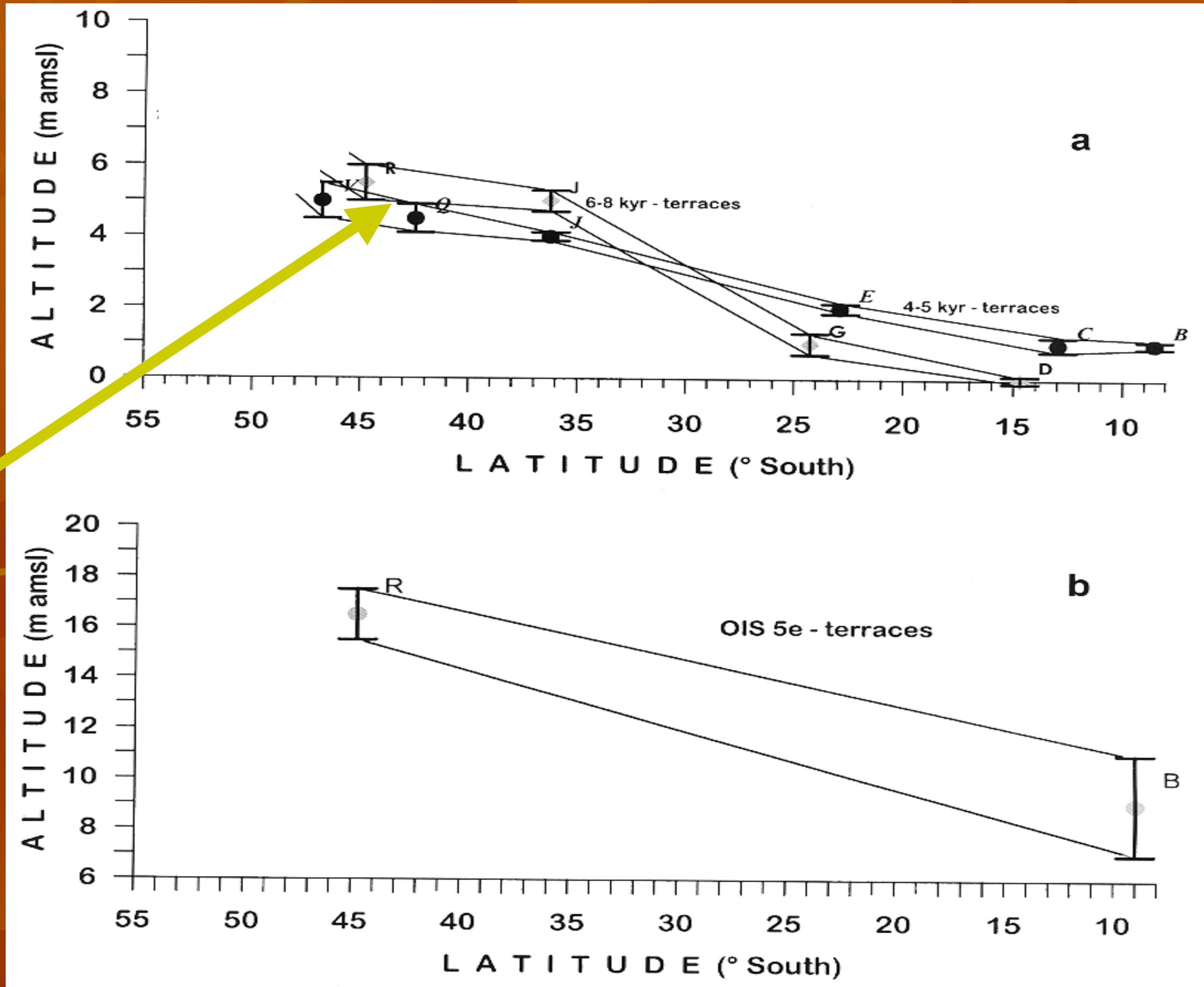


Sea level histories along the east coast of the South American continent reveal the presence of a mid-Holocene high stand of the sea, a feature that is found to be characteristic of the sea level record on most coastlines at sites in the far field of the Pleistocene ice sheets

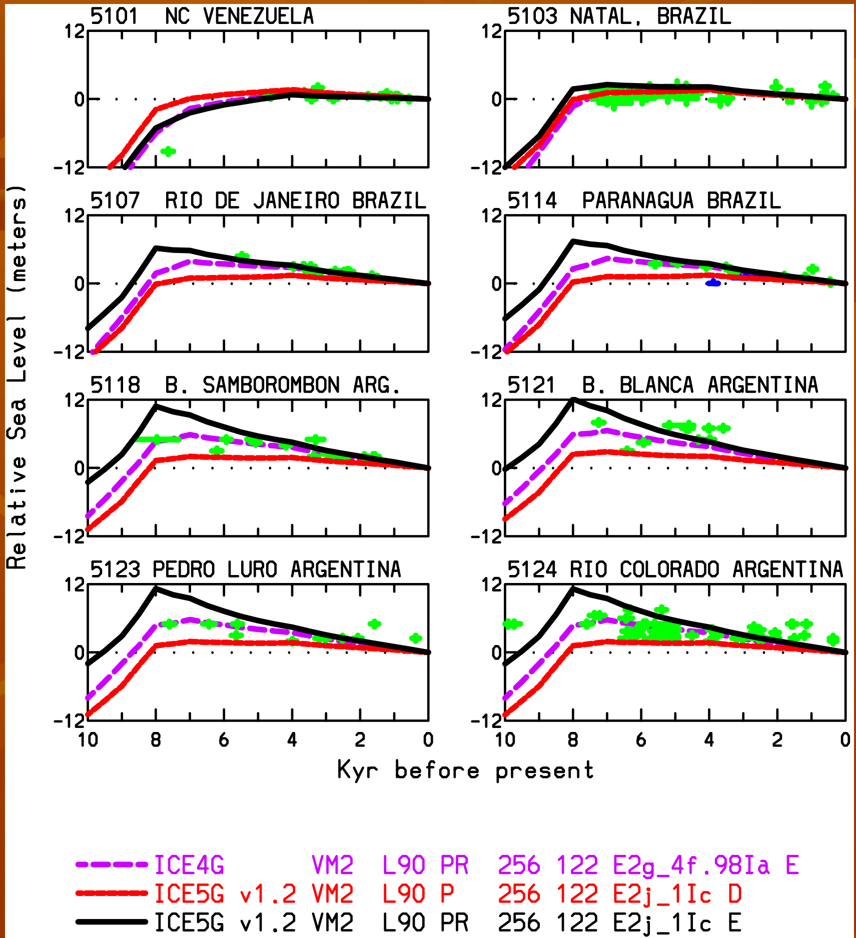


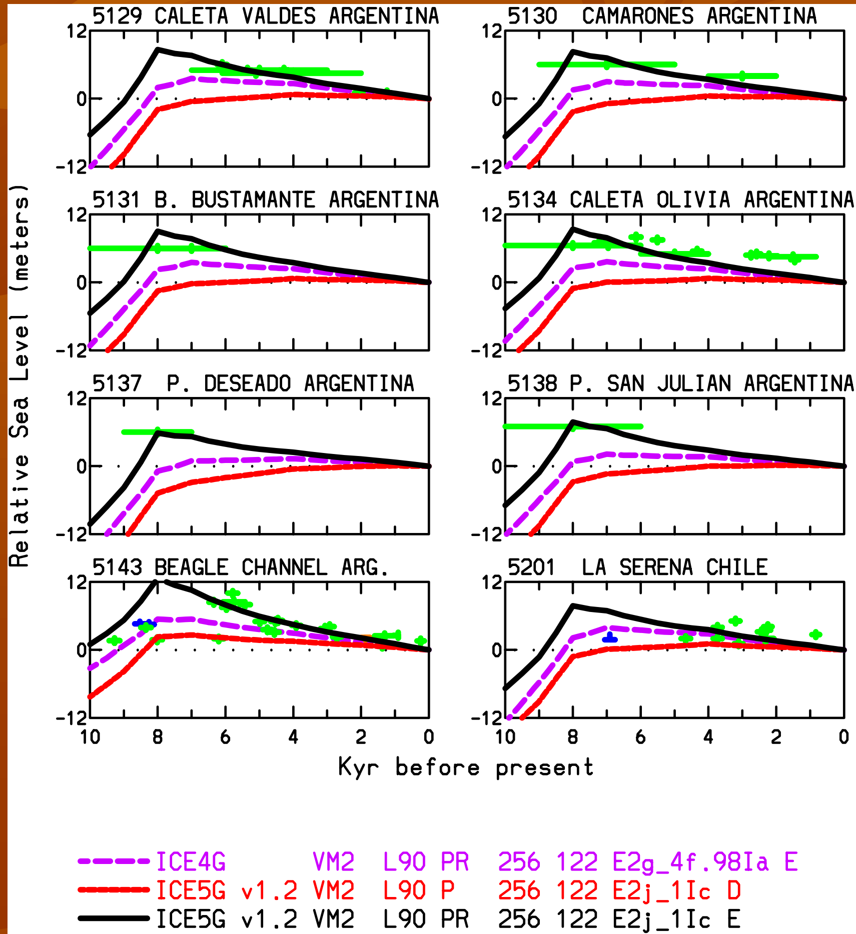
The primary observation reported in Rostami et al was of an extremely "odd" systematic increase with latitude of the elevation of the mid-Holocene high stand of the sea as a function of increasing southern latitude.

According to the Rostami et al analysis the mid-Holocene high stand increases in elevation from the north to the south by approximately 6m. This is the signal that provides us with a means to test the validity of the theory of rotational feedback.

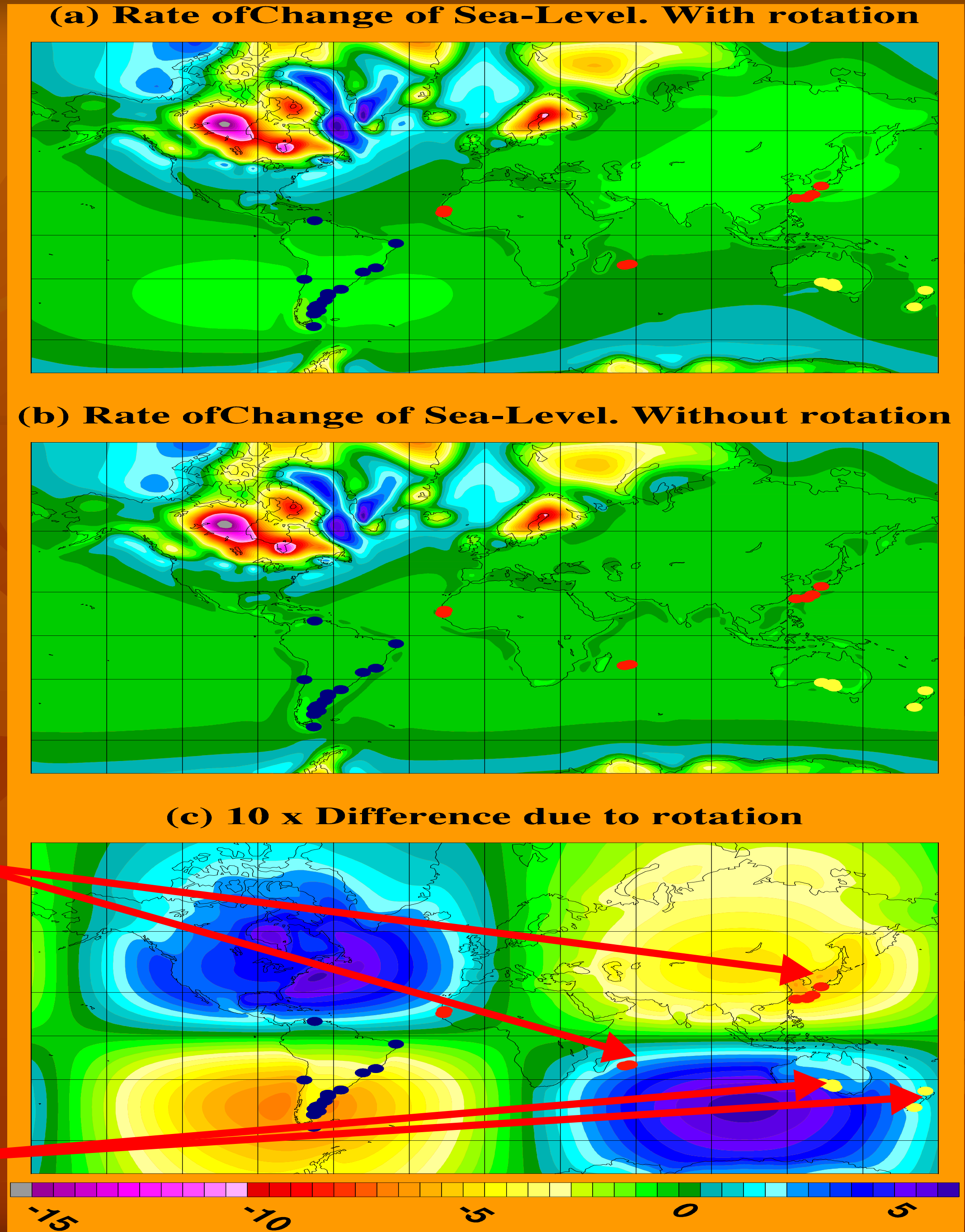


The east coast passive margin of South America





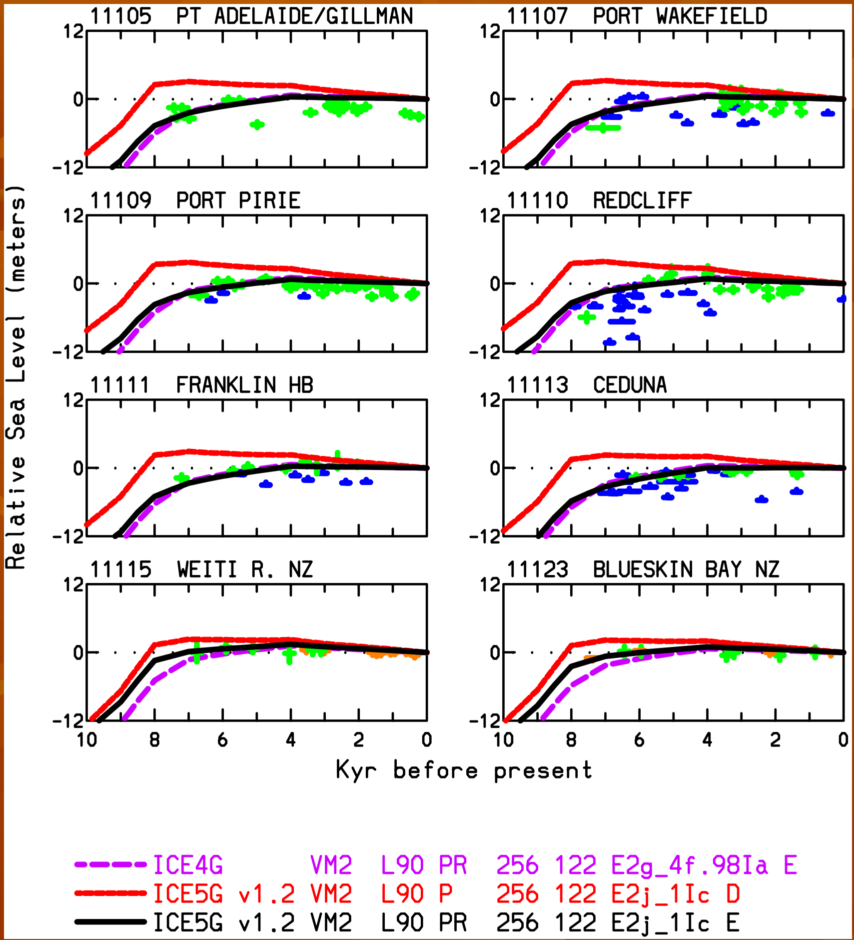
Further tests of the valididdty of the EQM hypothesis based upon RSL histories from the polar wander “bullseye’s”

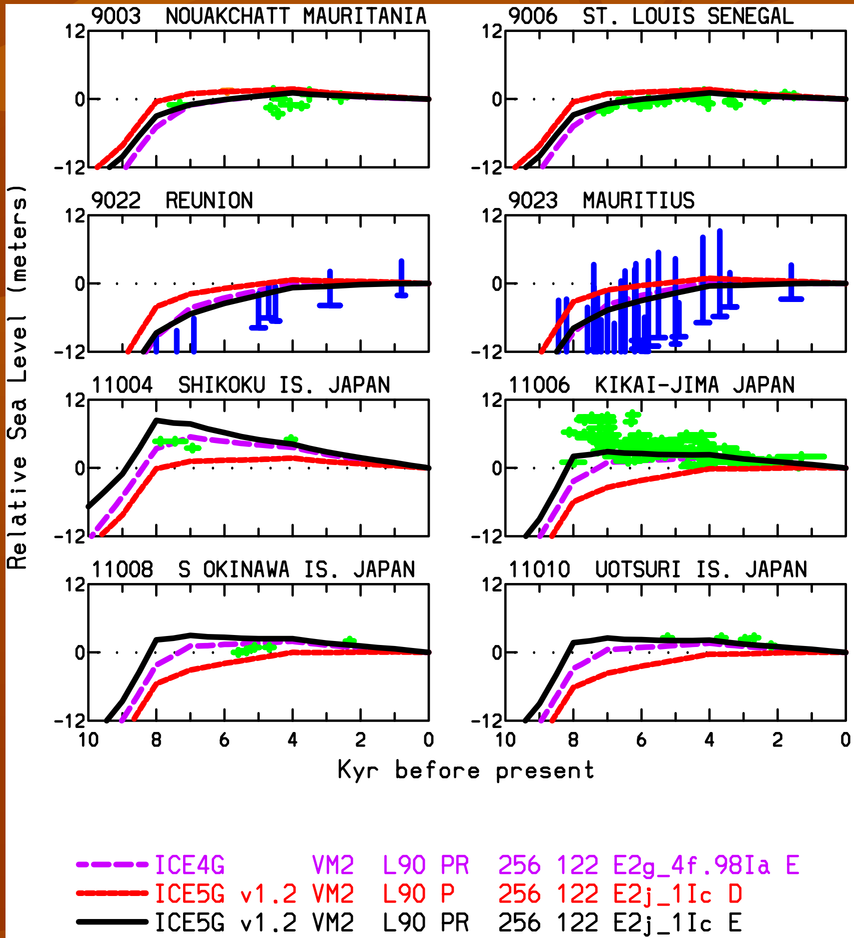


The Japanese Islands, Western Africa and the Indian Ocean

Australia-New Zealand

Data from the other extrema of the degree 2 and order 1 pattern





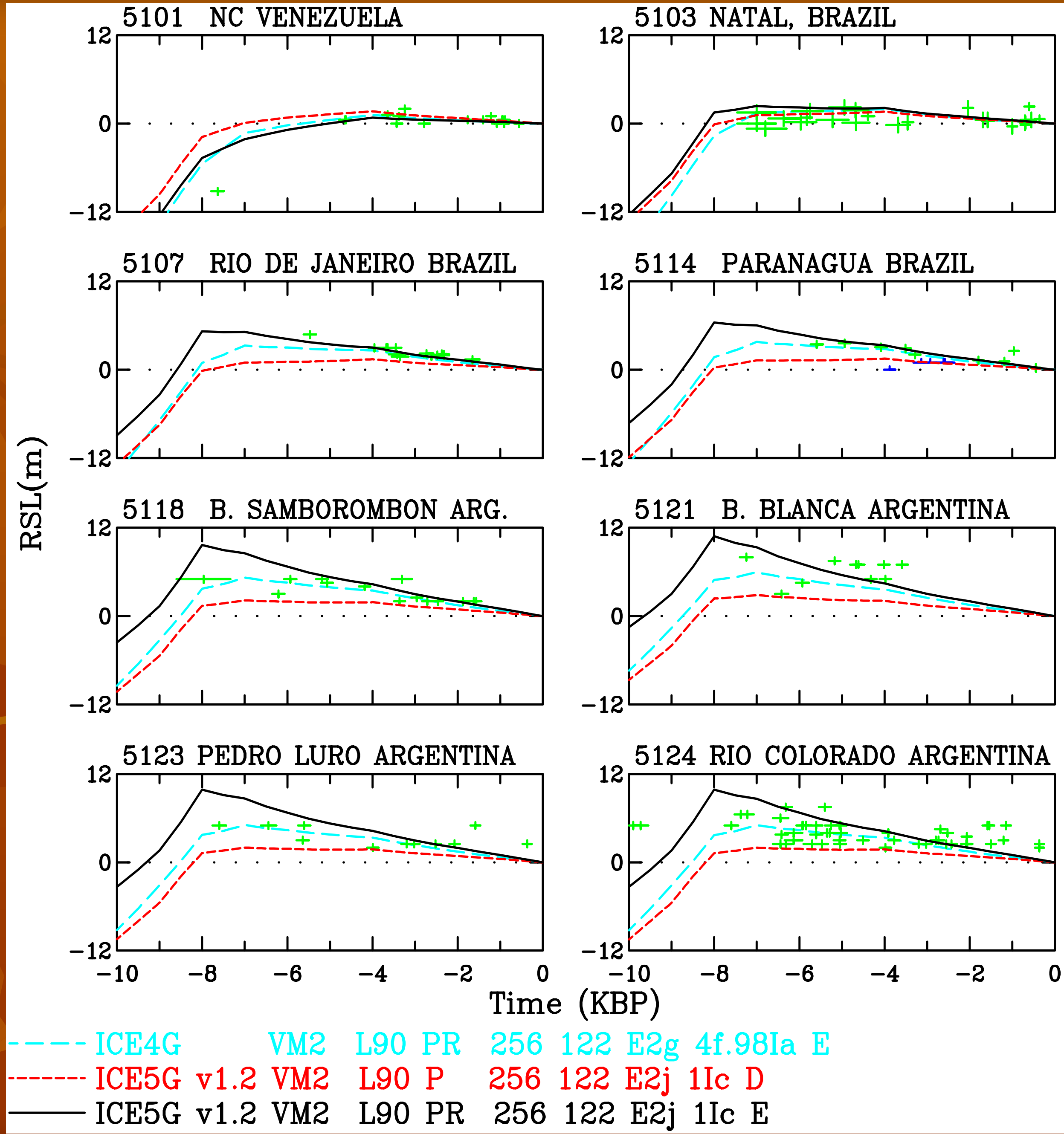
Australia and New Zealand

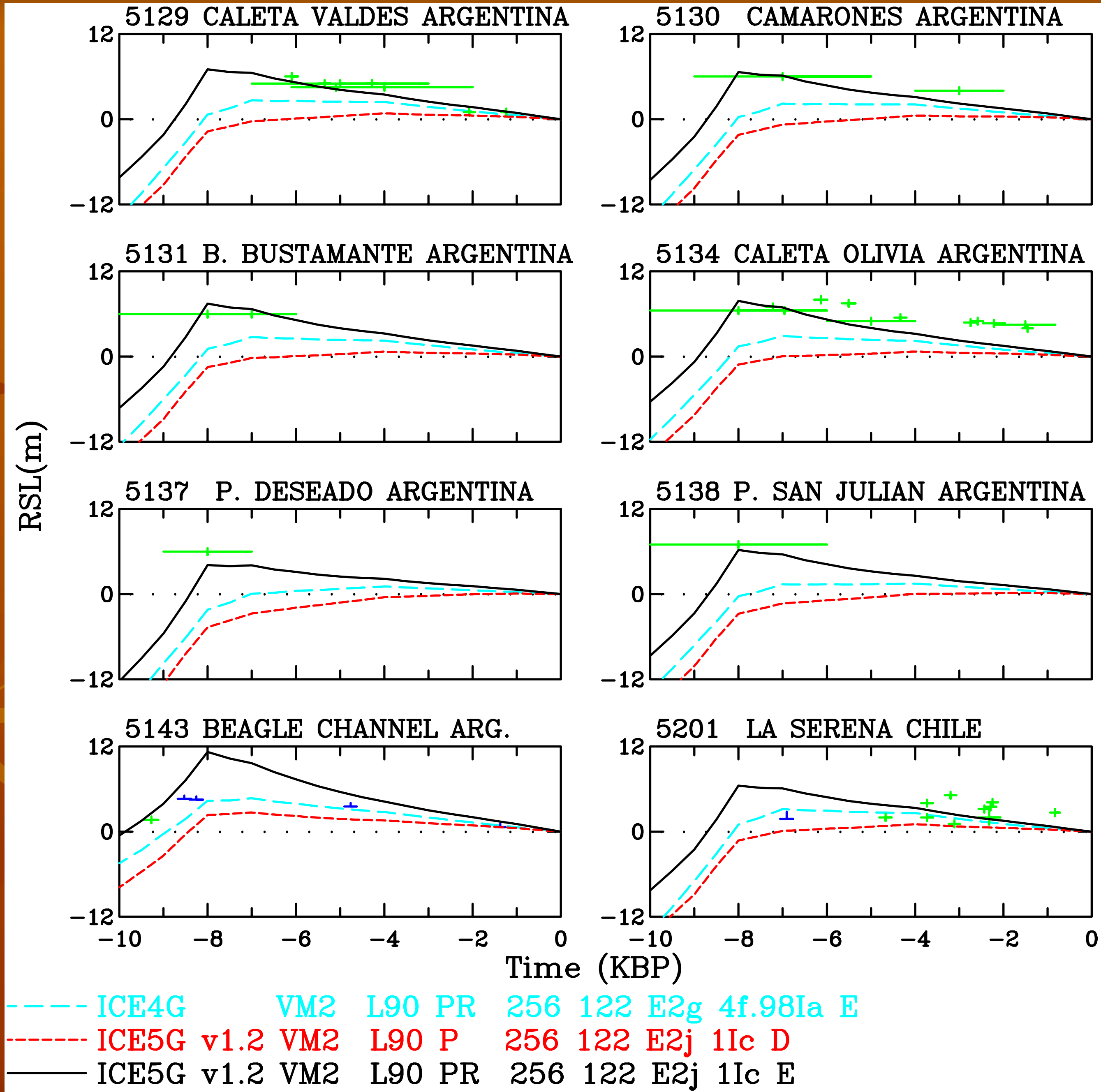
Additional Sites Confirming

Summary

- A detailed test has been performed of quality of the theory that has been produced on the basis of which the planet's rotational response to the GIA process is computed.
- This test proves that the "Equivalent Earth Model" hypothesis of Munk and MacDonald is the correct basis for the computation as previously assumed in Peltier (1982) AND Wu and Peltier (1984)
- The recent claim that this theory was in error (by Mitrovica, Wahr and others (GJI,2005)) is therefor erroneous.
- The theory allows for the modern melting of ice from Greenland and Antarctica at a rate that substantially explains the previously noted "missing source" of the meltwater required to explain the observed modern rate of global sea level rise as previously argued by Peltier (1998, Rev. Geophys.)

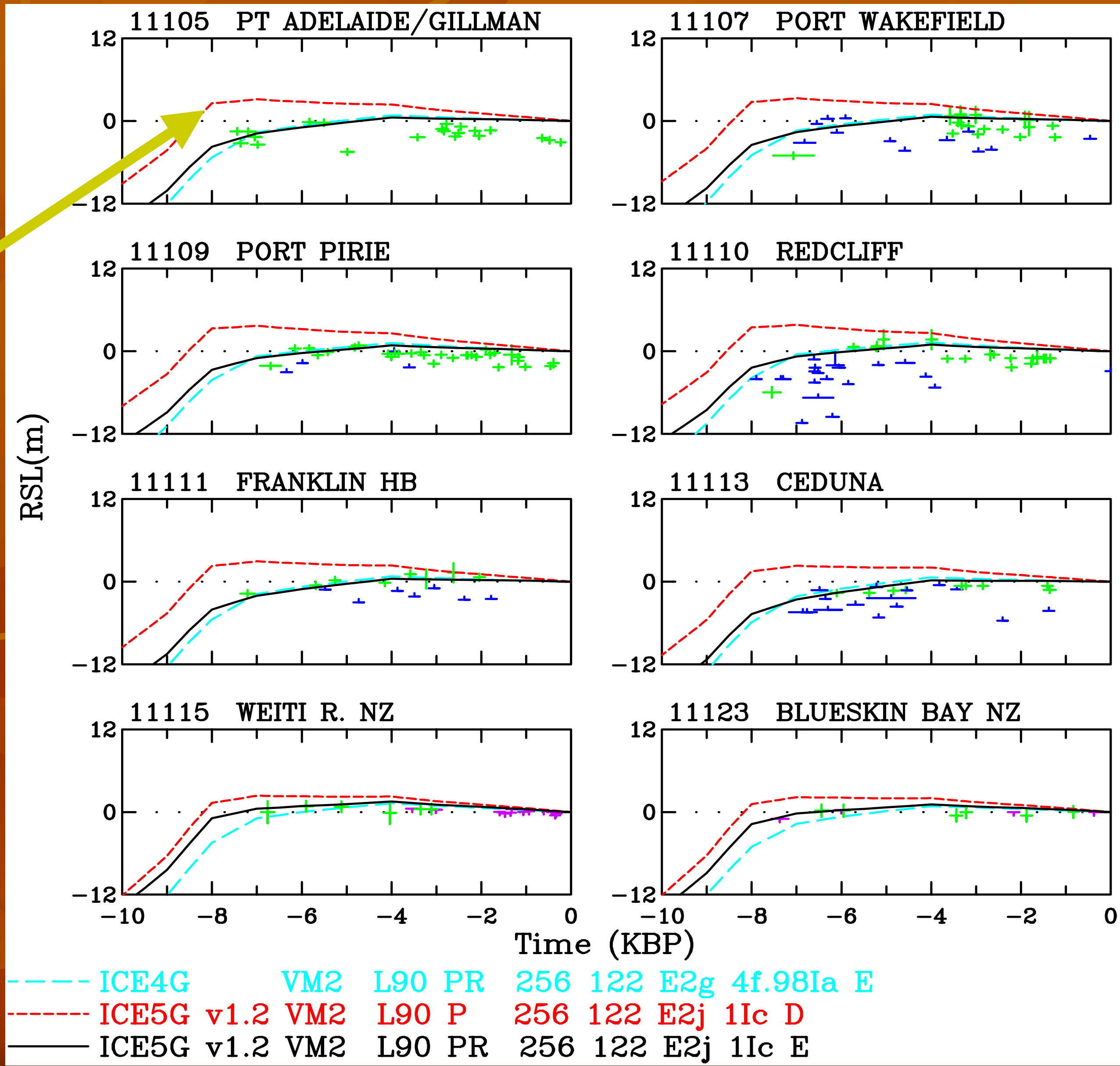
Moving from north to south along the coast the theory with rotational feedback predicts that the mid-Holocene high stand should be found at increasing elevation as latitude increases.




5129 CALETA VALDES ARGENTINA
5130 CAMARONES ARGENTINA
5131 B. BUSTAMANTE ARGENTINA
5134 CALETA OLIVIA ARGENTINA
5137 P. DESEADO ARGENTINA
5138 P. SAN JULIAN ARGENTINA
5143 BEAGLE CHANNEL ARG.
5201 LA SERENA CHILE
12
0
−12
RSL(m)
−10 −8 −6 −4 −2 0
Time (KBP)
ICE4G VM2 L90 PR 256 122 E2g 4f.98Ia E
ICE5G v1.2 VM2 L90 P 256 122 E2j 1Ic D
ICE5G v1.2 VM2 L90 PR 256 122 E2j 1Ic E

What about the other "bulls-eyes" in the degree 2 and order 1 pattern due to rotational feedback?

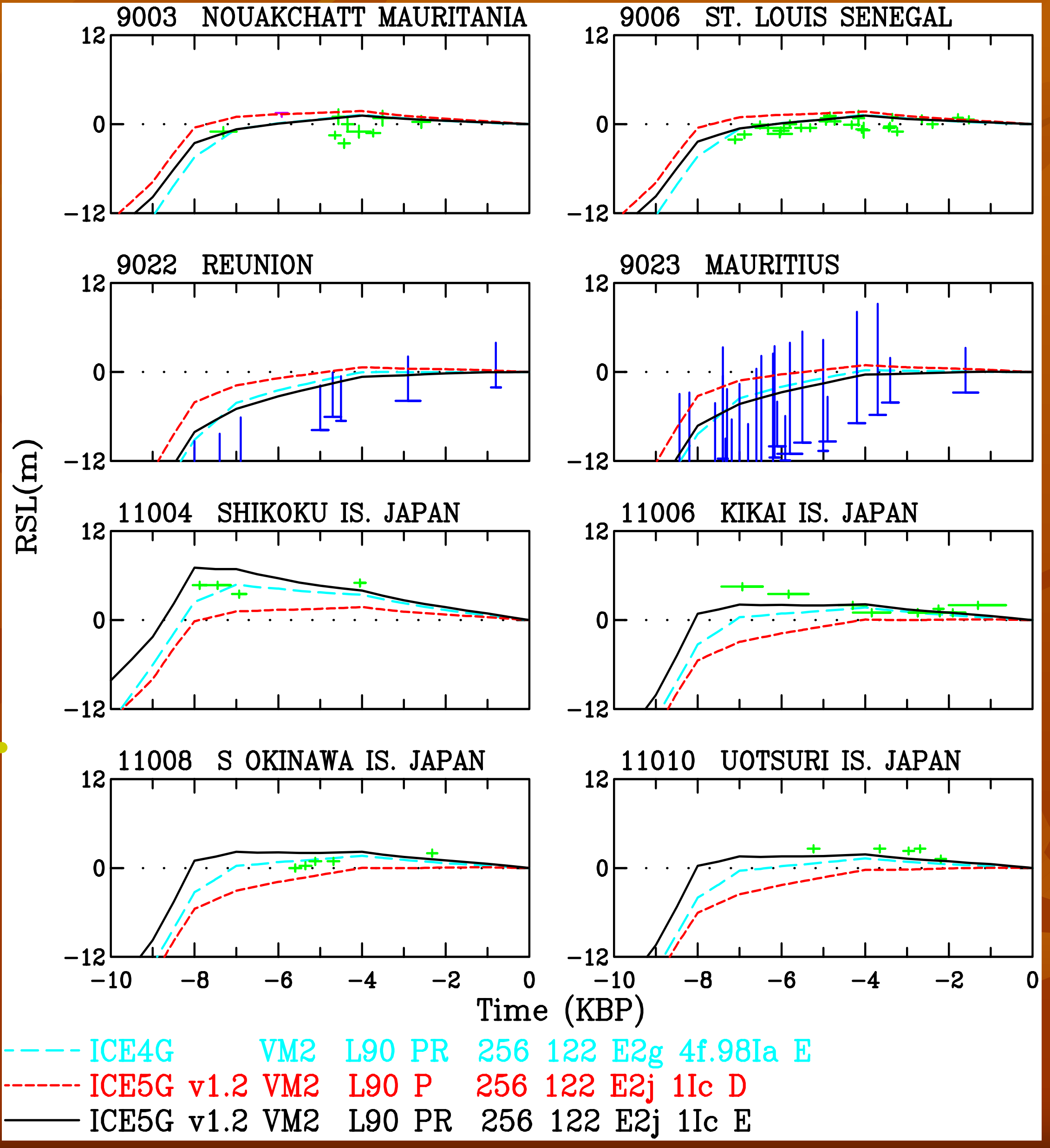
These sites are all from the Great Australian Bight, a region in which the influence of rotational feedback causes the removal of a mid-Holocene highstand that would otherwise exist.



Further examples from other rotation influenced regions

"Down sites" from the west Coast of Africa and from the Indian Ocean

"Up sites" from the Japanese Islands



(6) In the domain of the Laplace transform variable "s" , the polar wander equations may then be written in the form:

$$sm_1 + \sigma(1 - k_2^T / k_f) m_2 = \psi_1(s)$$
$$sm_2 - \sigma(1 - k_2^T / k_f) m_1 = \psi_2(s)$$
$$\sigma = \Omega(C - A)/A$$

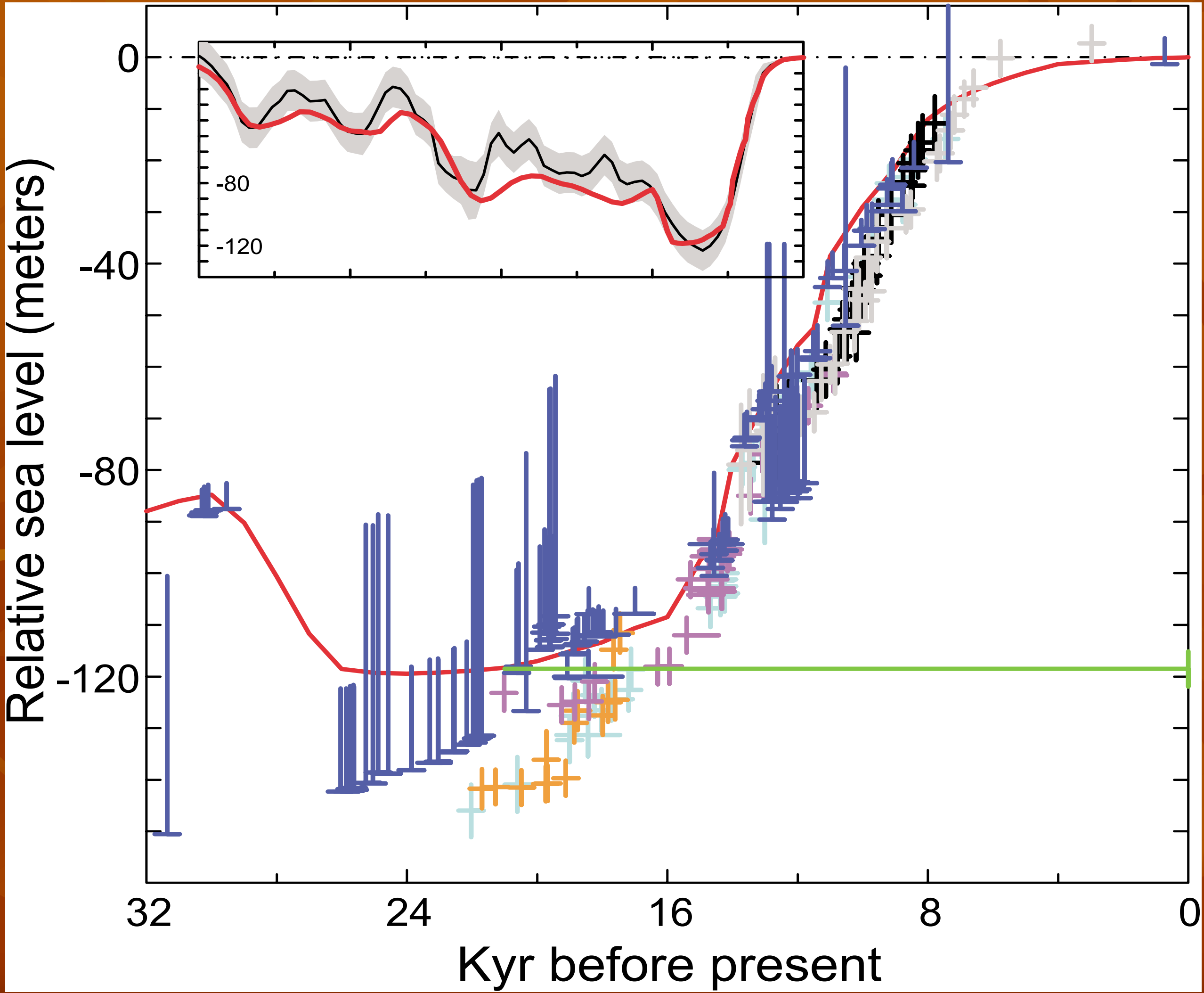
These constitute two inhomogeneous algebraic equations in the m_j

(7) The Laplace transforms of the excitation functions are simply:

$$\psi_1(s) = \left(\frac{\Omega}{A}\right) I_{23}(s) - \left(\frac{s}{A}\right) I_{13}(s)$$
$$\psi_2(s) = -\left(\frac{\Omega}{A}\right) I_{13}(s) - \left(\frac{s}{A}\right) I_{23}(s)$$
$$I_{ij}(s) = (1 + k_2^L(s)) I_{ij}^{Rigid}(s)$$

Barbados RSL constrains the total variation of continental ice mass through a glacial cycle

From Peltier and Fairbanks QSR 2006, in press.



What about the other element of the Earth's rotational response to the Glacial Isostatic Adjustment forcing?

Geoid Height Time Dependence and the $\dot{J}_{\ell m}$

(1) Geoid Height Time Dependence (take $G(\theta, \lambda, t)$ = geoid height)

$$\dot{G}(\theta, \lambda, t) = \dot{R}(\theta, \lambda, t) + \dot{R}SL(\theta, \lambda, t), \quad R = \text{local radius wrt com}$$

RSL = sea level wrt solid surface

(2)

$$\dot{G}(\theta, \lambda, t) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{+\ell} \dot{G}_{\ell m} Y_{\ell m}(\theta, \lambda)$$

(3) Definition of the $J_{\ell m}$

$$\dot{G}(\theta, \lambda, t) = -a \sum_{\ell=2}^{\infty} \sum_{m=0}^{\ell} (\dot{J}^1_{\ell m} \cos m\lambda + \dot{J}^2_{\ell m} \sin m\lambda) P^m_\ell (\cos\theta)$$

With the P^m_ℓ normalized such that

$$\underset{\text{unit sphere}}{\int\int} \left(P^m_\ell (\cos\theta) \begin{matrix}\cos m\lambda \\ \sin m\lambda\end{matrix}\right)^2 \sin\theta \, d\theta \, d\lambda = \frac{4\pi (\ell+m)!}{(2\ell+1)(\ell-m)!(2-\delta_{om})}$$

(4) Thus

$$\dot{J}^1_{\ell m} = \frac{-(2-\delta_{om})}{a} \sqrt{\frac{(2\ell+1)(\ell-m)!}{(\ell+m)}} \quad \mathrm{Re}[\dot{G}_{\ell m}]$$

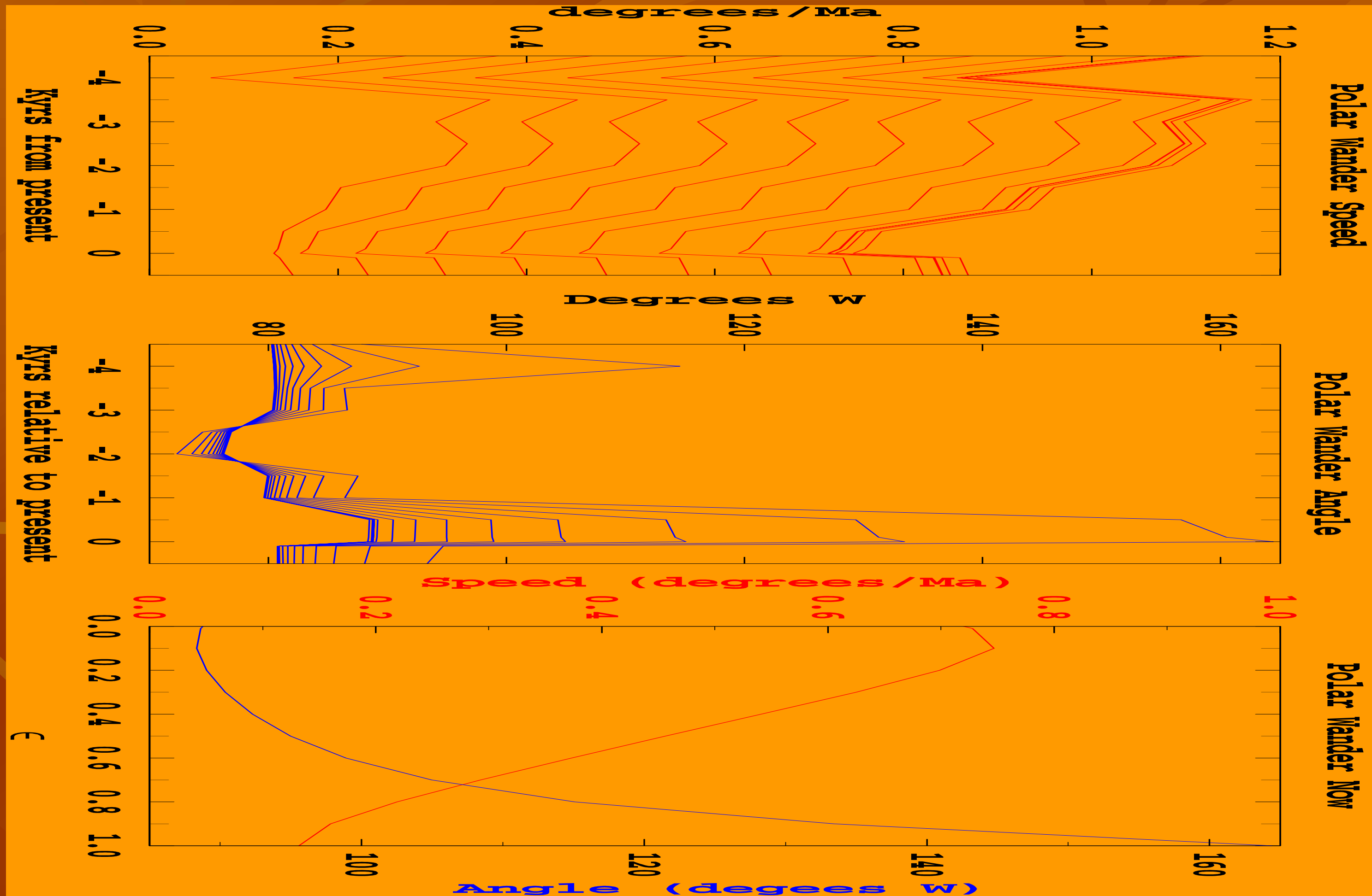
$$\dot{J}^2_{\ell m} = \frac{2}{a} \sqrt{\frac{(2\ell+1)(\ell-m)!}{(\ell+m)}} \quad \mathrm{Im}[\dot{G}_{\ell m}]$$

(5) Variance Spectra

$$\sigma^2_{\dot{J}} = \sum_{m=0}^{\ell} [(\dot{J}^1_{\ell m})^2 + (\dot{J}^2_{\ell m})^2]$$

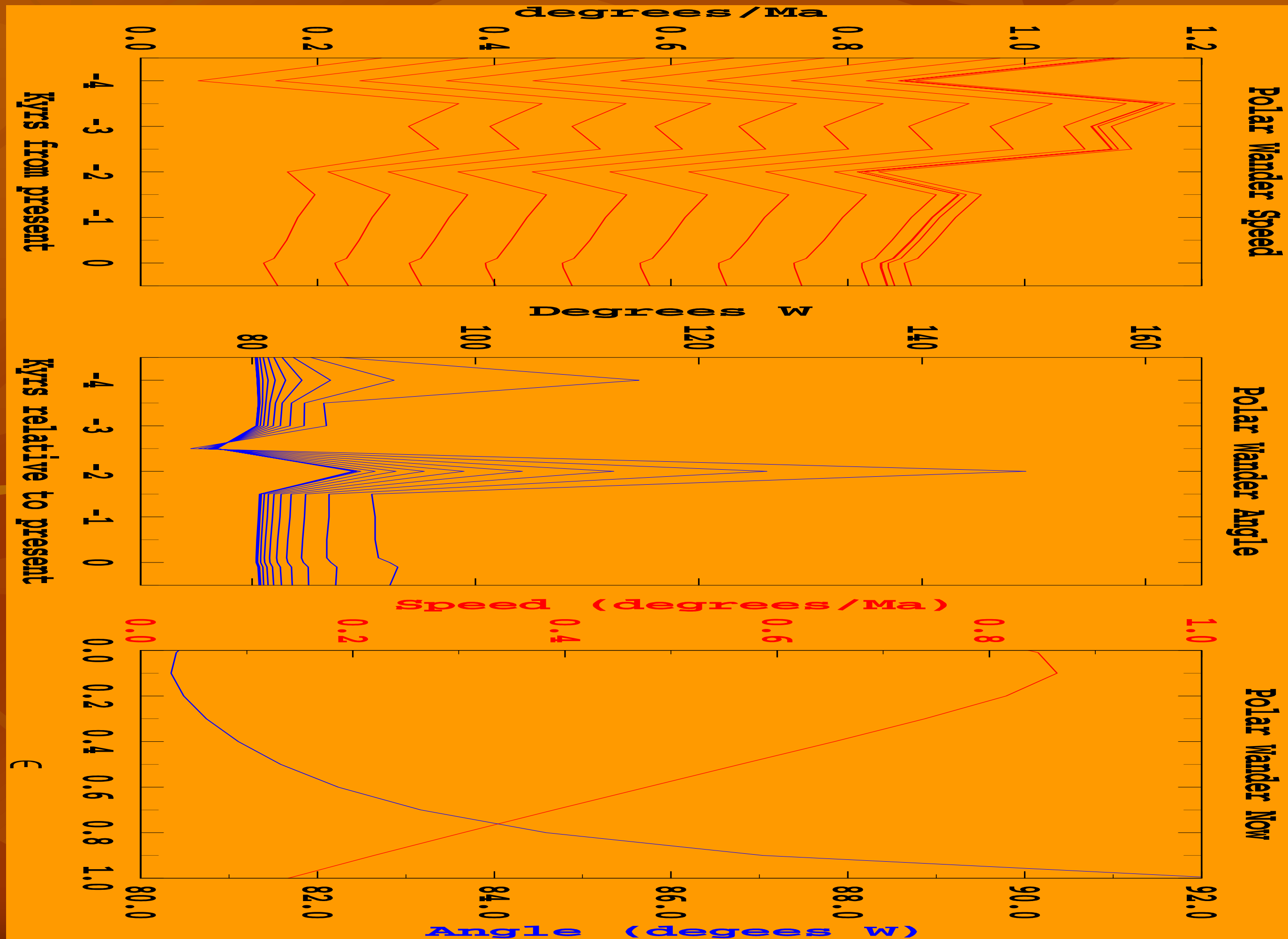
The influence of failure to adhere to the EQM hypothesis-1(ICE-5G senso stricto)

NOTE:
Incorporation of the epsilon effect reduces the speed prediction by a factor of ~4 and skews the polar wander angle westward by approximately 80 degrees. The Holocene records that record the influence of rotational feedback would therefor not be explicable



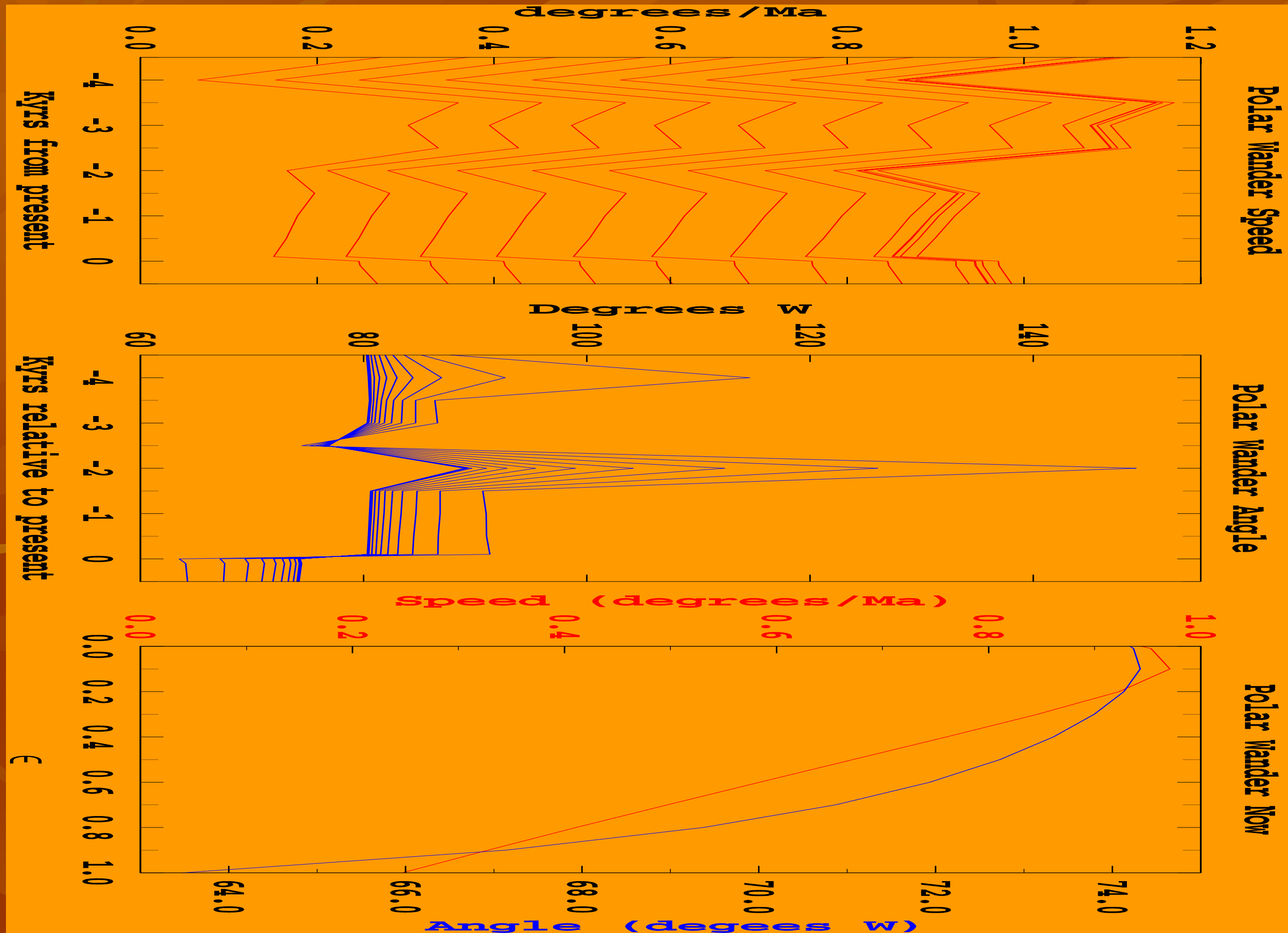
Now Ice-5G senso stricto includes continued growth of the Greenland Ice Sheet through the Holocene. If such Neoglacial growth is assumed to stop after 2 ka

If the influence of continuing Neoglacial growth of Greenland is eliminated, then the predicted polar wander angle prediction is strongly stabilized but the speed is still diminished by aproximately a factor of 4 and the data are still not reconciled.



The Influence of present day Greenland melting at a rate of 0.1 mm per year

Note: the addition of present day Greenland melting at a rate of 0.1 mm per year increases the speed prediction so that it now lies within the observed range while the angle remains stable and also in the observed range. A further increase in the Greenland melt rate by a factor of three remains allowed by the data.



(9) The complexity of the polar wander solution arises because of the fact that there exists a multiplicity of “normal modes of viscous gravitational relaxation” that govern the response of the planet to both surface mass and tidal load forcing. In Peltier (1976) it was first established that:

$$k_2^T(s) = k_2^{TE} + \sum_{j=1}^{N} \frac{t_j}{s+s_j} = k_2^T(0) - s\sum_{j=1}^{N} \frac{(t_j/s_j)}{(s+s_j)}$$

$$k_2^L(s) = k_2^{LE} + \sum_{j=1}^{N} \frac{r_j}{s+s_j} = (-1+l_s) - s\sum_{j=1}^{N} \frac{(r_j/s_j)}{(s+s_j)}$$

Note: it is a consequence of the presence of the surface lithosphere that the infinite time, zero “s”, limit of the surface load Love number is different from -1. I measure the strength of the “lithosphere effect” by the parameter

l_s

(10) In order to accurately construct a time domain solution we need to rewrite the system impulse response function H(s) following Peltier (1982), Wu and Peltier(1984) and Peltier and Jiang(1996) as:

$$H(s)=\left(\frac{\Omega}{A\sigma}\right)\left[l_s - s\sum_{j=1}^{N}\frac{r_j}{s_j(s+s_j)}\right]\prod_{j=1}^{N}\frac{s+s_j}{Q_N(s)}$$

$$Q_N(s)=\left(1+k_2^T(0)/k_f\right)\prod_{j=1}^{N}(s+s_j)+\left(\frac{s}{k_f}\right)\sum_{j=1}^{N}\frac{t_j}{s_j}\prod_{i\neq j}^{N}(s+s_i)$$

$$Q_N(s)=\prod_{j=1}^{N}(s+\lambda_j)$$

The sequence of lambdas consists of the roots of the function Q-N
In order to prepare for Laplace transformation of H(s), this function
Must be further manipulated as follows:

NOTE: In the analysis of the GIA influence upon rotation that I perform, I employ A fluid Love number that is identical to $k_2^T(0)$ This assumption has recently been questioned in Mitrovica et al (GJI, 161, 491-506, 2005). This issue will be a focus in what follows.

(11) In order to perform the inverse Laplace transform we further manipulate H(s) as:

$$H(s) = \left(\frac{\Omega l_s}{A\sigma}\right)\left[1 - \frac{q(s)}{Q_N(s)}\right] - \left(\frac{\Omega}{A\sigma}\right)\sum_{j=1}^{N}\frac{r_j}{s_j}\left[1 - \frac{R_j(s)}{Q_N(s)}\right]$$

$$q(s) = \prod_{j=1}^{N}(s+\lambda_j) - \prod_{j=1}^{N}(s+s_j)$$

$$R_j(s) = \prod_{j=1}^{N}(s+\lambda_j) - s\prod_{i\neq j}^{N}(s+s_i)$$

With this polynomial form for H(s) it is now amenable to to Laplace Inversion, as follows:

(12) Given these factorizations we may simply invert H(s) into the time domain to obtain:

$$H(t) = \left(\frac{\Omega}{A\sigma}\right)\left[l_s - \sum_{j=1}^{N}\frac{r_j}{s_j}\right]\delta(t) + \left(\frac{\Omega}{A}\right)\left[\sum_{j=1}^{N} E_j e^{-\lambda_i t}\right]$$

$$E_i = \left[l_s q(-\lambda_i) + \sum_{j=1}^{N}\frac{r_j}{s_j}R_j(-\lambda_i)\right] \Big/ \prod_{k\neq i}^{N}(\lambda_k - \lambda_i)$$

The solutions for the polar wander components then follow by convolution as:

$$m_1(t) = \left(\frac{\Omega}{A\sigma}\right)\left[l_s - \sum_{j=1}^{N}\frac{r_j}{s_j}\right] I_{13}^{Rigid}(t) + \left(\frac{\Omega}{A}\right)\sum_{i=1}^{N} E_i e^{-\lambda_i t} * I_{13}^{Rigid}(t)$$

$$m_2(t) = \left(\frac{\Omega}{A\sigma}\right)\left[l_s - \sum_{j=1}^{N}\frac{r_j}{s_j}\right] I_{23}^{Rigid}(t) + \left(\frac{\Omega}{A}\right)\sum_{i=1}^{N} E_i e^{-\lambda_i t} * I_{23}^{Rigid}(t)$$

Where the star indicates the time domain convolution operation

Remember that the PWS predicted by the ICE-5G(VM2) model fits the observational constraint whereas the prediction of the ICE-4G(VM2) model is too slow.

Note also that the Rostami et al observation is that the mid-Holocene high stand rises by about 6m from north to south. This fits the prediction of the ICE-5G(VM2) model only when the influence of rotational feedback is included, USING THE CONVENTIONAL THEORY.