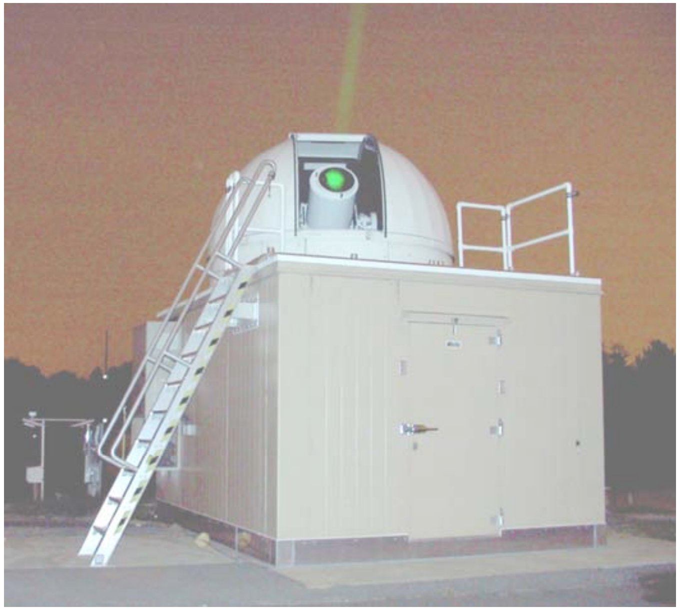
Ray Matrix Approach for the Real Time Control of SLR2000 Optical Elements



John J. Degnan

Sigma Space Corporation

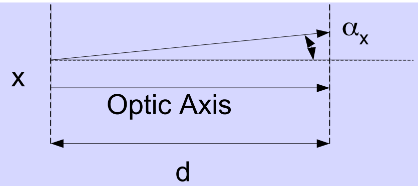
14th International Workshop on Laser Ranging

San Fernando, Spain

7-11 June 2004

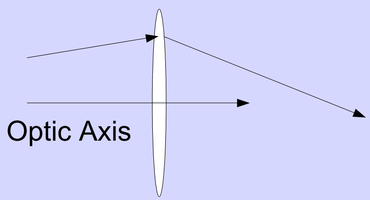
Basic 1D & 2D Ray Matrices*

1D



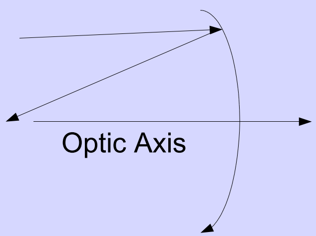
$$\begin{vmatrix} 1 & d \\ 0 & 1 \end{vmatrix}$$

Propagation over distance d



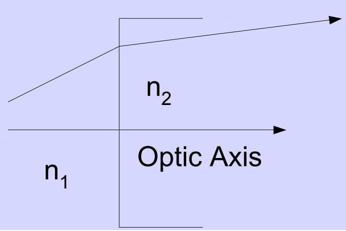
$$\begin{vmatrix} 1 & 0 \\ -1/f & 1 \end{vmatrix}$$

Thin Lens of Focal Length, f



$$\begin{vmatrix} 1 & 0 \\ -2/R & 1 \end{vmatrix}$$

Mirror Surface with Curvature R



$$\begin{vmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{vmatrix}$$

Dielectric Interface

2D

$$\begin{vmatrix} x \\ \alpha \end{vmatrix} \qquad \begin{vmatrix} x \\ y \\ \alpha_x \\ \alpha_y \end{vmatrix}$$

$$\begin{vmatrix} I & dI \\ 0 & I \end{vmatrix}$$

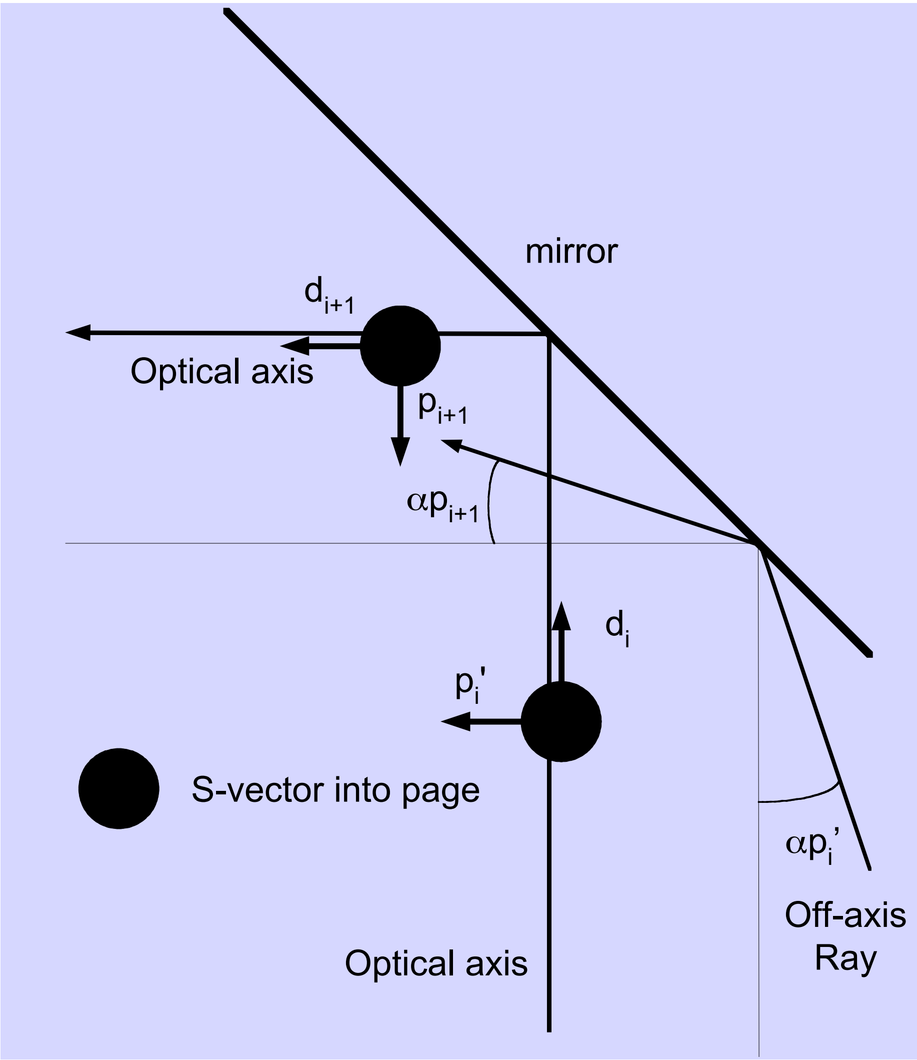
$$\begin{vmatrix} I & 0 \\ -\frac{1}{f} I & I \end{vmatrix}$$

$$\begin{vmatrix} I & 0 \\ -\frac{2}{R} I & I \end{vmatrix} \qquad I = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$\begin{vmatrix} I & 0 \\ 0 & \frac{n_1}{n_2} I \end{vmatrix}$$

*Valid only for a linear optical system. We need to perform a coordinate transformation whenever the beam changes direction

Example of Coordinate System Change: Canted Mirror



$$\hat{s}_{i+1} = \hat{s}'_i = \frac{\hat{d}_{i+1} \times \hat{d}_i}{\sin(2\theta_i)}$$

$$\hat{p}'_i = \hat{d}_i \times \hat{s}'_1$$

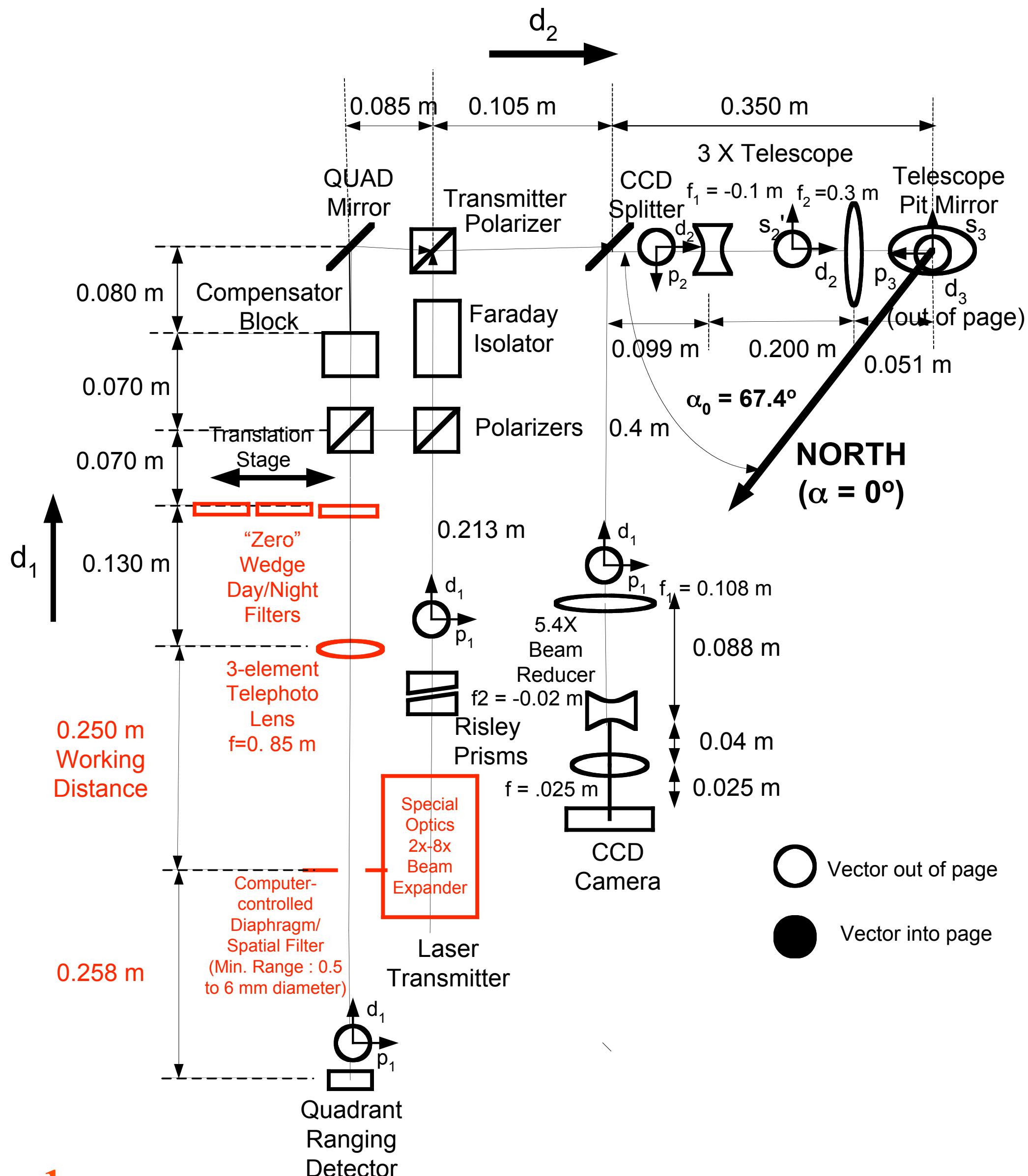
$$\hat{p}_{i+1} = \hat{d}_{i+1} \times \hat{s}_{i+1}$$

$$\begin{vmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x_p \\ x_s \\ \alpha_p \\ \alpha_s \end{vmatrix}$$

Simplified SLR2000 Transceiver*

Automated Devices

- Star CCD camera periodically updates mount model
- 3x telescope compensates for thermal drift in main telescope focus
- Beam magnifier controls laser spot size and divergence at exit aperture
- Risley prism pair controls transmitter point-ahead
- Variable iris controls receiver field of view (FOV)
- Quadrant detector provides fine pointing corrections



*Planned modifications in red

Transceiver Bench Matrices

Transmitter

$$M_{1a} = \begin{vmatrix} 3\Lambda & 2.267\Lambda \\ 0 & 0.333\Lambda \end{vmatrix}$$

Quadrant Detector

$$M_{1b} = \begin{vmatrix} 0.079\Lambda & 2.57\Lambda \\ -0.392\Lambda & -0.101\Lambda \end{vmatrix}$$

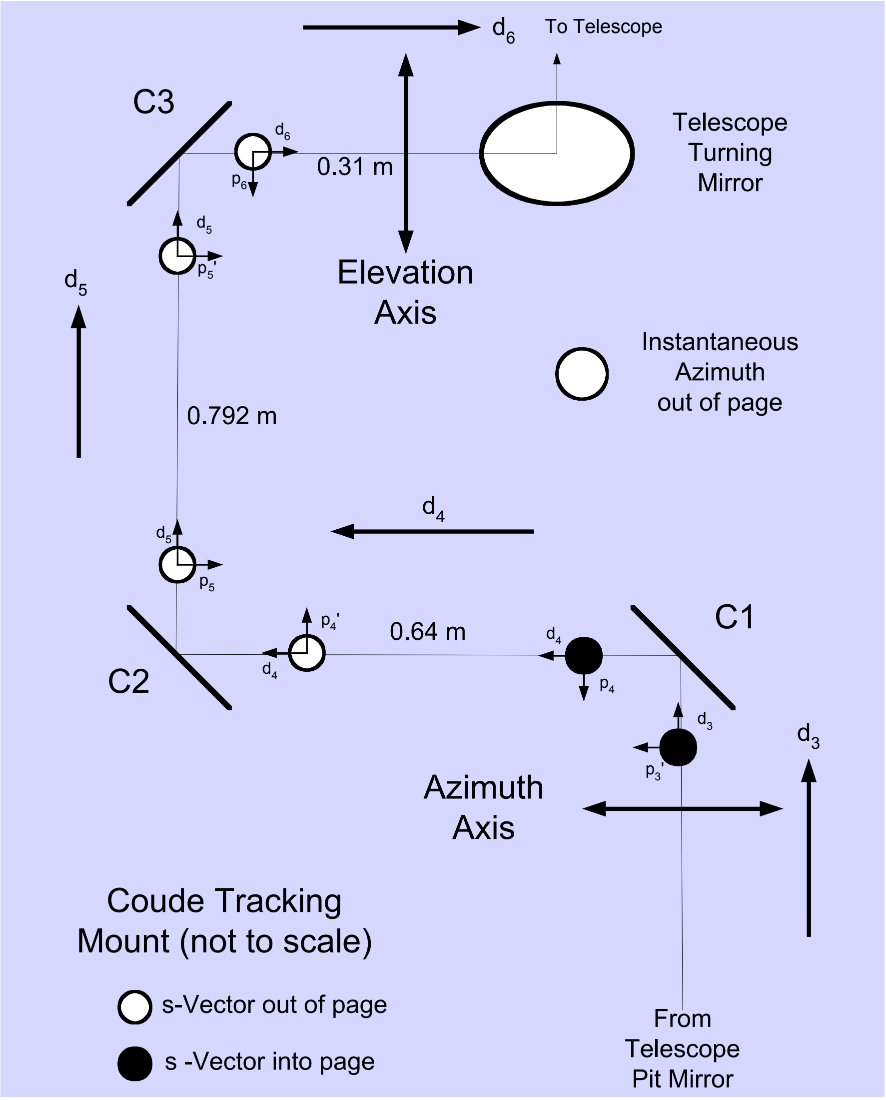
Star Camera

$$M_{1c} = \begin{vmatrix} -35.559\Lambda & 0.405\Lambda \\ -2.469\Lambda & 0 \end{vmatrix}$$

General Form

$$M_{1x} = \begin{vmatrix} a_x\Lambda & b_x\Lambda \\ c_x\Lambda & d_x\Lambda \end{vmatrix} \qquad \Lambda = \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix}$$

Coude Mount Matrix



$$M_2 = \begin{vmatrix} \Gamma & d_C \Gamma \\ 0 & \Gamma \end{vmatrix}$$

$$\Gamma = \begin{vmatrix} -\cos\gamma & -\sin\gamma \\ \sin\gamma & -\cos\gamma \end{vmatrix}$$

$$\gamma = \alpha - \alpha_0 - \varepsilon$$

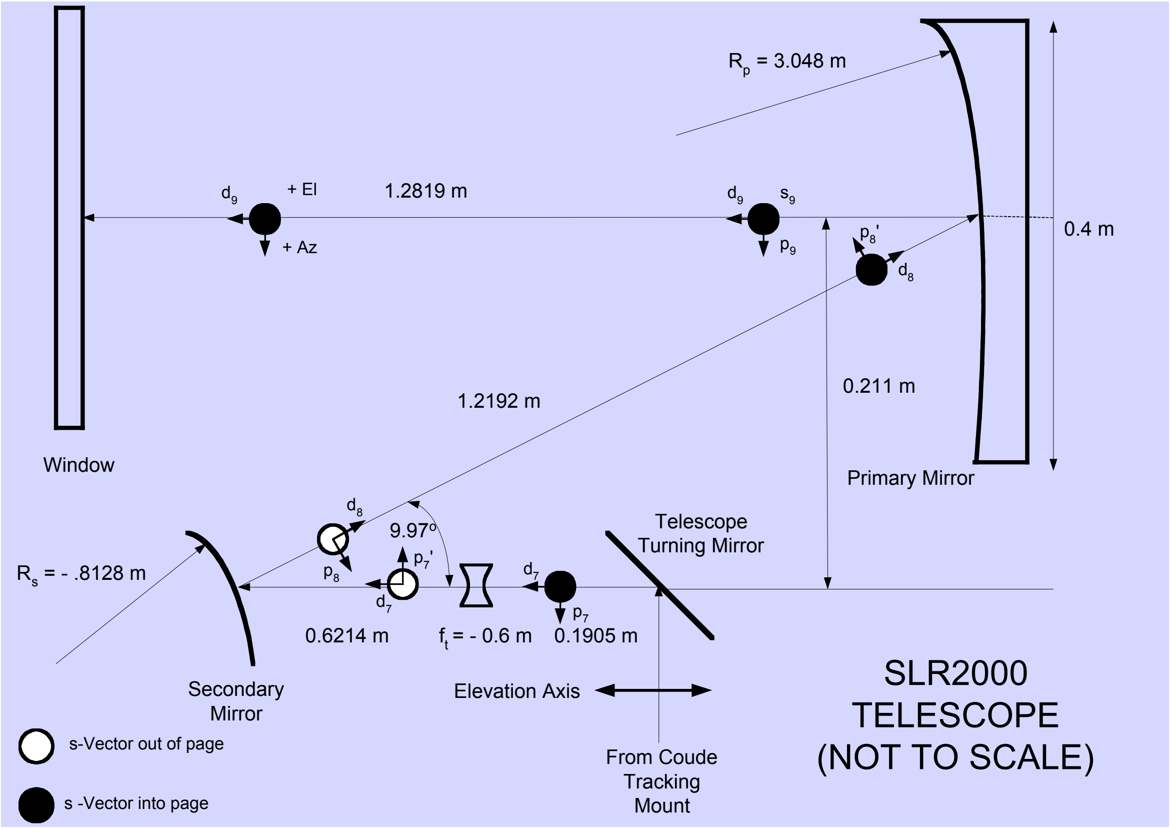
α= mount azimuth angle

ε = mount elevation angle

α_0 = azimuth angle of transceiver axis at the Coude pit mirror = 67.4° (SLR2000)

d_C = Coude path length = 1.742 m

Telescope Assembly Matrix



$$M_3 = \begin{vmatrix} m_T I & d_T I \\ 0 & \dfrac{1}{m_T} I \end{vmatrix}$$

m_T = telescope magnification = 10.16

d_T = 5.758 m

Total SLR2000 System Matrix

Outgoing Rays

$$M_x = M_3 M_2 M_{1x} = \begin{vmatrix} A_x\Gamma' & B_x\Gamma' \\ C_x\Gamma' & D_x\Gamma' \end{vmatrix}$$

$$\Gamma' = \begin{vmatrix} -\sin\gamma & \cos\gamma \\ -\cos\gamma & -\sin\gamma \end{vmatrix}$$

$$A_x = m_t\left(a_x + d_C c_x\right) + d_T c_x$$

$$B_x = m_t\left(b_x + d_C d_x\right) + d_T d_x$$

$$C_x = \frac{c_x}{m_t}$$

$$D_x = \frac{d_x}{m_t}$$

Incoming Rays

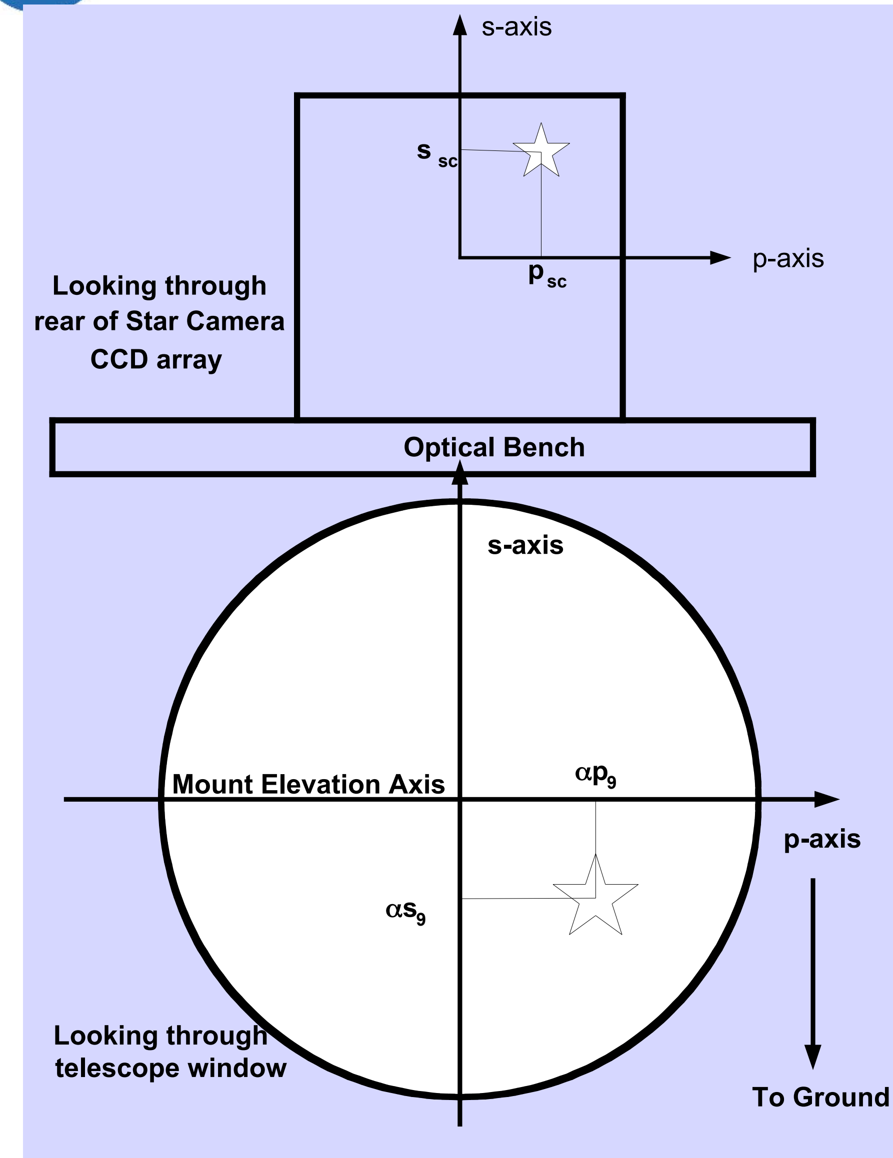
$$M_x^{-1} = \begin{vmatrix} D_x\Gamma'^T & -B_x\Gamma'^T \\ -C_x\Gamma'^T & A_x\Gamma'^T \end{vmatrix}$$

$$\Gamma'^T = \begin{vmatrix} -\sin\gamma & -\cos\gamma \\ \cos\gamma & -\sin\gamma \end{vmatrix}$$

$$M_{1x} = \begin{vmatrix} a_x\Lambda & b_x\Lambda \\ c_x\Lambda & d_x\Lambda \end{vmatrix}$$

x = a Transmitter
b Quadrant Detector
c Star Camera

Star Calibrations



$$\begin{bmatrix}\Delta\alpha\\ \Delta\varepsilon\end{bmatrix} = \frac{0.5\,arc\sec}{pixel}\begin{bmatrix}\sec\varepsilon\sin\gamma & -\sec\varepsilon\cos\gamma\\ \cos\gamma & \sin\gamma\end{bmatrix}\begin{bmatrix}n_p\\ n_s\end{bmatrix}$$

$$\gamma = \alpha - \alpha_0 - \varepsilon$$

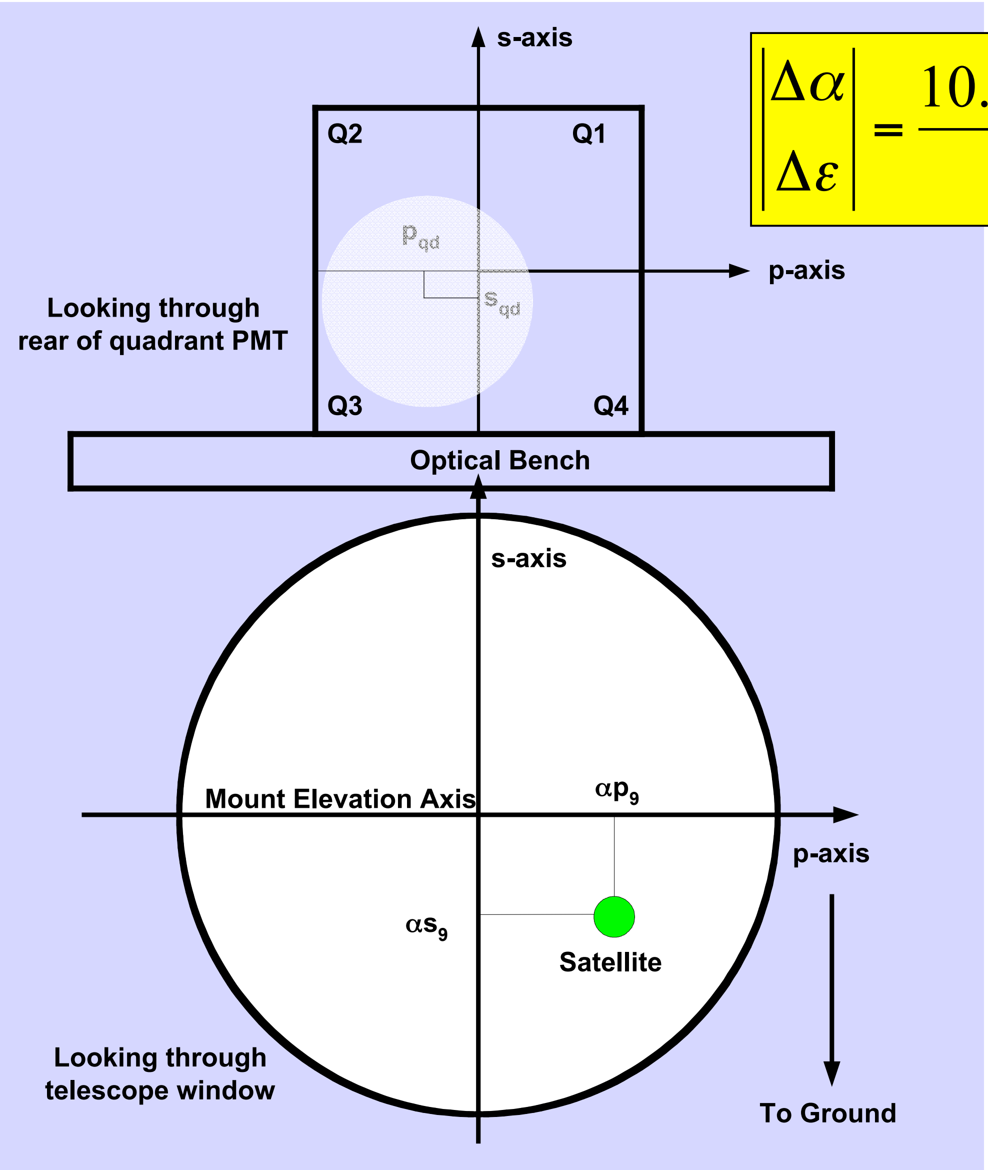
$\Delta\alpha$ = star azimuth offset

$\Delta\varepsilon$ = star elevation offset

n_p = CCD pixel column

n_s = CCD pixel row

Quadrant Pointing Correction



$$\begin{vmatrix}\Delta\alpha \\ \Delta\varepsilon\end{vmatrix} = \frac{10.5 arc\sec}{mm}\begin{vmatrix}\sec\varepsilon\sin\gamma & -\sec\varepsilon\cos\gamma \\ \cos\gamma & \sin\gamma\end{vmatrix}\begin{vmatrix}p_c \\ s_c\end{vmatrix}$$

$$\gamma = \alpha - \alpha_0 - \varepsilon$$

$\Delta\alpha$ = azimuth pointing correction

$\Delta\varepsilon$ = elevation pointing correction

p_c = horizontal centroid coordinate

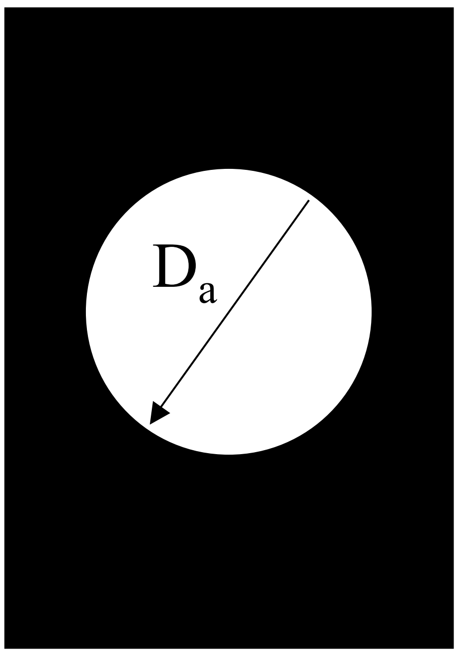
s_c = vertical centroid coordinate

Receiver Field of View

$$\begin{vmatrix}\vec{x}_a \\ \vec{\alpha}_a\end{vmatrix} = \begin{vmatrix} 0 & -25.908\Gamma'^T \\ 0.039\Gamma'^T & -24.387\Gamma'^T \end{vmatrix}\begin{vmatrix}\vec{x}_T \\ \vec{\alpha}_T\end{vmatrix}$$

$$\vec{x}_a = -\frac{25.908m}{rad}\Gamma'^T\,\vec{\alpha}_T$$

$$|x_a| = \sqrt{\vec{x}_a^{\,T}\vec{x}_a} = \frac{25.908m}{rad}|\alpha_T| = \frac{0.125mm}{arc\,\mathrm{sec}}|\alpha_T|$$



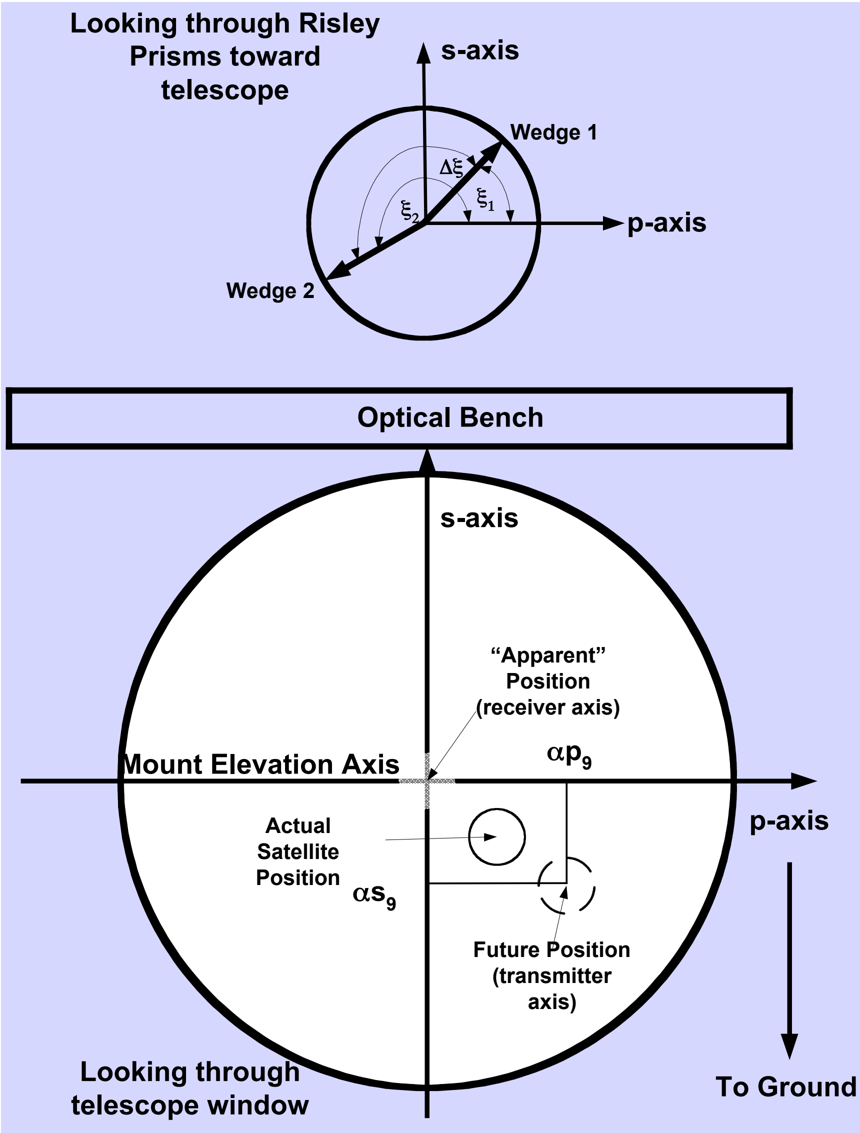
Stepper-controlled Iris

$$D_a = \frac{0.125mm}{arc\,\mathrm{sec}}FOV$$

D_a = iris diameter

FOV = Full Receiver Field of View in arcsec

Transmitter Point-Ahead



$$\begin{vmatrix} \alpha p_{rp} \\ \alpha s_{rp} \end{vmatrix} = m_T \tau_r \begin{vmatrix} -\sin\gamma\cos\varepsilon & -\cos\gamma \\ \cos\gamma\cos\varepsilon & -\sin\gamma \end{vmatrix} \begin{vmatrix} \dot{\alpha} \\ \dot{\varepsilon} \end{vmatrix}$$

$$\gamma = \alpha - \alpha_0 - \varepsilon$$

αp_{rp} =Risley prism output angle projected into p plane

αs_{rp} = Risley prism output angle projected into s plane

m_T = post-Risley magnification of transmitter =30.48

τ_r = pulse roundtrip time of flight

$\dot{\alpha}$ = azimuth rate

$\dot{\varepsilon}$ = elevation rate

Computing Risley Prism Orientations

$$\begin{vmatrix} \alpha p_{rp} \\ \alpha s_{rp} \end{vmatrix} = \begin{vmatrix} \delta_1 \cos\xi_1 + \delta_2 \cos\xi_2 \\ \delta_1 \sin\xi_1 + \delta_2 \sin\xi_2 \end{vmatrix} = m_T \begin{vmatrix} -\sin\gamma\cos\varepsilon & -\cos\gamma \\ \cos\gamma\cos\varepsilon & -\sin\gamma \end{vmatrix} \begin{vmatrix} \dot{\alpha}\tau_r \\ \dot{\varepsilon}\tau_r \end{vmatrix}$$

δ_1 = half cone angle traced by wedge 1
δ_2 = half cone angle traced by wedge 2
ξ_1 = wedge 1 angle relative to home position
ξ_2 = wedge 2 angle relative to home position

Solve above two equations for two unknown Risley orientations ξ_1 and ξ_2:

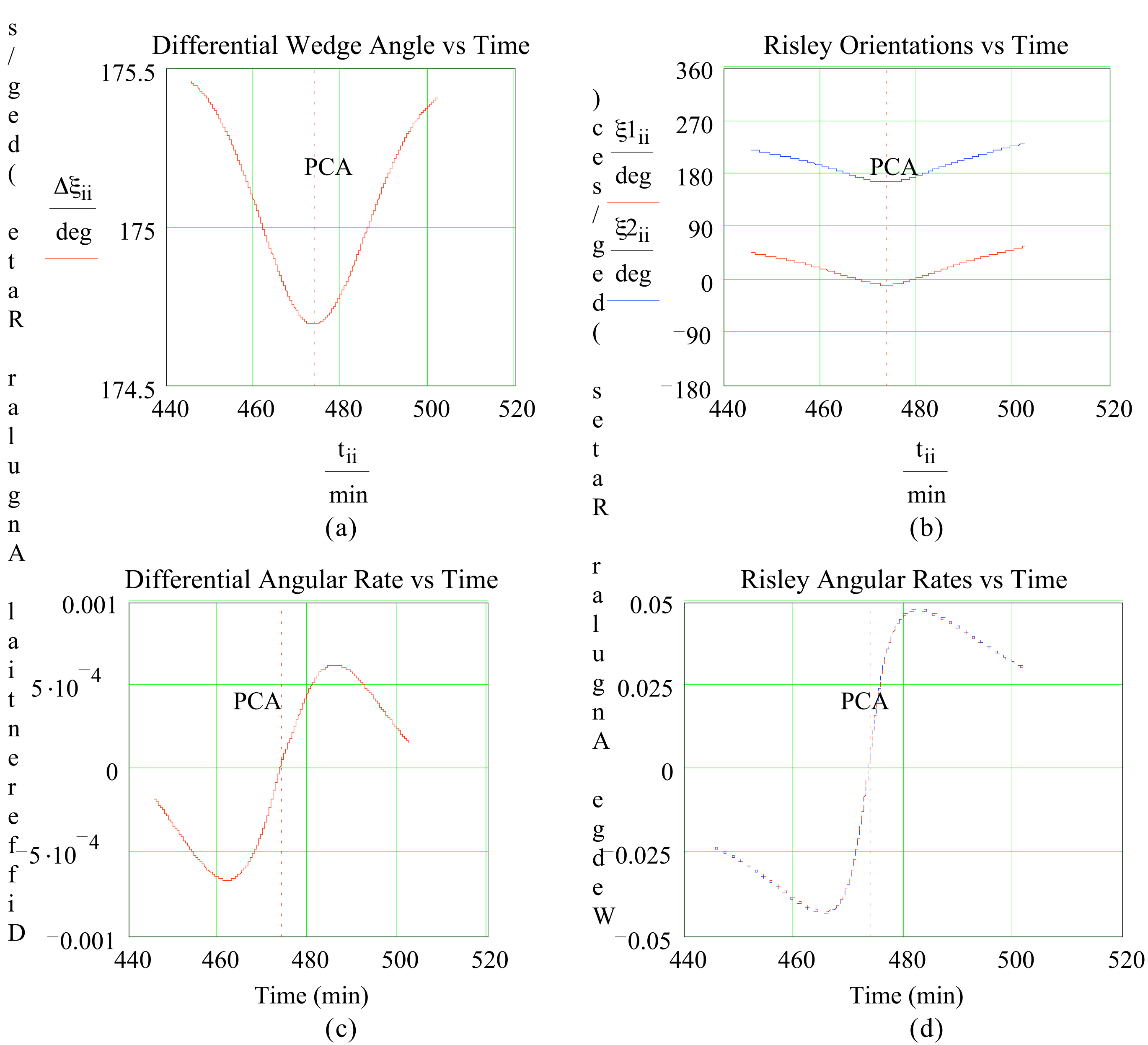
$$\Delta\xi \equiv \xi_2 - \xi_1 = \cos^{-1}\left\{ \frac{m_T^2 \left[(\dot{\alpha}\tau_r \cos\varepsilon)^2 + (\dot{\varepsilon}\tau_r)^2 \right] - \left(\delta_1^2 + \delta_2^2\right)}{2\delta_1\delta_2} \right\}$$

$$\cos(\xi_1) = \frac{-m_T \left[(\dot{\alpha}\tau_r \cos\varepsilon)(\delta_1 \sin\gamma + \delta_2 \sin(\gamma - \Delta\xi)) + (\dot{\varepsilon}\tau_r)(\delta_1 \cos\gamma + \delta_2 \cos(\gamma - \Delta\xi)) \right]}{\delta_1^2 + \delta_2^2 + 2\delta_1\delta_2 \cos(\Delta\xi)}$$

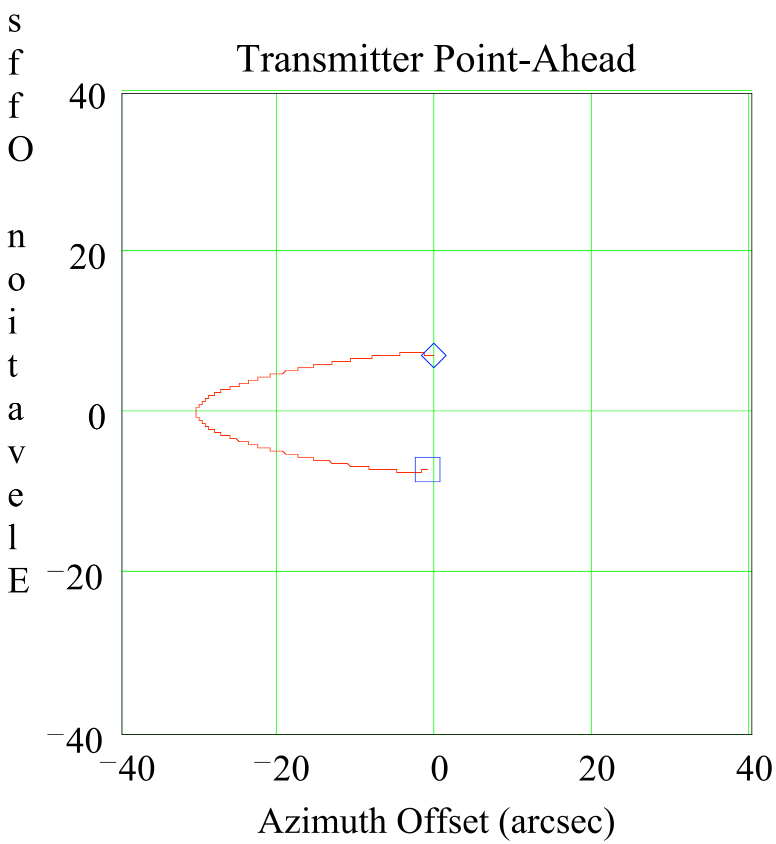
$$\sin(\xi_1) = \frac{m_T \left[(\dot{\alpha}\tau_r \cos\varepsilon)(\delta_1 \cos\gamma + \delta_2 \cos(\gamma - \Delta\xi)) - (\dot{\varepsilon}\tau_r)(\delta_1 \sin\gamma + \delta_2 \sin(\gamma - \Delta\xi)) \right]}{\delta_1^2 + \delta_2^2 + 2\delta_1\delta_2 \cos(\Delta\xi)}$$

$$\xi_2 = \xi_1 + \Delta\xi$$

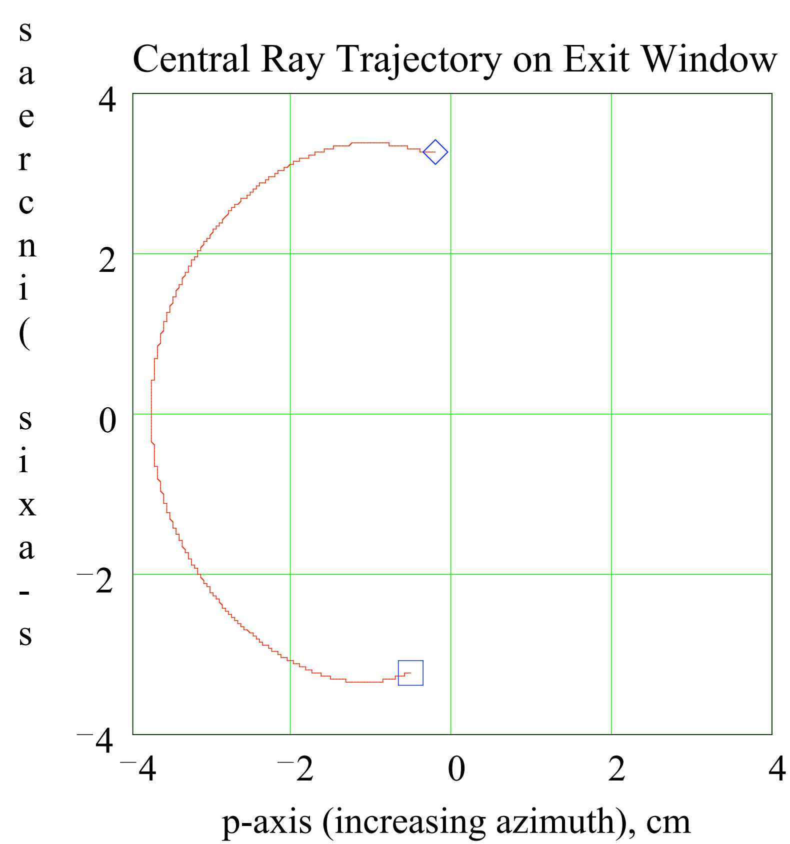
Simulated LAGEOS Pass



Azimuth- Elevation Offsets & Beam Centering



(a)



(b)

Ray Matrices and Gaussian Beams

Paraxial ray matrix theory can be applied to gaussian beam propagation if we define the following complex parameter:

$$\frac{1}{q(z)} = \frac{1}{R(z)} - j\frac{\lambda}{\pi\omega^2(z)}$$

If propagation from a point z_0 to z can be described by the ray matrix

$$M = \begin{vmatrix} A & B \\ C & D \end{vmatrix}$$

then the gaussian beam properties at z are given by

$$\frac{1}{q(z)} = \frac{C + D\dfrac{1}{q(z_0)}}{A + B\dfrac{1}{q(z_0)}}$$

Controlling Beam Divergence

The ray matrix which takes the transmitter beam from the Risley Prism output to the far field is of the form

$$FF = \lim_{r\to\infty}\begin{vmatrix} I & rI \\ 0 & I \end{vmatrix}\begin{vmatrix} m_t\Gamma' & d_t\Gamma' \\ 0 & \dfrac{1}{m_t}\Gamma' \end{vmatrix} = \begin{vmatrix} m_t\Gamma' & \dfrac{r}{m_t}\Gamma' \\ 0 & \dfrac{1}{m_t}\Gamma' \end{vmatrix}$$

where m_t is the total transmitter magnification. From our gaussian parameter, we obtain the following for the full beam divergence

$$\theta_t = 2\frac{\omega(r)}{r} = \frac{2\lambda}{\pi m_t \omega(z_0)}\sqrt{1+\left(\frac{\pi\omega^2(z_0)}{\lambda R(z_0)}\right)^2} = \theta_{\min}\sqrt{1+\left(\frac{\pi\omega^2(z_0)}{\lambda R(z_0)}\right)^2}$$

where $\omega(z_0)$ and $R(z_0)$ are the beam radius and phasefront radius of curvature out of the computer-controlled telescope in the transmit path. To first order, beam divergence varies linearly with the lens displacement from perfect focus.

Beam Divergence vs Phase Front Curvature at Risleys

Radius of SLR2000 primary, $a = 20$ cm

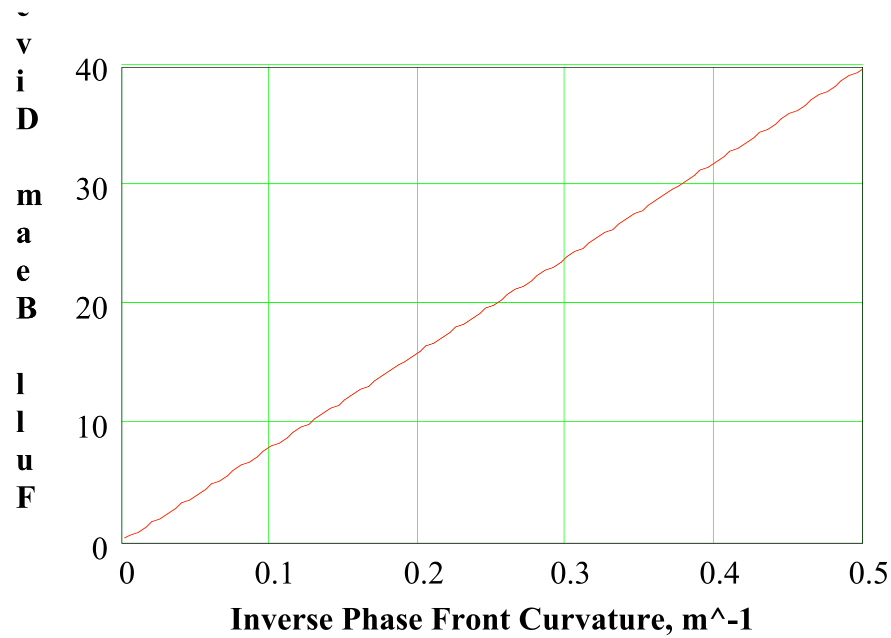
Optimum spot radius at window*, $\omega_{opt} = a/1.12 = 17.9$ cm

Post-Risley magnification, $m_t = 30.48$

Optimum beam radius at Risley, $\omega(z_o) = \omega_{opt}/m_t = 5.9$ mm

Minimum Divergence, $\theta_{min} = 0.388$ arcsec

$$\theta_{jj} := \theta_{min} \cdot \sqrt{1 + \left(\frac{\pi \cdot \omega_0^{\,2}}{\lambda} \cdot Rinv_{jj} \right)^2}$$



Summary

- Ray matrix approach provides us with the mathematical tools to calculate in real time:
  - Scale factor and angular rotation for converting star image offsets from center in the CCD camera to azimuth and elevation biases
  - Scale factor and angular rotation for converting quadrant centroid position to satellite pointing correction in az-el space
  - Transmitter point ahead as a function of round trip time-of-flight and the instantaneous azimuthal and elevation angular rates
  - Iris diameter (spatial filter) setting for a given receiver FOV
  - Transmitter beam size divergence as a function of transmit telescope defocus