Real-Time Separation of Atmospheric Tip-Tilt Signal from Lunar Surface

Xiong Yaoheng Guo Rui Yunnan Observatory, Chinese Academy of Sciences

14th International Laser Ranging Workshop, San Fernando, Cadiz, Spain June 8, 2004

1. Atmospheric Turbulence Effects on Laser Beam Propagation



Laser beam at far-field

Short-term beam wander:

$$\langle \rho_c^2 \rangle = \frac{10.22Z^2}{k^2 r_0^{\frac{5}{3}} D^{\frac{1}{3}}}$$

Short-term beam spread:

$$\left\langle \rho_{S}^{2} \right\rangle = \frac{4Z^{2}}{k^{2}D^{2}} + \frac{D^{2}}{4} \left(1 - \frac{Z}{F}\right)^{2} + \frac{17.6Z^{2}}{k^{2}r_{0}^{2}} \left[1 - 0.48 \left(\frac{r_{0}}{D}\right)^{\frac{1}{3}}\right]^{\frac{6}{5}}$$

Long-term beam spread:

$$\langle \rho_L^2 \rangle = \frac{4Z^2}{k^2 D^2} + \frac{D^2}{4} \left(1 - \frac{Z}{F}\right)^2 + \frac{17.6Z^2}{k^2 r_0^2}$$

• Here,

k wave number, D laser transmitter diameter Z laser propagation axis and coordinate F radius of curvature of laser beam r_o Fried's coherence length, $5 \sim 20$ cm

• Method:

Maxwell wave equation \rightarrow *Markov* approximation \rightarrow the second moment and the four moment (approximation) of the field \rightarrow mean square value of above terms



2. Returned Laser Photoelectrons N_r for one laser pulse firing:

$$N_r = \frac{4EN_0A_mA_rT_a^2T_tT_r\eta\alpha}{\pi^2(\theta_e^2 + \theta_s^2)\theta_m^2R^4} \exp\left(-\frac{\rho_c^2}{\rho_e^2 + \rho_s^2}\right)$$

Depend on turbulence terms to be considered

here: ρ_e laser beam radius at target, determined by laser divergence ρ_c short-term beam wander ρ_s short-term beam spread • If tilt is removed, the correction factor for the laser ranging is:

$$\frac{N_r}{N} = \frac{\theta_e^2}{4(\theta_e^2 + \theta_s^2)} \exp\left(-\frac{\theta_c^2}{\theta_e^2 + \theta_s^2}\right)$$

• $1/40 \sim 1/6$, depend on the turbulence

For Kunming station 1.2m laser ranging system on LLR: $N_r=0.17\times(1/40\sim 1/6)$ 3. Detection and Computation of Tip-Tilt from Lunar Surface

- Goal: <u>to compensate atmospheric tip-tilt effect in</u> <u>real- time on LLR</u>
- Object: the extended light source

Method: When performing the LLR, its telescope will point the retroreflector array on the moon surface. Atmospheric tip-tilt signal can only be obtained from a small area of the geomorphologic structure that is near the moon retroreflector. That is to track geomorphologic structure of the moon surface through the motion of the successive images. A $N \times N$ pixels reference image $I_R(x, y)$ that is within the isoplanatic angle is sampled, and a time series $I_1(x, y) \pounds I_2(x, y) \pounds \dots I_L(x, y)$ are sampled as live images.

To determine the displacement between reference image and live images by computing the sum of the absolute difference values of them. For each $N \times N$ pixels live image $I_{I}(x, y)$, a $M \times M$ pixels window is extracted. This central window of the live image is compared with the reference image. The absolute difference values $D \mathfrak{E} \mathfrak{B} \mathfrak{x}$, $\delta \mathfrak{y} \mathfrak{E} \mathfrak{B} \mathfrak{e}$ tween them are given through the absolute difference algorithm:

$$D(\delta x, \delta y) = \sum_{x=0}^{M-1} \sum_{y=0}^{M-1} |I_{R}(x + \delta x, y + \delta y) - I_{L}(x, y)|$$

The position $\mathfrak{L}(\mathbf{x}_{min}, \delta y_{min} \mathfrak{L}(\mathbf{w}))$ obtained where D $\mathfrak{L}(\mathbf{x}, \delta y \mathfrak{L}(\mathbf{w}))$ minimum. The tilt (T_x, T_y) can be determined using a parabolic interpolation.

$$T_x = \delta x_{\min} + \frac{1}{2} \frac{D(\delta x_{\min} - 1, \delta y_{\min}) - D(\delta x_{\min} + 1, \delta y_{\min})}{D(\delta x_{\min} - 1, \delta y_{\min}) + D(\delta x_{\min} + 1, \delta y_{\min}) - 2D(\delta x_{\min}, \delta y_{\min})}$$

$$T_{y} = \delta y_{\min} + \frac{1}{2} \frac{D(\delta x_{\min}, \delta y_{\min} - 1) - D(\delta x_{\min}, \delta y_{\min} + 1)}{D(\delta x_{\min}, \delta y_{\min} - 1) + D(\delta x_{\min}, \delta y_{\min} + 1) - 2D(\delta x_{\min}, \delta y_{\min})}$$

4. Experiment Results

- Using Kunming SLR station 1.2m telescope f = 6m
- 128×128 CCD, $16\mu \times 16\mu$ for one pixel
- About 0."55 /pixel Frame rate: 419
- Sampling area: near the moon retroreflector array Apollo 11, Apollo14, Apollo15 and Lunakhod 2





Apollo11





Apollo14

Lunakhoo 2----→





y component of tip-tilt, 16°16pixels Apollo15



x component of tip-tilt, 16° 16 pixels Apollo15



y component of tip-tilt, $32^{\circ}32$ pixels Apollo15



x component of tip-tilt, $32^{\circ}32$ pixels Apollo15

Real-Time:

32° β 2 pixels window, take 25 seconds to compute 2000 images, \Rightarrow 12.5ms/one image

16° 16 pixels window, take 6 seconds to compute 2000 images, \Rightarrow 3ms/one image

Conclusion:

Detection and separation of atmospheric Tip-Tilt signal real-time from Lunar surface is within the present technique, especially within the atmospheric turbulence scale.

5. Next Plan

• Apply grant for LLR (<u>important</u>)

• Combine the LR system and the AO system at 1.2m telescope, try to achieve real-time tip-tilt compensation for the laser beam on the LLR.



Optical Scheme of Kunming 1.2m LR System for Tilt Correction



