Improvement of Current Refraction Modeling in Satellite Laser Ranging (SLR) by Ray Tracing through Meteorological Data

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Abstract

The accuracy of current modern space-based geodetic systems such as Satellite Laser Ranging (SLR), Very Long Baseline Interferometry (VLBI), the Global Positioning System (GPS), and satellite altimetry all suffer from limitations in the modeling of atmospheric refraction corrections. The current modeling of atmospheric refraction in the analysis of SLR data comprises the determination of the atmospheric delay in the zenith direction and subsequent projection to a given elevation angle, using a mapping function (MF). Recently a new zenith delay (ZD) model of sub-millimeter accuracy [Mendes and Pavlis, 2004] and a new MF of sub-centimeter accuracy [Mendes et al., 2002] were developed, applicable to the wavelengths used in modern SLR instrumentation.

We have already assessed and validated the new ZD model and MF's using 2-d ray tracing and globally distributed data from the Atmospheric Infrared Sounder (AIRS), the European Center for Medium Weather Forecasting (ECMWF) and the National Center for Environmental Prediction (NCEP). However, the models still remain far from the required sub-millimeter accuracy goal for future SLR analysis standards as set forth by the International Laser Ranging Service (ILRS) based on the requirements place on SLR by the Global Geodetic Surveying System (GGOS) [Pearlman et al., 2005].

To further improve atmospheric delay modeling, we need to look at the application of ray tracing and horizontal refractivity gradients on SLR data collected at the core SLR sites around the globe. We have found horizontal gradient delays of up to 5 cm at an elevation angle of 10° at certain times of year and SLR site locations. The effects of applying ray tracing results, including horizontal gradients to a set of global SLR geodetic data resulted in reduction of the observation residuals by up to 45% in variance, and 3 mm in RMS. This is a highly significant contribution for the SLR technique's effort to reach an accuracy at the 1-mm level this decade.

Introduction

All current models of atmospheric delay for SLR observations assume a spherically symmetric atmosphere, ignoring horizontal gradients in the refractive index of the atmosphere. In order to improve models of atmospheric delay, horizontal gradients in the atmospheric refractive index need to be understood and modeled on a global scale. Currently, ignoring horizontal gradients is the largest source of error in atmospheric delay models for SLR at low elevation angles. We have demonstrated that the contribution of horizontal gradients to the total atmospheric delay is primarily at the few-centimeter level at 10° elevation, and can be as large as 5 cm at certain locations (where SLR stations operate) and times of year. Although centimeter delay corrections seem small, horizontal gradients need to be taken into account because they can lead to significant errors in estimated vertical and to a lesser extent, horizontal station coordinates, which in turn affect the accuracy of the scale and origin of the International Terrestrial Reference Frame (ITRF) [*Altamimi et al.*, 2002].

Presently, we are attempting to develop the infrastructure and enabling science that will allow us to develop future ITRF's with an origin accurate to 1 mm at its epoch of definition and a stability of 0.1 mm/year or better, a tenfold improvement over our current capabilities that are no better than 0.4 parts per billion (~3 mm) in origin stability. Part of this effort requires the improvement of our atmospheric delay corrections to the SLR data with an accuracy of 1 mm or better. In the past, VLBI groups used NCEP fields to calculate refractivity gradients in order to make comparisons with results obtained from their VLBI geodetic data. However, we are entering a new era where global snapshots are available from satellite-borne instruments on a daily basis and at much higher spatial resolution than weather models. We will primarily be using atmospheric profiles from the AIRS instrument on NASA's AQUA Earth Observing System (EOS) platform in order to compute the atmospheric delay by ray tracing and including horizontal refractivity gradient contributions. We also use global data sets from ECMWF and NCEP to supplement, compare, and validate the AIRS results.

Methodology

The optical path length between the tracking station and satellite is defined as the integral of the group refractive index along the path of the ray. We define the atmospheric delay as the difference between the optical path length and the geometric path length:

$$d_{atm} = \int_{ray} n ds - \int_{vac} ds \quad (1)$$

where n is the group refractive index, and $ds = dr/sin\theta$ is a differential element of length along the path of the ray. The subscripts ray and vac in the integral indicate the actual ray path and vacuum path of the signal. If we express the group refractive index in terms of the group refractivity, N

$$n = 1 + 10^{-6} N \quad (2)$$

then the atmospheric delay can be expressed as:

$$d_{atm} = 10^{-6} \int_{ray} N ds + \left[\int_{ray} ds - \int_{vac} ds \right] \quad (3)$$

where the first term represents the excess path delay or velocity error, and the bracketed term is the delay due to the bending of the ray, called the geometric delay (d_{geo}).

By expanding the refractivity, N, in a Taylor's series expansion around the laser site [*Gardner*, 1977], the total atmospheric delay including gradients, can be written as:

$$d_{atm} = 10^{-6} \int_{r_s}^{r_a} \frac{N(r)}{sin\theta} dr + d_{geo} + \left[\int_{r_s}^{r_a} \frac{N_{ns}(r)\rho}{sin\theta} dr \right] cos\alpha + \left[\int_{r_s}^{r_a} \frac{N_{ew}(r)\rho}{sin\theta} dr \right] sin\alpha \quad (4)$$

where θ is the elevation angle at altitude calculated using Snell's law, $\rho = r\phi$ represents horizontal arc distance from the station, r_s is the geocentric radius of the station, and r_a is the geocentric radius at the top of the atmosphere. The third and fourth terms are the contribution to the total delay from horizontal gradients, where N_{ns} and N_{ew} are the North-South (NS) and East-West (EW) components of the horizontal refractivity gradient. The $cos\alpha$ and $sin\alpha$ terms project the NS and EW gradient components onto the azimuth of the observation.

Ray Tracing

The most accurate and comprehensive way of calculating the atmospheric delay is by using a technique known as ray tracing. The computation process is based on geometric optics theory applied over a series of thin spherical shells, concentric with the earth, within which a constant refractivity is assumed. Using Snell's law to calculate elevation changes and horizontal refractivity gradients to calculate azimuth changes along the ray's path, one can trace the ray accurately through the atmosphere in two or three dimensions and calculate the total delay by integrating the incremental delay at each atmospheric layer until the top of the atmosphere using equation (4).

Atmospheric delay modeling has been neglected for decades, with the official model for SLR being that of Marini and Murray [1973], developed in the early 70's. Only in recent years, has an improved ZD model [*Mendes and Pavlis*, 2004] and MF [*Mendes et al.*, 2002] been developed, applicable to the wavelengths used in present day SLR. The new ZD model and MF, called the Mendes-Pavlis (M-P) model, was adopted for the reanalysis of all SLR data from 1976 till present, and in the production of the weekly operational products, beginning January 1, 2007. However, these are still models and the assumption of uniform, spherically symmetric refractive index layers made in their development is unreasonable as it makes the delay only dependent on elevation and not on azimuth. We now have the capability to use atmospheric fields from AIRS that are available at near-real time, twice-daily (day and night), and on a global scale. This enables us to compute the total delay, including gradients, by ray tracing at any elevation and azimuth using real-time atmospheric conditions at any chosen SLR site on the globe. Although ray tracing can be computationally expensive and involves many steps, the results are more physically meaningful than those calculated from delay models, and with the computing facilities available today, the benefits far outweigh the costs. Furthermore, the process can be highly automated at a single, "clearinghouse" type location, with the results disseminated to the users via Internet services and the World Wide Web.

Horizontal Refractivity Gradients

Until now, the contribution from horizontal refractivity gradients to the total atmospheric delay has essentially been ignored in the analysis of SLR data. Previous studies of horizontal gradients (see, for example, *Gardner et al.*, 1978; *MacMillan*, 1995; *Chen and Herring*, 1997) were all based on developing models to account for the gradient delay. We have found these models to be unreasonable in estimating the delay for several reasons: The mapping function used by *Chen and Herring* [1997] ignores higher order terms in the expansion of the continued fraction used in calculating the mapping function, and the development is based on the fact that the gradients have the same direction at all levels in the atmosphere. The model developed by *MacMillan* [1995] includes an extra term, $cot(e)$, that accounts for larger gradient changes at low elevation angles, but the delay becomes infinite at small elevation angles as a result. The *Gardner* [1978] gradient model is dependent on surface gradient values of temperature and pressure, thereby ignoring gradient values at higher altitudes that could introduce significant errors in the magnitude and sign of the gradient delay.

We calculate the gradients in a more direct and accurate way by ray tracing using the third and fourth terms in equation (6) combined with atmospheric profiles from AIRS, ECMWF, and NCEP. Our initial results show that the largest gradient variations occur as a result of seasonal and diurnal changes. Stations situated in mountainous regions, such as McDonald, TX and Monument Peak, CA had larger horizontal pressure gradients, while stations in close proximity to large bodies of water such as Yarragadee, Australia, had larger horizontal temperature gradients. No significant non-hydrostatic (wet) gradients were found, with maximum wet delays only reaching a few tenths of a millimeter during the summer at Greenbelt, MD. Maximum NS gradient delays of up to 5 cm were found at Yarragadee and Herstmonceux, UK, at an elevation angle of 10°, while standard deviations ranged from 6-12 mm depending on location and time of year. The EW gradients were smaller in magnitude and variability than the NS gradients.

Results

We now look at the impact of using ray tracing with AIRS, ECMWF and NCEP data on the analysis of a set of real SLR data for the geodetic satellite LAGEOS 1 during 2004 and 2005 and for 10 of the globally distributed core SLR stations. We analyze our results by looking at the RMS and variance percent difference between the 'corrected' SLR residuals with the atmospheric delay estimated by ray tracing and including horizontal gradients, and the 'original' residuals, that use the M-P model for calculating the atmospheric delay. The total number of observations used in the statistics for all stations is 47664. Positive values of RMS and variance indicate improvement in the results.

The results when including the gradients in Figure 1 (i.e. delay = model + gradients) show that the residual variances when using AIRS data are reduced by up to 10-15% in variance when only gradient corrections are applied. ECMWF and NCEP results also show improvement with residual reductions ranging from 5-10%. AIRS ray tracing results had a greater improvement in RMS and variance when compared to

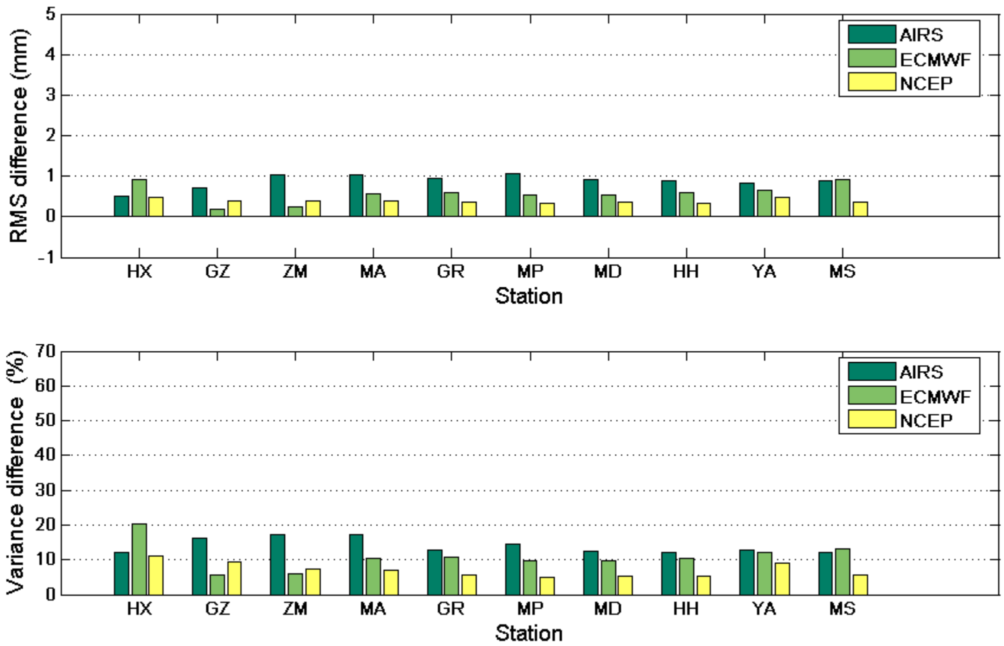

Figure 1. *RMS (top) and variance (bottom) differences between the original residuals (model) and the gradient-corrected residuals (model + gradients) for stations: HX (Herstmonceux, UK), GZ (Graz, Austria), ZM (Zimmerwald, Switzerland), MA (Matera, Italy), GR (Greenbelt, MD), MP (Monument Peak, CA), MD (McDonald, TX), HH (Hartebeesthoek, South Africa), YA (Yarragadee, Australia), and MS (Mt. Stromlo, Australia).*

NCEP and ECMWF results for all stations. This can probably be attributed to the higher resolution of the AIRS data, providing the ability to calculate the gradients on a much finer scale.

When the total correction is applied (i.e. delay = ray tracing + gradients) with no dependence on the model, the NCEP results actually show larger improvements than AIRS and ECMWF (Figure 2). However, it is interesting to note that there are instances where we see negative RMS differences for NCEP at Herstmonceux, Graz and Greenbelt, even though the corresponding variances show improvement. This is most likely due to either a large positive or negative bias in the mean of the corrected residuals. There is an overall greater improvement in the results when the total correction is applied, and this can be seen as an increase in variance percent difference from Figure 1 to Figure 2. However, at Yarragadee and Mt Stromlo, AIRS total correction actually does slightly worse than the gradient correction. AIRS variances decrease from 12.8% for the gradient correction, to 12.4% for the total correction at Yarragadee and from 12.3% to 9.8% at Mt Stromlo. High AIRS variabilities in boundary layer pressure and temperatures on the interface between land and ocean at these stations could be a factor in this case.

Summary and future plans

Our current and near-term plans are to improve and generalize our 3-d ray tracing process and to include as many sources as presently available. In a second step, we plan to establish an automated daily service for all SLR-tracked targets with high-accuracy requirements (i.e. those used for the ITRF, sea-level monitoring, etc.), and provide the community with value-added data sets including these improved atmospheric delay corrections.

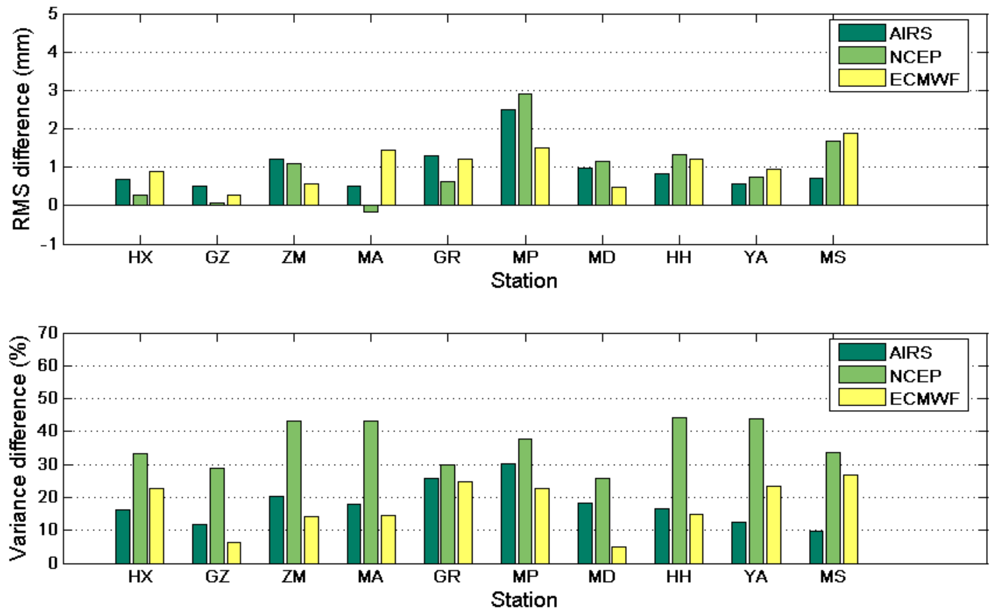

Figure 2. *Differences between the original residuals (model) and the total-corrected residuals (ray-tracing + gradients).*

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