# MOUNT MODEL STABILITY

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*The most important factor in mount model stability is to have a stable mount.*

## ABSTRACT

The ability to acquire an invisible satellite target with minimum delay is vital in autonomous SLR systems as well as those manually operated. It is a particularly stringent requirement in LLR. Yet we don't want to waste hours doing star calibrations too often, so a Mount Model which yields 1 second of arc absolute accuracy for many months is needed.

Since the coefficients of the usual analytically-based Mount Model can yield useful diagnostic engineering information, it is important that they truly represent the errors being modeled, un-corrupted by aliasing from the other errors. There is a great deal of correlation between terms of the usual model, resulting in absurdly high condition numbers in the solution algorithm. For example, terms in secE and tanE are necessarily almost perfectly correlated.

The stability of the model is assessed firstly by the smallness of the pre-fit residuals of subsequent star calibrations, secondly by the variability of the solution coefficients over long periods of time, and thirdly by the condition numbers of the solutions. Some methods for reducing these are given.

The possibility of adding (not replacing) an orthogonal polynomial solution to the usual model is briefly discussed, using even-only or odd-only Legendre Polynomials. If anyone knows of a set of polynomials which are orthogonal over a hemisphere, please let me know!

Preliminary results from the new Mount Stromlo 1-metre telescope suggest that 1 second of arc stability is likely, which is probably due more to its excellent mechanical construction than to any algorithm improvements.

## INTRODUCTION

Computer control of astronomical telescopes large and small, optical and radio blossomed in the 1970s when computer speeds and interfacing capabilities made it feasible. Before then, pointing and tracking were done by such means as, in 1964: "The telescope for the GODLAS system . . was pointed by a modified Nike-Ajax missile tracking mount controlled by two operators guiding on a sunlit satellite under joystick control. One operator controlled azimuth and the other controlled elevation." (*Degnan, 1996*). See also (*Mueller 1964*). The need for rapid acquisition of faint objects and for perfect tracking in astronomy using ever more automated instrumentation, and for tracking invisible objects day and night in space geodesy, made absolute pointing to a few seconds of arc essential. Pioneering work was done in Australia at Mount Stromlo Observatory (*Hovey, 1974*) and the Anglo-Australian Observatory (*Wallace, 1976; Straede and Wallace, 1976)*) in which both surface-fitting methods and the now-commonplace model-fitting methods were developed (although not necessarily invented) and published. Parallel investigations included use of spherical harmonic functions *(Powell, 1977*) and application to X-Y mounts (*Matzke, 1976*). The first published work in SLR I could find was by Randy Ricklefs (*Ricklefs, 1982*) who applied the model-fitting method to arbitrary mount configurations - some of his old Fortran code survives to this very day.

More recently, the explosion in automated small telescopes has produced popular works on the subject (*Trueblood and Genet, 1997; Wallace, 2004*). And last year Bob Meeks of EOS

Technologies in Tucson completed a detailed critique of model fitting (*Meeks, 2003*) aimed at 1 second-of-arc absolute pointing accuracy.

From very limited data under testing conditions, the <u>precision</u> of absolute pointing on both new telescopes at the EOS Space Research Centre on Mount Stromlo is better than 1.5 seconds of arc, assessed from star calibrations and a 23-coefficient model. Anecdotally (*Moore, 2004*), the <u>accuracy</u> is comparable because tracking biases seldom need to be applied while tracking satellites on the SLR 1.0 metre system.

In this paper, the <u>numerical stability</u> of solutions to the standard model plus variants and competitors is discussed, and various tricks to improve confidence in the coefficient values are studied.

**THE MOUNT MODEL**
The alignment errors commonly modeled to describe errors observed during star calibrations, whatever the mount configuration, are:
- Encoder zero-point displacements;
- Encoder scales;
- Tilt of the major axis, e.g. the azimuth axis;
- Non-orthogonality of the secondary axis (e.g. the elevation axis) to the major axis;
- Collimation error, i.e. non-perpendicularity of the optical axis to the secondary axis;
- Bending (flexure) in the telescope tube;
- Bending or torsion of the mount, where applicable (e.g. X-axis in alt/alt mount).

Subject to taste, to these are added "empirical terms" describing repeatable patterns of unidentified or speculated physical origin. It is not common to include electronic effects such as servo, following error which should be physically adjusted during installation, or misalignments of the mirrors in the Coude path because they are not mathematically describable in a solvable way. Great care must be exercised when including empirical terms - they must be repeatable night after night, and even so can easily degrade the final solution.

Rigorously, the model is developed by applying a series of rotation matrices to the incoming beam from a star, that is, to the actual direction that the telescope sees after refraction has been applied to the direction in vacuum. Each rotation matrix describes the effect of a particular misalignment. The outputs are the actual encoder readings for the directions seen at the eyepiece or camera when the star is <u>centered</u> by some means. In practice, the mathematics are made more tractable by <u>linearizing</u>, i.e. by assuming that $sin\theta = \theta$, $\cos\theta = 1$ where $\theta$ is the misalignment under consideration. Of course, this implies that the optical and mechanical alignments are sufficiently good that the products of terms neglected in the approximations really are negligible. This always seems to be assumed, and will be here, too. Thus, the model is algebraically linear in its coefficients, and solution by least squares is simple and un-iterated.

The result is a set of Model Equations (Observation Equations if the random error terms were included) in the form suitable for azimuth/elevation mounts:

$$\delta A_i = \sum_{j=1}^{m} c_j F_j\left(A_i, E_i\right), \quad i = 1, \ldots, n$$

$$\delta E_i = \sum_{j=1}^{m} c_j G_j\left(A_i, E_i\right), \quad i = 1, \ldots, n \tag{1}$$

where:

$\delta A_i$ = [encoder reading - refracted position] is the observation residual in azimuth for star $i$, and similarly for the observation residual in elevation $\delta E_i$;

$c_j$, $j = 1, \ldots, m$ are the model coefficients for the $m$ terms (in our case, $m = 23$);

$F_j\left(A_i, E_i\right)$ is the function in azimuth residual describing the $j^{\text{th}}$ misalignment, depending upon the azimuth $A_i$ and elevation $E_i$ of star $i, i = 1, \ldots, n$; and similarly for the function in elevation residual $G_j\left(A_i, E_i\right)$. The full model used at Stromlo is given in Table 1.

**Table 1**: The Stromlo Mount Model.

| Term | Description | Azimuth Function (F) | Elevation Function (G) |
|------|-------------|----------------------|------------------------|
| 1 | Azimuth encoder offset | $1$ | - |
| 2 | Elevation encoder offset | - | $1$ |
| 3 | Azimuth axis tilt about North | $-\cos A \tan E$ | $\sin A$ |
| 4 | Azimuth axis tilt about East | $-\sin A \tan E$ | $-\cos A$ |
| 5 | Collimation (optical axis misalign) | $\sec E$ | - |
| 6 | Non-orthogonality of Az & El axes | $-\tan E$ | - |
| 7 | Azimuth bearing ellipticity (sin) | $\sin A$ | - |
| 8 | Azimuth bearing ellipticity (cos) | $\cos A$ | - |
| 9 | Elevation bearing ellipticity (sin) | - | $\sin E$ |
| 10 | Elevation bearing ellipticity (cos) | - | $\cos E$ |
| 11 | Telescope tube flexure | - | $\cot E$ |
| 12 | Azimuth encoder scale error | $A/2\pi$ | - |
| 13 | Elevation encoder scale error | - | $E/2\pi$ |
| 14 | Bi-periodic in azimuth (empirical) | $\sin 2A$ | - |
| 15 | Bi-periodic in azimuth (empirical) | $\cos 2A$ | - |
| 16 | Elevation encoder stiction (sin) | - | $\sin A$ |
| 17 | Elevation encoder stiction (cos) | - | $\cos A$ |
| 18 | Elevation bearing stiction (sin) | - | $E \sin A$ |
| 19 | Elevation bearing stiction (cos) | - | $E \cos A$ |
| 20 | Scaled bi-periodic in azimuth (sin) | $\sin 2A \sec E$ | - |
| 21 | Scaled bi-periodic in azimuth (cos) | $\cos 2A \sec E$ | - |
| 22 | Bi-periodic in elevation (sin) | - | $\sin 2A$ |
| 23 | Bi-periodic in elevation (cos) | - | $\cos 2A$ |
| (24) | Observing clock error (not used) | $\sin \phi \cos E$ $-\cos \phi \sin E \cos A$ | $\cos \phi \sin A$ |

Note that the azimuth and elevation residuals are coupled through the coefficients which have effect in both equations, such as the azimuth axis tilt terms. An alternative formulation would describe different coefficients in each of the two residual sets, so that each of the

ModelEquations is completely independent of the other. This was done for the X-Y mount at Orroral (*Luck, 1993*) and worked better there for some unknown reason.

**STATISTICS**

The statistics of interest are:

$N = 2n$ : Number of observations from $n$ stars successfully observed;

$\hat{\sigma}_0$ : Standard error of unit weight = post-fit RMS of residuals if observations are unweighted;

$\sigma_j$ : Standard error of coefficient $j$;

$\rho_{jk}$ : Correlation coefficient between coefficients j and k, from the variance-covariance matrix of solved coefficients;

$\kappa$ : Condition number of the Normal Matrix **N**. $\kappa$ is defined (*Dahlquist and Bjorck, 1974*) by:

$$\kappa(\mathbf{N}) = \|\mathbf{N}\| \|\mathbf{N}^{-1}\| \tag{2}$$

where $\|\mathbf{N}\|$ is a given norm of **N** in the solution for coefficients $\underset{\sim}{\mathbf{c}}$ in linear equations $\mathbf{N}\underset{\sim}{\mathbf{c}} = \underset{\sim}{\mathbf{u}}$.

Then the perturbations $\delta\underset{\sim}{\mathbf{c}}$ arising from input perturbations $\delta\underset{\sim}{\mathbf{u}}$ are characterized by:

$$\frac{\|\delta\underset{\sim}{\mathbf{c}}\|}{\|\underset{\sim}{\mathbf{c}}\|} \leq \kappa(\mathbf{N}) \frac{\|\delta\underset{\sim}{\mathbf{u}}\|}{\|\underset{\sim}{\mathbf{u}}\|} \tag{3}$$

so $\kappa$ is a measure of the instability of the solution. A perfectly stable solution gives $\kappa = 1$. Large correlations will give large condition numbers, as the Normal Matrix then tends to singularity.

**Table 2**: Mount Model solution from 29 star Star Calibration of 23 March 2004.

```
Number  29.  Sigma-Hat 1.29 arcsec     Condition Number  0.6577D+06
Term Description                       δ _Parameter   Sigma
                                          (arcsec)  (arcsec)

  1 (Az) Az encoder offset:  1            4686.38      2.31
  2 (El) El encoder offset:  1            -507.56    194.71
  3 (Both) Az tilt about N:  cosA.tanE      15.17      0.29
  4 (Both) Az tilt about E:  sinA.tanE      32.98      0.45
  5 (Az) Collimation error:  secE         -125.20      3.09
  6 (Az) Non-orthogonality:  tanE           -1.11      2.37
  7 (Az) Az bearing ellipt:  sinA          -26.59      0.56
  8 (Az) Az bearing ellipt:  cosA          -15.24      0.46
  9 (El) El bearing ellipt:  sinE          116.61     79.27
 10 (El) El bearing ellipt:  cosE         -216.26    147.41
 11 (El) Tube flexure:       cotE          -18.36      8.94
 12 (Az) Az encoder scale:   A/twopi         0.87      0.74
 13 (El) El encoder scale:   E/twopi     -1924.44   1088.51
 14 (Az) Az encoder double-cycl: sin2A      -0.28      0.65
 15 (Az) Az encoder double-cycl: cos2A       2.01      1.80
 16 (El) El encoder stiction:sinA           -6.56      0.98
 17 (El) El encoder stiction:cosA          -45.57      0.92
 18 (El) El bearing stiction:E.sinA         10.64      1.34
 19 (El) El bearing stiction:E.cosA         29.84      1.02
 20 (Az) Double periodic :  sin2A/cosE      -0.51      0.35
 21 (Az) Double periodic :  cos2A/cosE      -1.48      1.37
 22 (El) Double periodic :  sin2A           -0.34      0.36
 23 (El) Double periodic :  cos2A           -0.99      0.39
```

## RESULTS

From a single 29-star calibration on the Stromlo 1.0-metre SLR telescope, the simple solution is given in Table 2:

Note the raw post-fit RMS of residuals, 1.3 arcseconds, which is far better than anything I have ever seen before. However, the condition number of $6.6*10^5$ is huge, as are the highlighted standard errors of coefficients 2, 9, 10 and 13, so big correlations between some of the coefficients are indicated. Accordingly, some of the correlation matrix is shown in Table 3.

**Table 3**: Correlation coefficients between parameters (coefficients) in Table 2. Extremely high correlations are highlighted.

```
CORRELATION MATRIX
==================    Condition Number : 0.658D+06

Term   1      2      3      4      5      6      7      8      9     10     11     12
  2   0.00
  3   0.40   0.00
  4  -0.68   0.00  -0.40
  5  -0.97   0.00  -0.28   0.64
  6  -0.90   0.00  -0.10   0.57   0.97
  7  -0.41   0.00  -0.26   0.68   0.38   0.33
  8   0.16   0.00   0.61  -0.27  -0.08   0.00  -0.15
  9   0.00   0.94   0.00   0.00   0.00   0.00   0.00   0.00
 10   0.00  -1.00   0.00   0.00   0.00   0.00   0.00   0.00  -0.96
 11   0.00  -0.92   0.00   0.00   0.00   0.00   0.00   0.00  -0.72   0.88
 12   0.00   0.00   0.05   0.14   0.02   0.02  -0.14  -0.03   0.00   0.00   0.00
 13   0.00  -0.99   0.00   0.00   0.00   0.00   0.00   0.00  -0.97   1.00   0.87  0.00
```

There are some correlations with effectively $\left|\rho_{jk}\right| = 1.00$, hence the Normal Matrix is singular so should not be inverted as it stands. This has got nothing whatsoever to do with floating point precision; it shows true intrinsic instability.

## STABILITY IMPROVEMENT

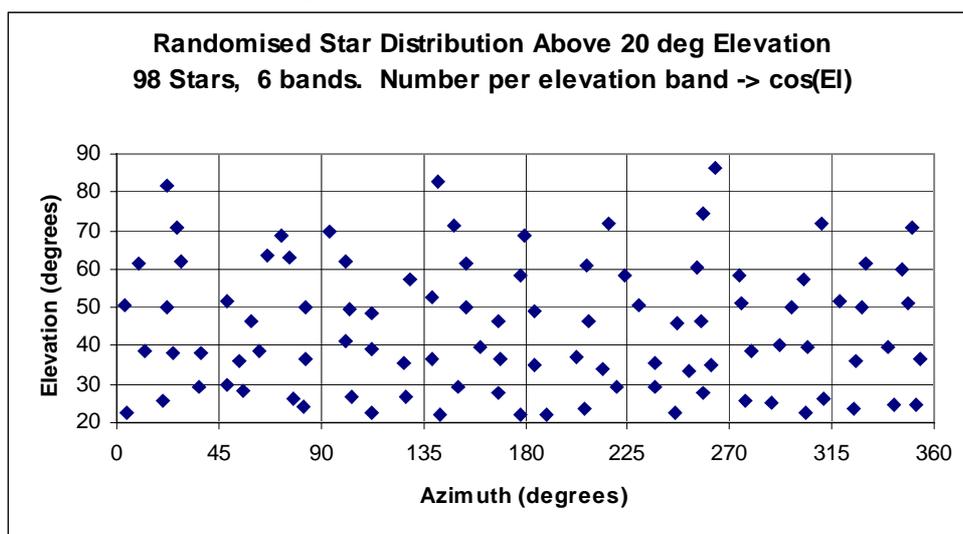Four ways have been tested to improve the stability of mount model solutions.



**Figure 1**: Proposed centres of search regions for equal-sky-area star distribution.

## 1. Star Distribution

Obviously, it is necessary in the star calibration to sample the entire region of the sky over which satellites are going to be tracked. It is particularly important in alt-az systems to observe stars as close to the zenith as possible because of the "keyhole" there, leading to terms like $\tan E$, and for all mounting systems close to the horizon because of $\cot E$ terms as with tube flexure. The implications of observing in azimuth outside the range $0 \le A \le 2\pi$ must be carefully considered, too, for this is generally done to cope with cable wrap-around strategies and could introduce discontinuities in the azimuth scale - especially if there is a gap in the encoder tape. (There were no such problems with the Orroral X-Y mount: the "keyholes" were on the un-reachable horizon, both axes were restricted to only half a revolution, and cable wrap-around was non-existent. Perfect, really.)

An algorithm has been devised to produce star distributions covering equal areas of sky for any selected number of stars between nominated elevation limits. The results for 98 stars between $20^o$ and $90^o$ elevation are illustrated in Fig. 1, and could be used to choose the centres of little regions from which real stars are selected from the catalogue. Details may be obtainable from the author.

It hardly needs saying that as many stars as possible should be observed, the only constraint being the observing time available. Centering on the "cross-hairs" must be free of parallax, which is not always possible even when using a camera and display screen; and the cross-hair or fiducial spot must be bore-sighted with the transmitted laser beam. (Once again, at Orroral the fiducial spot was the small fraction of the laser beam reflected from the Spider Retroreflectors (*Luck, 2004a*) through the dichroic tertiary mirror into an eyepiece at the Cassegrain focus.)

## 2. Parameter Deletion

One of the perils of using least-squares is over-fitting in an attempt to reduce the overall RMS. Terms 5 and 6 are collimation error and axes non-orthogonality whose respective functions in azimuth are $\sec E$ and $\tan E$. Their correlation coefficient in Table 3 is 0.97. Now:

$$\sec E = \pm \sqrt{\left[ 1 + \tan^2 E \right]}, \text{ i.e. } \sec E \approx \tan E \text{ as } E \to \pi/2 \tag{4}$$

so they are virtually indistinguishable at high elevation angles, hence really cannot be separated. Another example is terms 2 (elevation encoder offset) and 11 (tube flexure) in elevation, whose functions are 1 and $\cot E$, respectively, and whose correlation coefficient is -0.92. *Prima facie* they should be well separable, but both are influenced jointly by other correlations.

Some of these coefficients can be deleted from the solution - but great care is needed. Incorrect choices can lead to considerable blow-out in the RMS of residuals with consequent degradation in the predictive power of the solution, i.e. the model will then be under-fitting the observations. One way around this is to delete first the coefficient which has the largest number of large correlations. In Table 3, parameters 2 and 13 each have 4 huge correlations. We would choose to delete 13 (elevation encoder scale) first because parameter 2, the elevation encoder offset, is physically a much more fundamental item. The choice is actually made easier by the methods of the next section.

## 3. Normalizing to the Means

As a simple introduction, suppose a set of observations $(t_i, y_i), i = 1, \ldots, n$ are related by the straight line $y_i = a + bt_i$. Imagine that the times $t_i$ are expressed in full Julian Dates, but the data only span this year. Then the intercept $a$ gives the value at Jan 0.5, 4713BC which is of no interest to anyone. Further, there is a great correlation between $a$ and $b$ because a small change in the slope $b$ will produce a huge change in $a$ so far back in history. The correlation coefficient is numerically real but totally misleading (except perhaps to archaeologists . . .). However, if the model is expressed as $y_i = a + b(t_i - \overline{t})$ where $\overline{t}$ is the mean of this particular data set then there is zero correlation between the solution values of $a$ and $b$. On the other hand, if the observations are repeated in some other time span, the new value of $a$ will be different, so this trick is a mixed blessing.

This trick can be applied to star calibrations, with the advantage that the data span is always (nominally) the same because the same region of sky is sampled every time. In analogy to $\overline{t}$ above, define for each $F_j$ about its mean $\overline{F}_j$ over the observable cap of sky:

$$\overline{F}_j = \int_{A=0}^{2\pi} \int_{E=E_0}^{\pi} F_j(A, E) \cos E \, dE \, dA \Big/ \int_{A=0}^{2\pi} \int_{E=E_0}^{\pi} \cos E \, dE \, dA \tag{5}$$

and similarly with each $\overline{G}_j$, $j = 3, \ldots, m$ (the constant term in each series is not modified). Each of these means is a number, not a function. The model equations (1) then become:

$$\delta A_i = \sum_{j=1}^{m} c'_j \left[ F_j(A_i, E_i) - \overline{F}_j \right], \quad i = 1, \ldots, n$$

$$\delta E_i = \sum_{j=1}^{m} c'_j \left[ G_j(A_i, E_i) - \overline{G}_j \right], \quad i = 1, \ldots, n \tag{6}$$

It turns out that $c'_j = c_j, j = 3, \ldots, m; \quad c'_1 = c_1 + \sum_{j=3}^{m} c_j \overline{F}_j; \quad c'_2 = c_2 + \sum_{j=3}^{m} c_j \overline{G}_j$. All the variation due to differing star distributions from star cal to star cal are then absorbed into the constant terms 1 and 2 which thus become the prime criteria for the numerical stability of successive solutions.

This concept was tested by simulations, including some with perturbations in the "true" star positions arising from interpolation errors as found in (*Luck, 2004b*). As will be seen in Table 4, it reduces the condition number by a factor of about 10, i.e. from huge to large. It also reduces the number of large correlation coefficients substantially, making it much easier to select terms for deletion. On the basis of the simulations and of this "normalization to the means", a good selection of terms to delete is 6, 9, 11, 13 and 15 which results in a negligible increase in residual RMS from 1.29 to 1.32 seconds of arc for the real data. Deletion of these terms also yields a dramatic decrease in condition number from 658,000 to 37 and removal of even more terms from the list of large correlation coefficients.

## 4. Use of Prior Information

The local tie surveys give results *inter alia* for the non-orthogonality between axes and for the components of azimuth axis tilt away from vertical, and their variances (*Dawson et al, 2004*). They can be used as weighted constraints upon these parameters which is a simple application of Bayesian inference using prior information. In this case the weights must be proportional to the inverse variances both in the constraints and in the original observations,

**Table 4**: Comparison of solutions with and without normalization and deletions.

| Item | 23-Term Solutions | | |
|---|---|---|---|
| *Solution Type* | *Raw* | *Normalized* | |
| Terms Deleted | 0 | 0 | 6,9,11,13,15 |
| | | | |
| Az Post-fit RMS | 0.8 | 0.8 | 0.8 |
| El Post-fit RMS | 1.2 | 1.2 | 1.3 |
| Total Post-fit **RSS** | 1.3 | 1.3 | 1.5 |
| Residuals RMS | 1.29 | 1.29 | 1.32 |
| Condition Number | 6.60E+05 | 6.80E+04 | 32.4 |
| | | | |
| **Solution Values** | | | |
| 1 Az Encoder | 4686.38 | 4459.23 | 4458.75 |
| 2 El Encoder | -507.56 | -837.63 | -838.01 |
| 3 Az Axis Tilt @ N | 15.17 | 15.17 | 15.50 |
| 4 Az Axis Tilt @ E | 32.98 | 32.98 | 32.92 |
| 5 Collimation | -125.20 | -125.20 | -124.82 |
| 6 Non-Orthogonality | -1.11 | -1.11 | - |
| 7 Az Bearing sinA | -26.59 | -26.59 | -26.64 |
| 8 Az Bearing cosA | -15.24 | -15.24 | -15.03 |
| 9 El Bearing sinE | 116.61 | 116.61 | - |
| 10 El Bearing cosE | -216.26 | -216.26 | 45.76 |
| 11 Tube Flexure | -18.36 | -18.36 | - |
| 12 Az Encoder Scale | 0.87 | 0.87 | 0.92 |
| 13 El Encoder Scale | -1924.44 | -1924.44 | - |
| 14 Az Enc.Bi-periodic | -0.28 | -0.28 | 0.20 |
| 15 Az Enc.Bi-periodic | 2.01 | 2.01 | - |
| 16 El Encoder Periodic | -6.56 | -6.56 | -6.68 |
| 17 El Encoder Periodic | -45.57 | -45.57 | -44.95 |
| 18 El Bearing Stiction | 10.64 | 10.64 | 10.19 |
| 19 El Bearing Stiction | 29.84 | 29.84 | 28.91 |
| 20 Az Enc.Bi-modified | -0.51 | -0.51 | -0.88 |
| 21 Az Enc Bi-modified | -1.48 | -1.48 | -0.04 |
| 22 El Enc.Bi-Periodic | -0.34 | -0.34 | -0.33 |
| 23 El Enc.Bi-periodic | -0.99 | -0.99 | -1.14 |

and the standard error of unit weight ($\hat{\sigma}_0$) becomes a measure of how well the weights have been assigned, the goal being 1.0.

Table 5 summarizes the inputs and statistical results of applying prior information in several ways, as well as the effects of "normalizing to the means" and of deleting the five parameters from the solution. In all cases the initial guesses of the solution values are all zero, so deleted coefficients remain zero. The software can handle incremental updating wherein previously determined values are held fixed, but that feature might give too optimistic a picture here. The results in Table 5 show that the changes in parameter values caused by the available constraints are not especially severe even when the constraints are "tight", i.e. have relatively large weights. All that this really shows is that the constraints chosen are consistent with the observations.

## SPHERICAL HARMONIC MODEL
An alternative approach to equation (1), which uses physically identifiable models, is to fit a surface model which is completely empirical. This might satisfy the primary purpose of

**Table 5**: Solutions with prior information. Results for constrained parameters are highlighted. The columns labeled "Prev Sol'n" use values and standard errors from an unconstrained solution, as a sanity check. Units are seconds of arc.

| | Full Sol'n | Constrained Solutions of 23-Term Model | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| *Solution Type* | *Raw* | *Normalized* | | | | | | | |
| Terms Deleted | Nil | Nil | | | | 6, 9, 11, 13, 15 | | | |
| Constraints | Nil | Nil | GA Survey | | Prev. Sol'n | Nil | GA Survey | | Prev .Sol'n |
| | | | Loose | Tight | Tight | | Loose | Tight | Tight |
| **Prior Information Input** | | | | | | | | | |
| S.E. of star obs | | | 1.5 | 1.32 | 1.32 | | 1.5 | 1.32 | 1.32 |
| (3) Tilt @ N | | | 21.0 | 21.0 | 15.5 | | 21.0 | 21.0 | 15.5 |
| s.e. | | | 30.0 | 1.5 | 1.3 | | 30.0 | 1.5 | 1.3 |
| (4) Tilt @ E | | | 42.4 | 42.4 | 32.9 | | 42.4 | 42.4 | 32.9 |
| s.e. | | | 30.0 | 1.5 | 1.3 | | 30.0 | 1.5 | 1.3 |
| (6)Non-orthogonality | | | 26.1 | 26.1 | -1.1 | | 26.1 | 26.1 | -1.1 |
| s.e. | | | 10.0 | 1.5 | 1.3 | | 10.0 | 1.5 | 1.3 |
| **Statistics Output** | | | | | | | | | |
| Az post-fit RMS | 0.8 | 0.8 | 2.1 | 5.2 | 1.0 | 0.8 | 2.1 | 10.1 | 1.7 |
| AzcosE post-fit RMS | 0.6 | 0.6 | 0.6 | 1.8 | 0.6 | 0.7 | 0.6 | 1.1 | 0.6 |
| El post-fit RMS | 1.2 | 1.2 | 1.2 | 1.2 | 1.2 | 1.3 | 1.3 | 1.3 | 1.3 |
| Total post-fit **RSS** | 1.3 | 1.3 | 1.4 | 2.1 | 1.3 | 1.5 | 1.5 | 1.7 | 1.5 |
| Sigma Unit Weight $\left(\hat{\sigma}_0\right)$ | 1.29 | 1.29 | 0.88 | 1.63 | 0.89 | 1.32 | 0.89 | 2.96 | 0.92 |
| Condition Number $\left(\kappa\right)$ | 658000 | 68300 | 68300 | 68300 | 68300 | 32.4 | 18.0 | 13.3 | 12.5 |
| Num.Correlations>0.8 | 15 | 9 | 10 | 8 | 8 | 2 | 3 | 1 | 1 |
| **Solution Values** | | | | | | | | | |
| 1 Az encoder offset | 4686.38 | 4459. | 4459. | 4461. | 4459. | 4458. | 4458. | 4459. | 4458.7 |
| 2 El encoder offset | -507.56 | -837.6 | -837.6 | -837.6 | -837.6 | -838.0 | -838.0 | -838.0 | -838.0 |
| **3 Az axis tilt about N** | **15.17** | **15.17** | **15.17** | **17.11** | **15.15** | **15.50** | **15.37** | **16.77** | **15.4** |
| **4 Az axis tilt about E** | **32.98** | **32.98** | **32.79** | **37.59** | **32.70** | **32.92** | **32.50** | **36.82** | **32.67** |
| 5 Collimation error | -125.20 | -125.2 | -118.8 | -92.27 | -125.1 | -124.8 | -124.7 | -124.4 | -124.7 |
| **6 Non-orthogonality** | **-1.11** | **-1.11** | **3.68** | **23.64** | **-1.04** | **-** | **-** | **-** | **-** |
| 7 Az bearing (sin A) | -26.59 | -26.59 | -26.78 | -23.73 | -26.84 | -26.64 | -26.96 | -24.20 | -26.85 |
| 8 Az bearing (cos A) | -15.24 | -15.24 | -15.24 | -13.93 | -15.28 | -15.03 | -16.17 | -14.31 | -15.16 |
| 9 El bearing (sin E) | 116.61 | 116.6 | 116.6 | 116.6 | 116.6 | - | - | - | - |
| 10 El bearing (cos E) | -216.3 | -216.3 | -216.3 | -216.3 | -216.3 | 45.76 | 45.76 | 45.76 | 45.76 |
| 11 Tube flexure | -18.36 | -18.36 | -18.36 | -18.36 | -18.36 | - | - | - | - |
| 12 Az encoder scale | 0.87 | 0.87 | 0.81 | 1.15 | 0.77 | 0.92 | 0.81 | 0.98 | 0.82 |
| 13 El encoder scale | -1924.4 | -1924. | -1924. | -1924. | -1924. | - | - | - | - |
| 14 Az encoder (sin 2A) | -0.28 | -0.28 | 0.10 | 1.32 | 0.16 | 0.20 | 0.56 | 1.80 | 0.61 |
| 15 Az encoder (cos 2A) | 2.01 | 2.01 | 3.57 | 8.35 | 2.14 | - | - | - | - |
| 16 Az bearing in El (sinA) | -6.56 | -6.56 | -6.56 | -8.50 | -6.54 | -6.88 | -6.55 | -7.95 | -6.58 |
| 17 Az bearing in El (cos A) | -45.57 | -45.57 | -45.76 | -40.97 | -45.86 | -44.95 | -45.37 | -41.05 | -45.20 |
| 18 El stiction (Esin A) | 10.64 | 10.64 | 10.64 | 10.64 | 10.64 | 10.19 | 10.19 | 10.19 | 10.19 |
| 19 El stiction (Ecos A) | 29.84 | 29.84 | 29.84 | 29.84 | 29.84 | 28.91 | 28.91 | 28.91 | 28.91 |
| 20 Az mod.bi-period (sin) | -0.51 | -0.51 | -0.71 | -1.65 | -0.78 | -0.88 | -1.11 | -2.12 | -1.15 |
| 21 Az mod.bi-period (cos) | -1.48 | -1.48 | -2.77 | -6.88 | -1.57 | -0.04 | 0.10 | 0.10 | 0.10 |
| 22 Az bearing in El (sin2A) | -0.34 | -0.34 | -0.34 | -0.34 | -0.34 | -0.33 | -0.33 | -0.33 | -0.33 |
| 23 Az bearing in El (cos2A) | -0.99 | -0.99 | -0.99 | -0.99 | -0.99 | -1.14 | -1.14 | -1.14 | -1.14 |

Mount Modelling, which is to provide the best possible pointing predictions for satellite tracking. The natural functions to use over a sphere such as the whole sky are the Legendre Polynomials $P_{nm}\left(A, E\right)$, which are orthogonal over the whole sphere given a uniformly dense set of observations (*Heiskanen and Moritz, 1972*). However, there are at least four problems:

1) I am not sure how well they cope with the zenith "keyhole".
2) The available sky is rather less than a hemisphere, so not all the polynomials will be orthogonal hence there will necessarily be correlations between some of the

coefficients. If only odd-degree terms are used, they are orthogonal over a hemisphere but the risk of under-fitting is severe.

3) The number of terms increases rapidly with the maximum degree chosen, so one could easily run out of degrees of freedom.

4) There can be no cross-coupling between the azimuth and elevation residuals such as occurs through the tilt terms in the physical model, so completely independent solutions are required on each axis, which halves the number of observations available for each solution.

Nevertheless, some study was made of this approach on the same realistically simulated data sets as used for testing the physical model approach. A more extensive report will be prepared later, but the preliminary results suggest that the spherical harmonic approach is considerably inferior. A possibility is to fit spherical harmonics to the post-fit residuals after a physical-model solution - provided that the coefficients thereby produced are repeatable over many data sets. It is emphasized that such coefficient repeatability is also required for the empirical and exotic terms included in the physical model approach.

**REPEATABILITY OF SOLUTIONS**
The acid test is to compare the coefficient values obtained from one night to the next, and month after month. Regretfully, only one data set has become available in usable form (computer crashes have spoilt May's data), so this assessment must wait until another day. (Subsequent star cals indicate the need for even deeper investigations.)

**CONCLUSIONS**
This study in effect has been a search for the perfect (matrix) inversion which will give a mount model solution which can be used with absolute confidence to predict celestial positions at all azimuths and at all elevations from the horizon right up to the zenith, year after year, from limited numbers of stars observed in each calibration session.

One of the real practical difficulties is finding and observing stars close to the zenith. The model is therefore often required to extrapolate towards zenith. This aspect alone, in my opinion, justifies the effort required in this study. Otherwise, surface-fitting techniques such as fitting Legendre Polynomials might be considered purely as an interpolation strategy, but they have some conceptual problems and simulations suggest that they are less than satisfactory.

By far the largest problem with the physical model approach is trying to fit too many terms. The process of "normalizing to the means" greatly facilitates identification of superfluous terms which, when deleted, hugely improves the Normal Matrix condition for inversion, i.e. takes it well away from being singular, with consequent massive increase in confidence in the solution values. The correct choice of terms will barely affect the residual RMS, which is only one of the criteria upon which the accuracy of the model should be judged.

Despite the aids mentioned above, the selection of terms for deletion was still somewhat arbitrary. The experience gained when more data become available will guide a maybe even better selection.

A pointing precision of 1.5 seconds of arc can certainly be claimed for the Stromlo SLR 1.0 metre telescope. There is high confidence that accuracy and stability will be at the same level, and that all will produce sub-arcsecond absolute pointing in the very near future.

None of which would be possible without the extraordinary mechanical stability of the telescope.

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