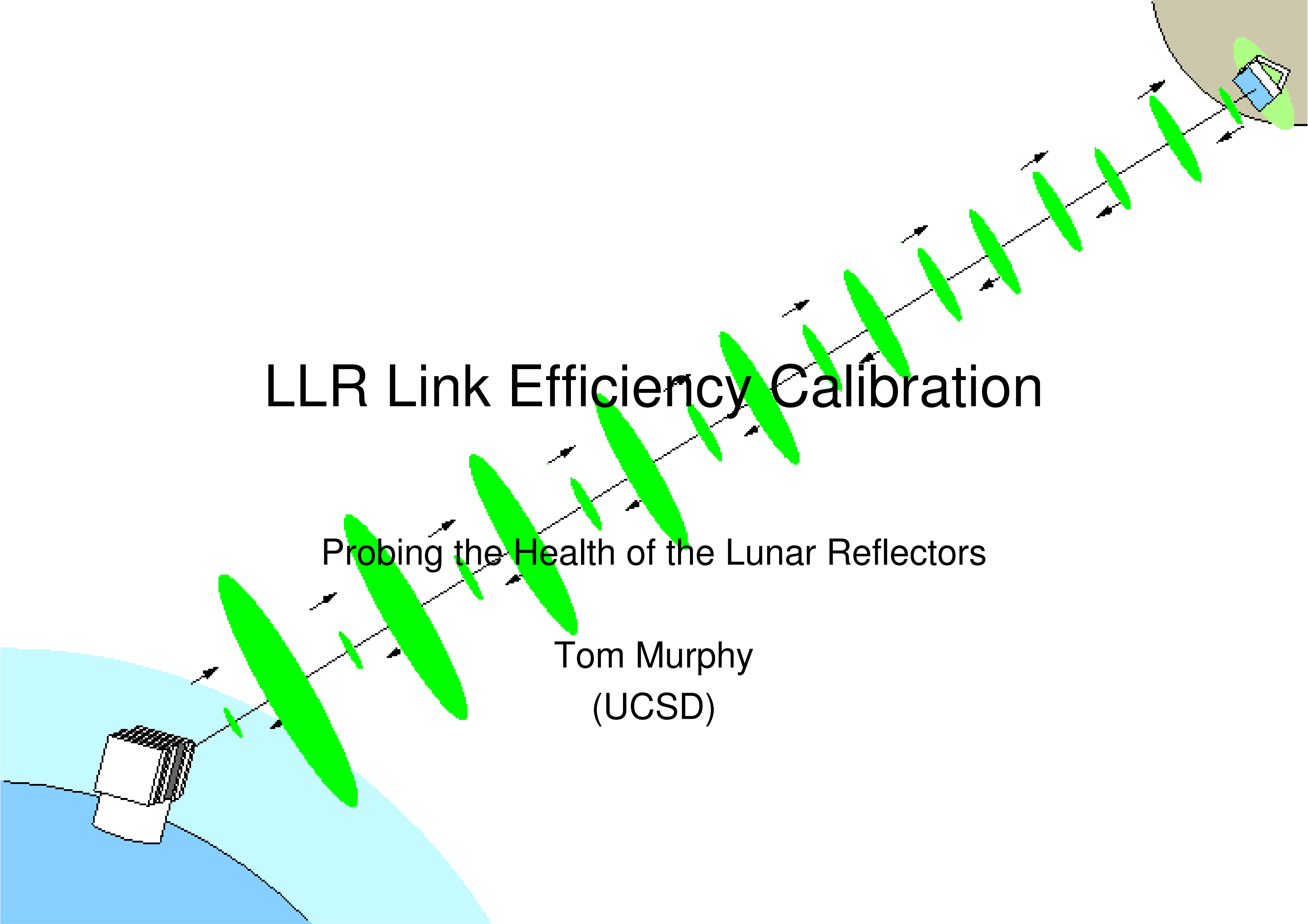


LLR Link Efficiency Calibration

Probing the Health of the Lunar Reflectors

Tom Murphy
(UCSD)

The Basic Link Equation

$$N_{\rm rx} = N_{\rm tx}\eta_c^2\eta_r Q n_{\rm refl}\left(\frac{d}{\phi r}\right)^2\left(\frac{D}{\Phi r}\right)^2$$

η_c = one-way optical throughput (encountered twice)
η_r = receiver throughput (dominated by narrow-band filter)
Q = detector quantum efficiency
$n_{\rm refl}$ = number of corner cubes in array (100 or 300)
d = diameter of corner cubes (3.8 cm)
ϕ = outgoing beam divergence (atmospheric “seeing”)
r = distance to moon
Φ = return beam divergence (diffraction from cubes)
D = telescope aperture (diameter; 3.5 m)

$$N_{\rm rx} = 5.4\left(\frac{E_{\rm pulse}}{115\ {\rm mJ}}\right)\left(\frac{\eta_c}{0.4}\right)^2\left(\frac{\eta_r}{0.25}\right)\left(\frac{Q}{0.3}\right)\left(\frac{n_{\rm refl}}{100}\right)\left(\frac{1\ {\rm arcsec}}{\phi}\right)^2\left(\frac{10\ {\rm arcsec}}{\Phi}\right)^2\left(\frac{385000\ {\rm km}}{r}\right)^4$$

- APOLLO should see 5 photons per pulse on Apollo 11 & 14; 15 on Apollo 15

Refining the Estimates, Part I

Terms contributing to the common (two-way) path, $\eta_c \approx 0.51$

symbol	value	fractional error	# occur	description
η_{atmos}	0.87	0.03	1	atmospheric transmission
η_1	0.825	0.03	1	Aluminum-coated primary mirror
η_2	0.875	0.03	1	Aluminum-coated secondary mirror
η_3	0.825	0.03	1	Aluminum-coated tertiary mirror
η_4	0.992	0.01	1	High-power dielectric turning-mirror
η_5	0.998	0.01	1	High-power dielectric turning-mirror
η_L	0.996	0.01	3	AR-coated lens

Refining the Estimates, Part II

Terms contributing to the receiver, $\eta_r \approx 0.43$

symbol	value	fractional error	# occur	description
η_{TR}	0.998	0.01	1	AR-coated Transmit/Receive optic
η_6	0.995	0.01	1	broad-band dielectric turning-mirror
η_7	0.995	0.01	1	broad-band dielectric turning-mirror
η_{BS}	0.93	0.01	1	uncoated fused-silica beam splitter
η_D	0.998	0.01	3	AR-coated variable attenuator disks
η_L	0.996	0.01	1	AR-coated lens
$\eta_{\mu L}$	0.95	0.04	1	microlens: uncoated epoxy plus AR side
$f_{\mu L}$	0.67	0.04	1	measured microlens efficiency
f_{APD}	0.5–1.0	0.10	1	APD fill factor: seeing/source dependent

Refining the Estimates, Part III

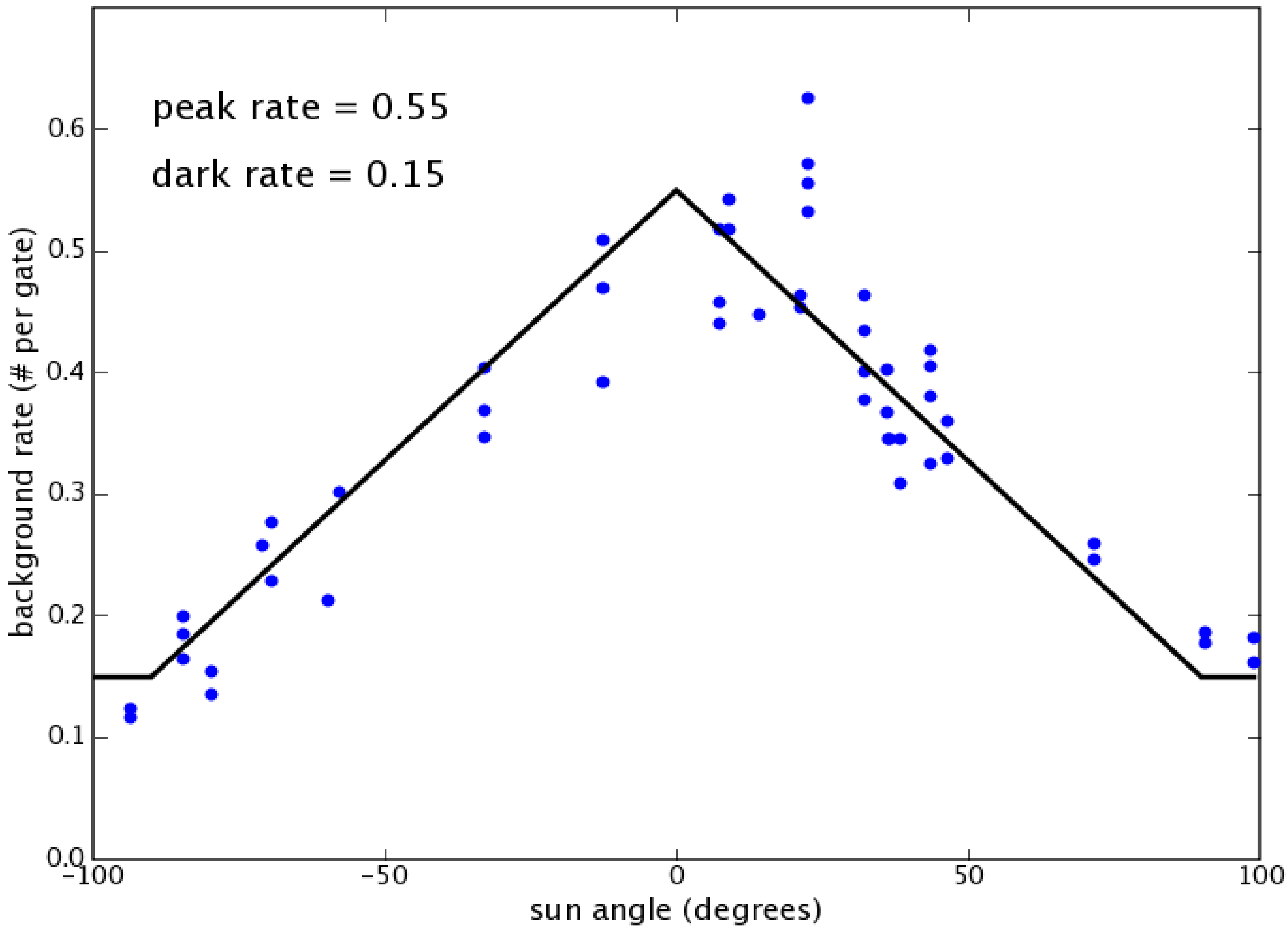
Other terms contributing to flux check

symbol	value	fractional error	# occur	description
Q	0.30	0.12	1	APD photon detection efficiency
D	3.26 m	0.01	2	effective aperture: $A = \pi D^2/4$
$\Delta\lambda_{NB}$	0.95 nm	0.05	—	effective filter bandpass
Δt_{APD}	95 ns	0.03	—	APD integration time per gate
F_0	3.9×10^{-11} W/m^2/nm	0.03	—	zero-magnitude flux calibration

Checking the one-way flux

- Two ways: use star, or use moon
  - both give consistent results
- Moon around Apollo 15 is 3.60 magnitudes per square arcsec
  - at full-moon illumination
  - 2.87 mag into 1.4×1.4 arcsec APD field of view
  - fill factor (f_{APD}) is 13/16 (three channels missing)
- Measure 0.40 background photons per gate at full moon on Apollo 15
- Calculate 0.40 ± 0.08 using numbers presented above
  - *Q* was allowed to vary to match condition
  - came out right at expected value (30%) for uncoated APDs of this structure
- Thus much of link equation is confirmed
  - one-way photon detection efficiency: 2.3%

Apollo 15 Background Count Rate



Additional Parameters for Ranging

symbol	value	fractional error	# occur	description
η_{NB}	0.35	0.07	1	narrow-band filter throughput
f_{launch}	0.60	0.05	1	central obstruction on Gaussian beam
E_{pulse}	0.100 J	0.07	1	typical pulse energy
n_{refl}	300	—	1	# cubes in Apollo 15 (largest) array
η_{refl}	0.93	0.01	1	double-pass through corner-cube face
d	0.038 m	—	2	diameter of individual corner cube
r	3.85×10^{8} m	0.02	–4	typical earth-moon distance
ϕ	0.8 arcsec	0.15	–2	best outbound (seeing) divergence
Φ	10 arcsec	0.15	–2	divergence from return

Results of simple link equation

$$N_{\rm rx} = N_{\rm tx}\eta_c^2\eta_r Q n_{\rm refl}\left(\frac{d}{\phi r}\right)^2\left(\frac{D}{\Phi r}\right)^2$$

- Using parameters from previous tables, expected *average* return from Apollo 15 array is:

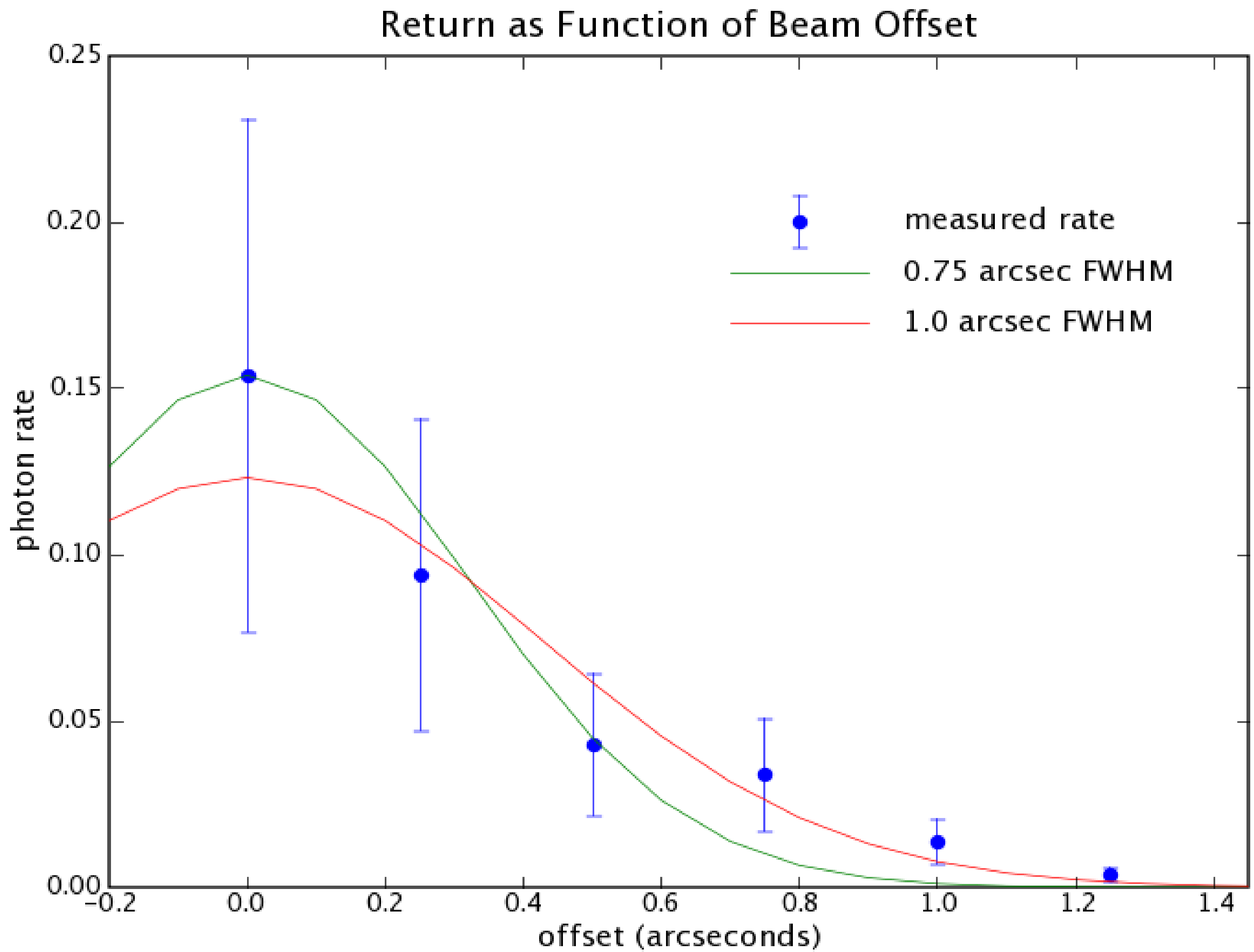
  12 ± 6 photons per pulse
- Example best ranges (December 2005, January 2006) were ~0.5 photons per pulse for brief periods (~30 sec)
  - best average rate over several minutes is 0.25 photons/pulse
- Ratio is 12/0.5 = 24
- Estimated uncertainty is 50% of average
  - would have to apply this (multiplicatively) 4.5 times to satisfy result
  - $12 \cdot 0.5^{4.5} \rightarrow 0.5$ ($12 \rightarrow 6 \rightarrow 3 \rightarrow 1.5 \rightarrow 0.75$… in successive factors of two)
  - thus this result is approximately 4.5σ in significance

A more sophisticated approach

- Many intricacies brushed under the rug:
  - outgoing beam profile (*not* tophat)
  - theoretical corner cube diffraction pattern
  - manufacturing tolerance of corner cubes
  - shadowing of recessed cubes in palette
  - velocity aberration
  - thermal degradation of cubes in sunlight
- A second stage of analysis treats these deficient

Outgoing Beam Profile

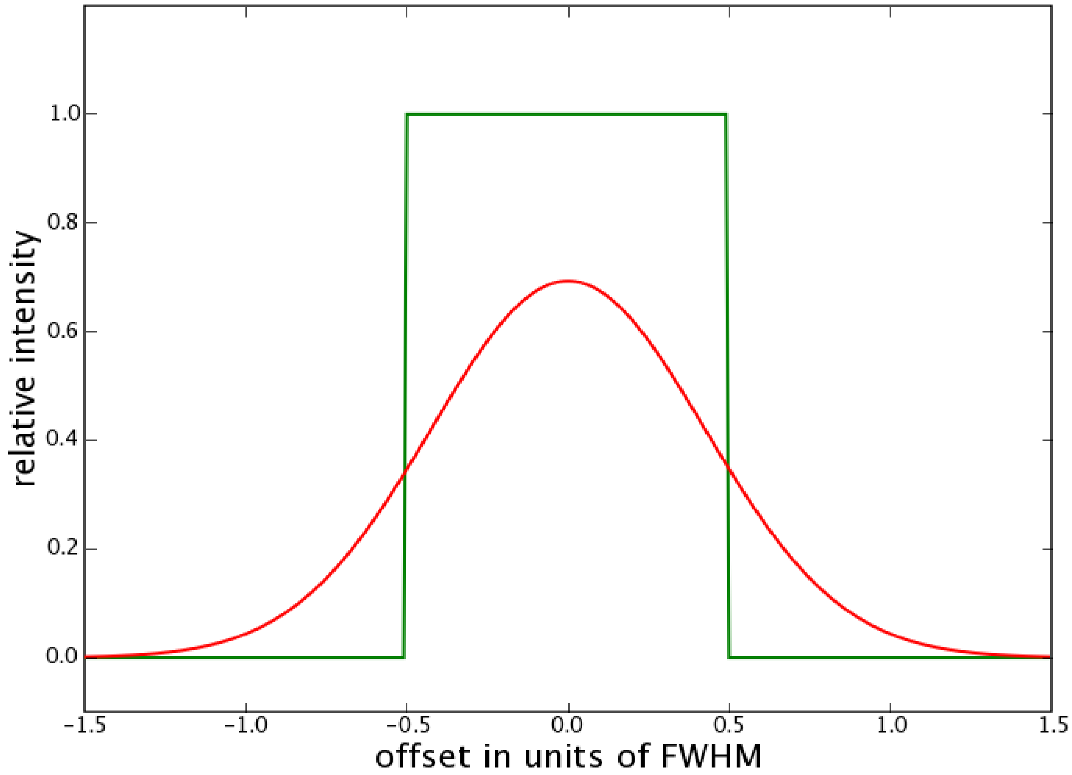
- Confidence in our seeing-limited outgoing beam comes from:
  - shear plate on collimated beam allows tuning of divergence at a level corresponding to 0.04 arcsec outside the telescope
  - corner cubes at telescope exit aperture can test divergence: see no divergence at < 0.5 arcsec level of confidence
  - rastering the transmit/receive offset while keeping the receiver fixed (i.e., slewing beam on moon with receiver fixed) has signal disappearing if we move the beam by more than about one arcsecond
- Should we really use 0.8 arcseconds?
  - Our CCD measures seeing consistent with other instruments on the telescope (thus APOLLO optics are not bad)
  - In good seeing, we see starlight concentrated on central 4 pixels of APD array (2×2 box is 0.7 arcsec on a side)
  - The median seeing for this telescope is 1.1 arcsec
    - thus best APOLLO performance likely better than this
  - The 0.5 photon per pulse results were obtained in very good seeing



Walking the beam toward optimal signal gives idea of beam profile
In this case, less than or about 1 arcsec FWHM fits reasonably well

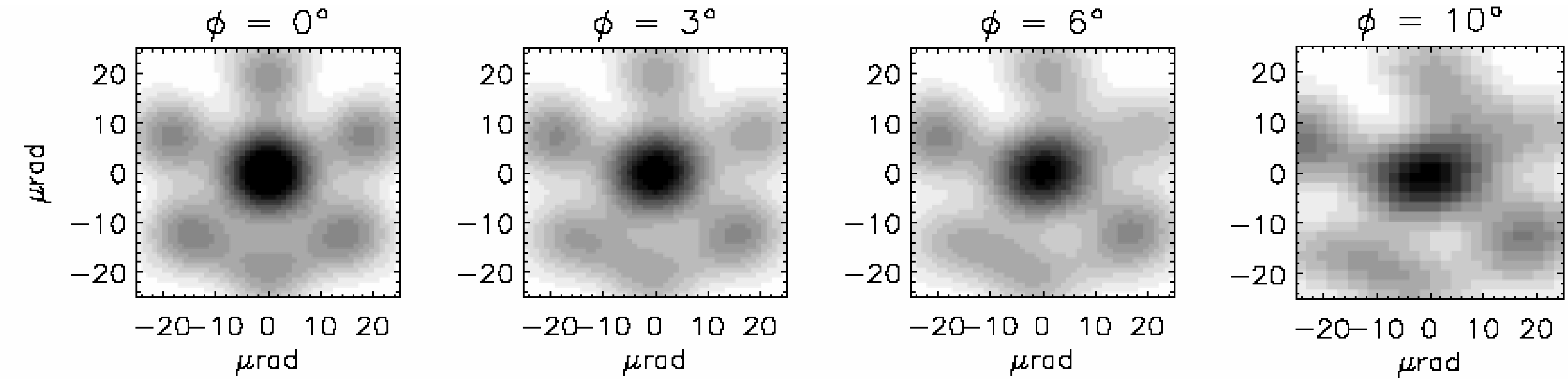
Correction for Gaussian Beam

- The simple form of the link equation assumes a “tophat” intensity distribution
- Gaussian distribution with same FWHM (full-width at half-max) as tophat has central intensity 0.69 (ln(2)) times that of tophat with same total intensity



same total intensity when revolved and summed in azimuthal axis

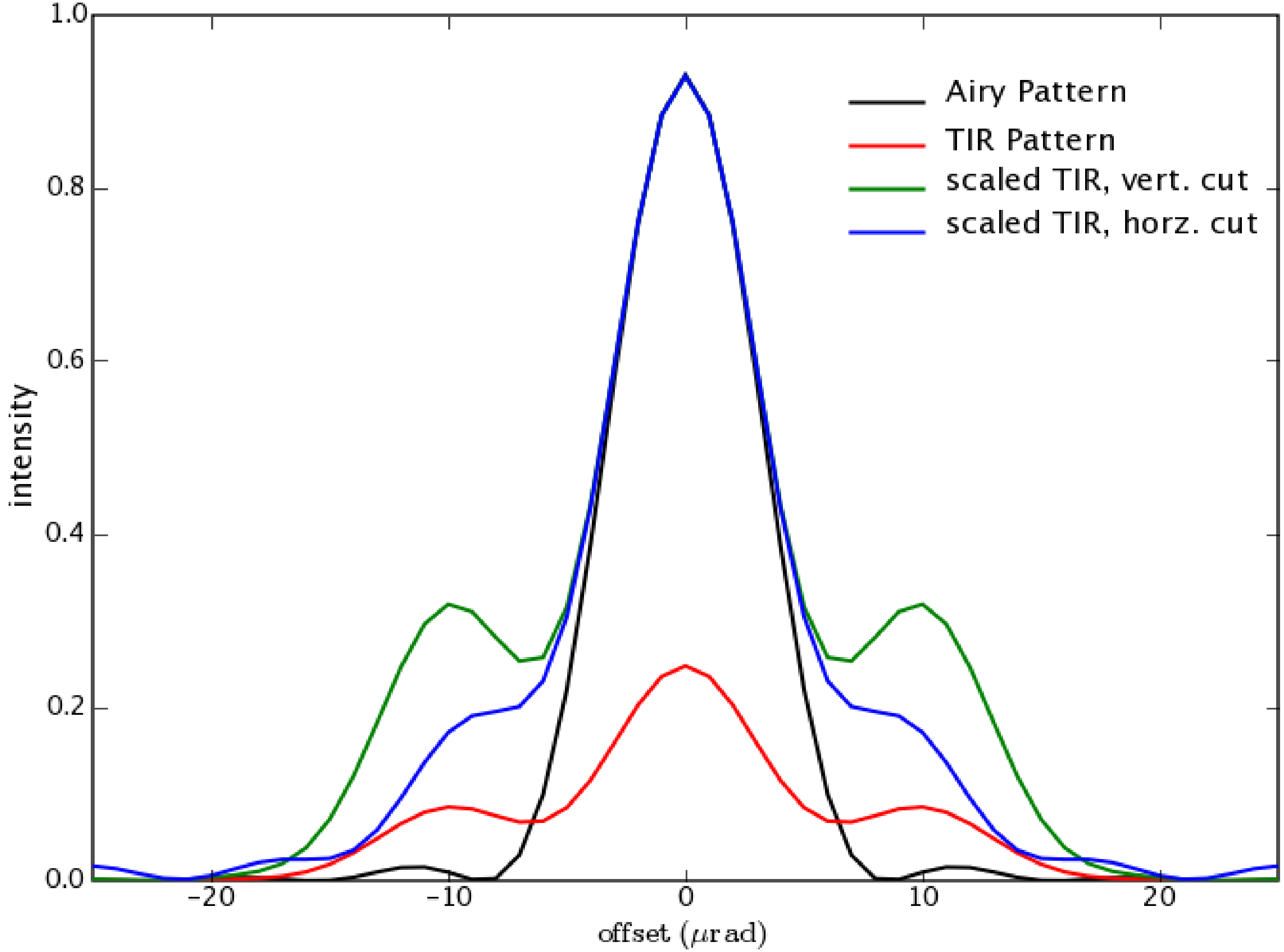
Proper treatment of C.C. diffraction



images courtesy David Arnold

- The diffraction pattern from an uncoated total internal reflection (TIR) corner cube is far from the tophat pattern used in the link equation
- The core follows the full-diameter Airy pattern
- But the wings contain significant power
- Peak is about 0.25 of perfect Airy pattern
- 36% of energy inside first Airy ring (84% for perfect Airy)

Airy vs. TIR


Airy Pattern
TIR Pattern
scaled TIR, vert. cut
scaled TIR, horz. cut
intensity
offset (μrad)
1.0
0.8
0.6
0.4
0.2
0.0
−20
−10
0
10
20

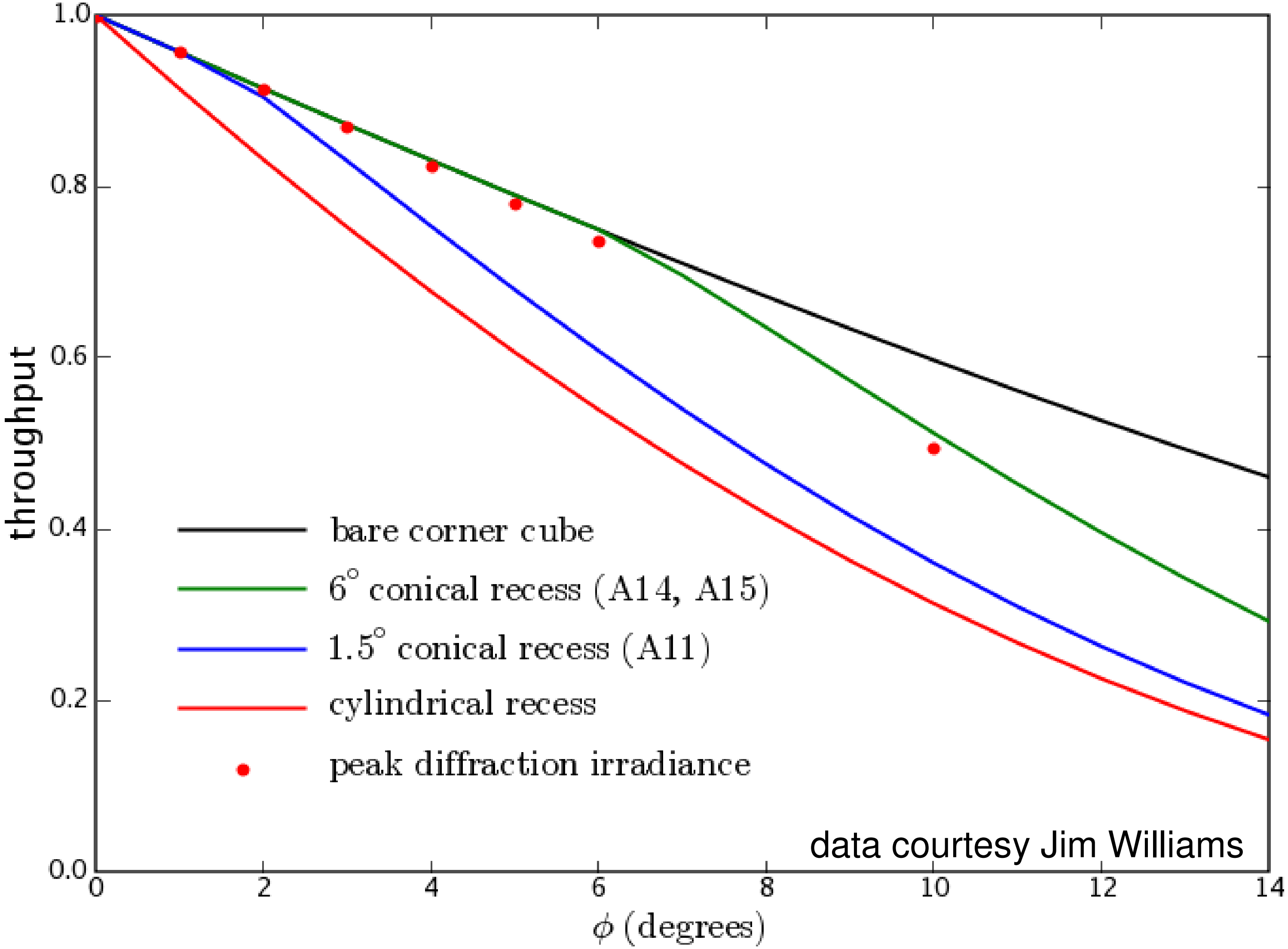
Central Irradiance Compared to Tophat

- Compared to a tophat with diameter λ/D, what is the central irradiance of an uncoated (total internal reflection) corner cube?
- Relative to perfect Airy pattern, central irradiance is:
  - 0.278 if no reflective loss at front surface
  - 0.248 if uncoated fused silica front surface
- Central Airy irradiance from diameter D is reduced from λ/D tophat by factor of 0.68
- Composite reduction of central irradiance is:
  - 0.182 if no reflective loss at front surface
  - 0.169 if uncoated fused silica front surface
- Recipe: treat return as tophat (Φ=2.89 arcsec at 532 nm) de-rated by 0.182
  - will apply 0.93 reflection loss separately

Velocity Aberration and Recessed Cubes

- Velocity of lunar orbit is about 1000 m/s
- Earth rotation is 400 m/s
- Typical velocity offset is 600–900 m/s
  - $2v/c \rightarrow$ 4–6 μR = 0.8–1.2 arcsec
- Results in de-rating irradiance:
  - typically factor of 0.64–0.86 for 532 nm [avg = 0.75]
- Apollo 11 cubes recessed by half-diameter with 1.5° half-angle conical opening (6° for Apollo 14 and 15 arrays)
- Lunar libration (~7° in both longitude and latitude) presents angular offsets as high as 10°
  - typical angle is 6.5°
- Central irradiance down as much as 0.50 (at 10°)

Recessed Cube Influence



All recesses are half-diameter, and throughput is total geometrical flux. central diffraction irradiance is reduced from this, but not much at first

Manufacturing Tolerance

- Nominal angular tolerance on Apollo cubes is ±0.3 arcsec
- The cubes that were selected for flight all demonstrated at least 90% the theoretical central irradiance
  - Use factor of 0.93 to account for typical manufacture error

Thermal Impact

- Detailed thermal conductivity/radiation studies predict degradation of central irradiance at a range of sun angles
  - most of effect is thermal gradient of refractive index

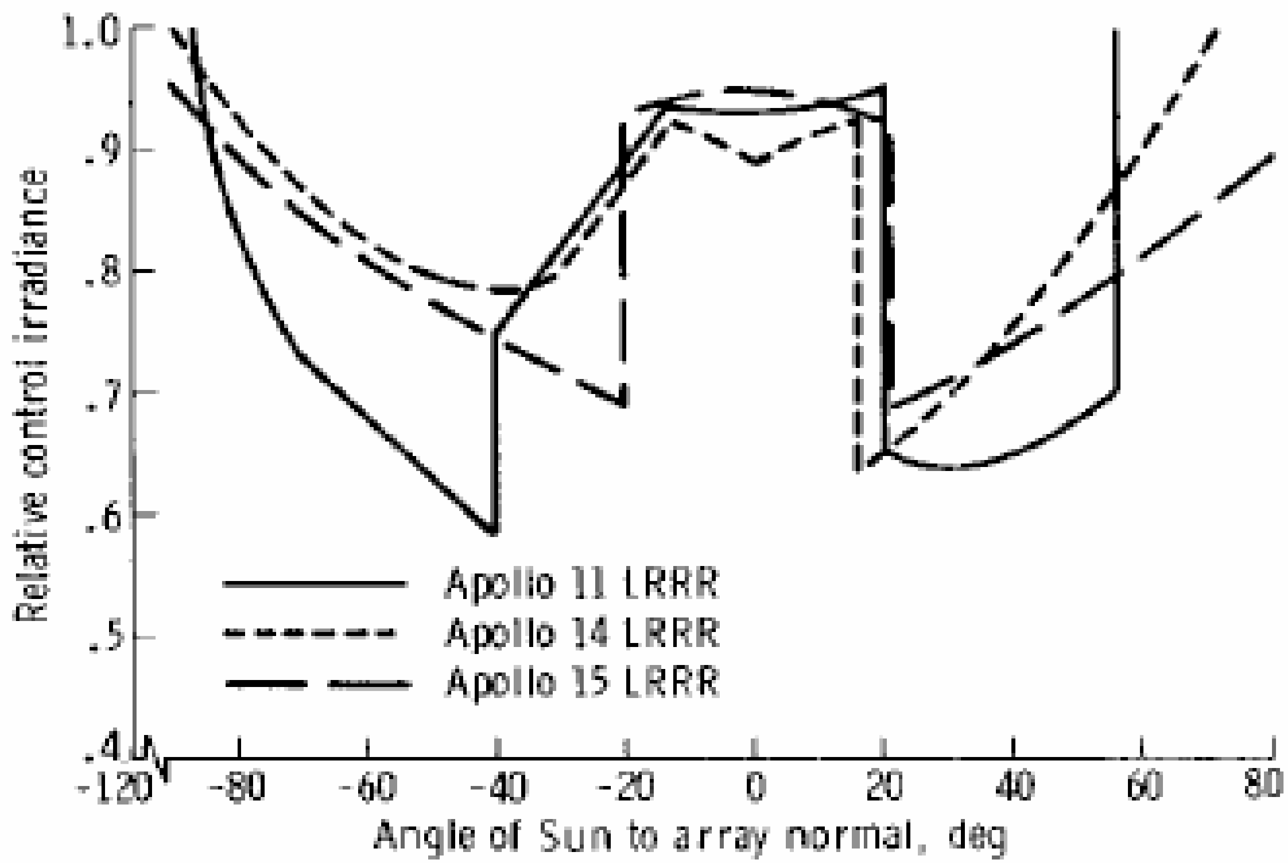
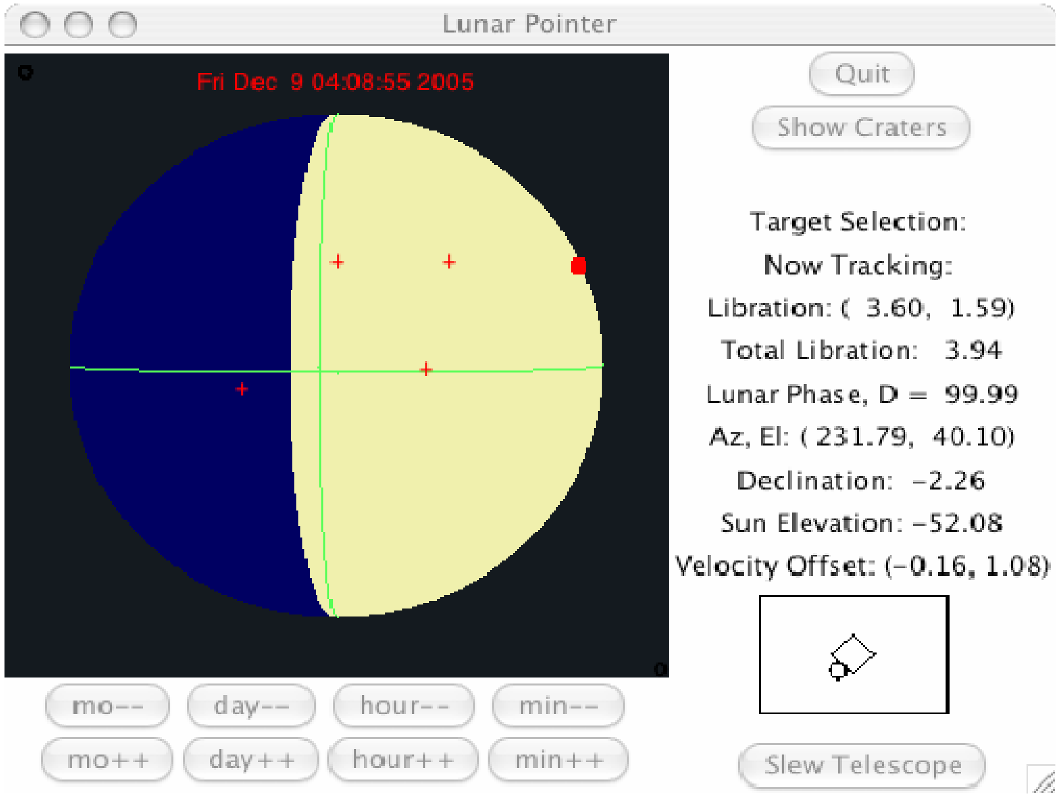


FIGURE 14-4.–Comparison of calculated thermal performance expected from Apollo 11, 14, and 15 LRRR arrays.

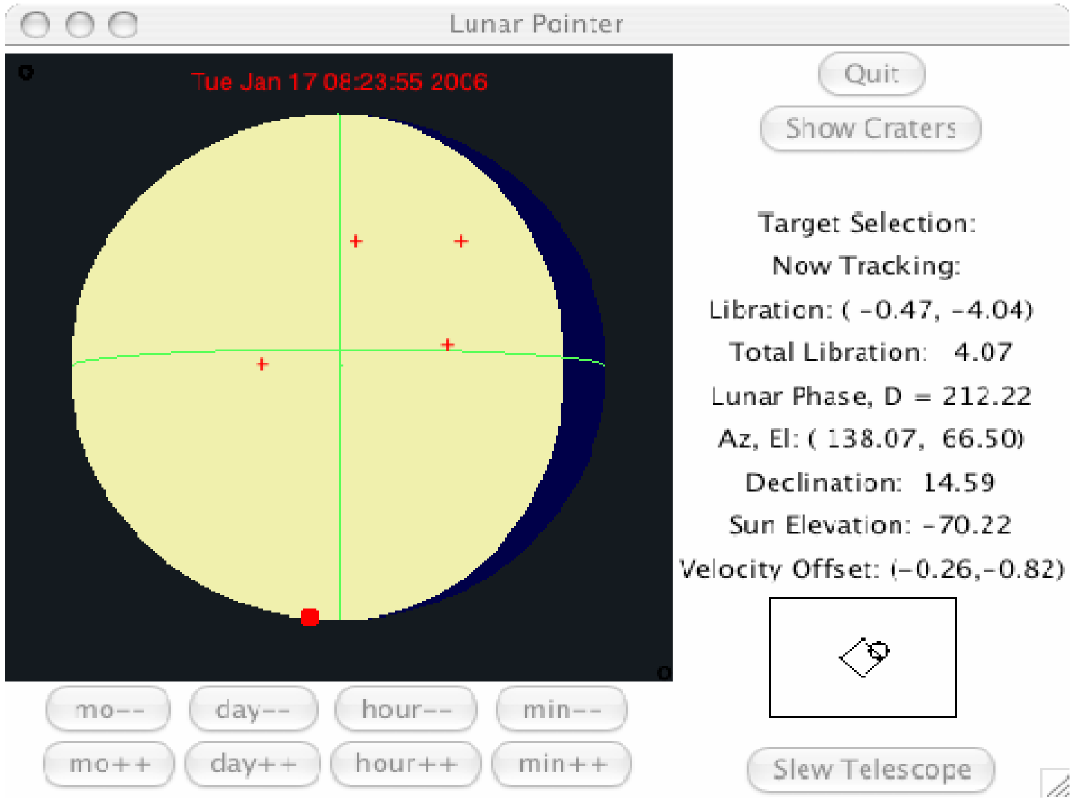
Putting it all together

- Shortfall from normal-incidence central irradiance due to:
  - velocity aberration: 0.64–0.86
  - angular offset: 0.5–1.0
  - thermal degradation: 0.7–1.0 (for Apollo 15)
  - manufacturing tolerance: 0.90–1.0
- amounts to 0.20–0.86
- Now using a tophat with angular diameter λ/D (Φ=2.89 arcsec at 532 nm) and associated TIR de-rating of 0.182, together with above detrimental effects, and 0.93 reflection loss from surface, we must de-rate the Apollo performance by a factor of 0.034–0.146
- Equivalent to tophat of 8–15 arcsec of uniform irradiance

Two cases



- Libration angle: 3.94° → 0.84
- Vel. aber.: 1.09 arcsec → 0.71
- Sun: –73° to normal → 0.85
- range: 371425 km
- expect 9.8 ± 4 photons/pulse
- 30 s peak was 0.5 photons/pulse
- ratio: 20



- Libration angle: 4.04° → 0.81
- Vel. aber.: 0.86 arcsec → 0.81
- Sun: +35° to normal → 0.70
- range: 404301 km
- expect 6.4 ± 2.7 photons/pulse
- 30 s peak was 0.5 photons/pulse
- ratio: 13

Scaling to Other LLR Stations

- A quick-and-dirty scaling of APOLLO to MLRS and OCA is interesting: assume similar detector/optical performance
- Use aperture, seeing (or image quality), and pulse energy alone
- MLRS: $$\left(\frac{3.5\text{ m}}{0.76\text{ m}}\right)^2\left(\frac{3\text{ arcsec}}{1\text{ arcsec}}\right)^2\left(\frac{100\text{ mJ}}{E}\right)\approx 180\left(\frac{100\text{ mJ}}{E}\right)$$
- So if APOLLO gets 1/4 photons per pulse, MLRS → 1/720
  - using E = 100 mJ
  - if we allow 2 arcsec for a "good" night, this goes to 1/320
- OCA: $$\left(\frac{3.5\text{ m}}{1.5\text{ m}}\right)^2\left(\frac{2\text{ arcsec}}{1\text{ arcsec}}\right)^2\left(\frac{100\text{ mJ}}{E}\right)\approx 22\left(\frac{100\text{ mJ}}{E}\right)$$
- So if APOLLO gets 1/4 photons per pulse, OCA → 1/40
  - using E = 200 mJ
  - if we allow 1 arcsec for a "good" night, this goes to 1/10

Source of Degradation

- To get a factor of 16 degradation at the array, we need a factor of 4 surface degradation (since light passes through twice)
- Dust is a very likely culprit
  - Apollo 17 astronauts saw glow & rays scattering at sunrise (from orbit)
  - Apollo 17 LEAM module saw tremendous dust activity at lunar sunrise/sunset, including horizontal transport
  - LEAM module began to overheat in lunar day: possibly albedo reduction due to dust plus thermal blanketing effect
  - Dynamic dust fountain model (Timothy Stubbs et al.) predicts many-kilometer ballistic lofting of dust due to charging (solar radiation and solar wind)
- Micrometeorites and meteoric ejecta can pit surface of glass
  - could have a frosted surface by now