RAY MATRIX APPROACH FOR THE REAL TIME CONTROL OF SLR2000 OPTICAL ELEMENTS

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Abstract

NASA's SLR2000 is an autonomous, eye-safe, photon-counting satellite laser ranging (SLR) system. As such, it requires some unique real-time control elements that are not generally found in conventional, high powered, manned systems. These include autonomous mechanisms and associated software for: (1) maintaining telescope focus over wide ambient temperature excursions; (2) conducting automated star calibrations and updating mathematical mount models; (3) centering the optical receiver field of view (FOV) on the satellite return based on single photon returns; (4) varying transmitter beam divergence and point ahead; and (5) controlling the receiver spectral bandwidths and spatial Field-of-View (FOV). Most of these real-time functions can be accomplished mathematically by utilizing a ray matrix approach. As an additional benefit, the ray matrix model can be used as a diagnostic tool to track the geometric size and orientation of beams and/or images anywhere in the system. Recent optical analyses of SLR2000 using the ray matrix model and experiences gained in satellite field experiments have led to some proposed modifications and simplifications to the optical transceiver, which will also be discussed.

INTRODUCTION

Paraxial ray matrices are a convenient and simple way to model ray propagation in optical systems [Kogelnik and Li, 1966]. In complex 3-dimensional systems, reflections off mirrors cause the propagation path to change direction several times, and one must go through a series of coordinate transformations where the precise form of the matrix depends on whether we are dealing with the p-component (in the mirror plane of incidence) or s-component (orthogonal to the plane of incidence) of the ray. In SLR2000, the ray changes direction 9 times before encountering the exit window of the telescope. If we consider a segment where propagation is along the local d-axis, any ray can be represented by the vector $r = \begin{pmatrix} p & \alpha_p & s & \alpha_s \end{pmatrix}$ where (p,s) is the transverse position of the ray at the optical element and (α_p, α_s) is the angle the ray makes with the d-axis when projected into the p-d and s-d planes respectively. Each optical element or propagation path between elements is represented by a 4x4 matrix, which operates on the position and direction of the ray. The SLR2000 optical train consists of three major subsystems: (1) optical transceiver; (2) the Coude mount; and (3) the main telescope. Each major subsystem can be represented by a single 4x4 matrix equal to the product of the element matrices making up that subsystem. From Figure 1, we see there are three parallel branches within the optical transceiver: (a) the transmitter/Risley prism path; (b) quadrant detector range receiver (two equal but orthogonally polarized paths) and (c) CCD star camera. In what follows, the initial propagation path for each branch (d-axis) runs vertically (down to up) in the figure, the positive p-axis on the transceiver bench points to the right of the figure (as if we were looking from behind the various elements toward the telescope), and the s-axis is directed out of the page toward the reader.

In this paper, we provide a summary of the analytical results. A detailed mathematical analysis of the SLR2000 system [Degnan, 2004] yields the following 4x4 matrices for rays propagating from the transceiver bench to the telescope exit window. The matrices describing outward propagation through the three branches of the transceiver have the general form

$$M_{1x} = \begin{vmatrix} a_x\Lambda & b_x\Lambda \\ c_x\Lambda & d_x\Lambda \end{vmatrix} \qquad \Lambda = \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix}$$

x = a = transmitter, b = quadrant detector, c = star camera

and the specific forms

$$M_{1a} = \begin{vmatrix} 3\Lambda & 2.267\Lambda \\ 0\Lambda & 0.333\Lambda \end{vmatrix} \quad M_{1b} = \begin{vmatrix} 0.079\Lambda & 2.57\Lambda \\ -0.392\Lambda & -0.101\Lambda \end{vmatrix} \quad M_{1c} = \begin{vmatrix} -35.559\Lambda & 0.405\Lambda \\ -2.469\Lambda & 0\Lambda \end{vmatrix}$$

All of the mount motion is contained in the matrix for the Coude mount given by

$$M_2(g) = \begin{vmatrix} \Gamma & d_C\Gamma \\ 0 & \Gamma \end{vmatrix} \qquad \Gamma = \begin{vmatrix} -\cos g & -\sin g \\ \sin g & -\cos g \end{vmatrix} \qquad g = a - a_0 - e$$

where d_c = 1.742 m is the total Coude path length, a is the azimuth angle, e is the elevation angle, and $a_o = 67.4^o$ (defined in Figure 1 where North corresponds to 0^o azimuth). Finally the telescope matrix is

$$M_3 = \begin{vmatrix} m_t I & d_t I \\ 0 & \dfrac{1}{m_t} I \end{vmatrix} \qquad I = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

and m_t = 10.16 is the main telescope magnification and d_t = 5.758 m.

For outgoing rays, one now multiplies the three matrices together to transfer a ray emanating from one of the branches of the transceiver, $\vec{r}_x$, to the output window of the telescope, i.e

$$\vec{r}_{out} = M_{tot}\,\vec{r}_{in} = M_3 M_2(g) M_{1x}\,\vec{r}_x = \begin{vmatrix} A_x\Gamma' & B_x\Gamma' \\ C_x\Gamma' & D_x\Gamma' \end{vmatrix}\vec{r}_x \qquad \Gamma' = \begin{vmatrix} -\sin g & \cos g \\ -\cos g & -\sin g \end{vmatrix}$$

The transfer of incoming rays, $\vec{r}_{in}$, from the telescope exit window to one of the three transceiver branches is given by the inverse of the matrix, i.e.

$$\vec{r}_x = M_{tot}^{-1}\,\vec{r}_{in} = M_{1x}^{-1} M_2^{-1}(g) M_3^{-1}\,\vec{r}_{in} = \begin{vmatrix} D_x\Gamma'^T & -B_x\Gamma'^T \\ -C_x\Gamma'^T & A_x\Gamma'^T \end{vmatrix}\vec{r}_{in} \qquad \Gamma'^T = \begin{vmatrix} -\sin g & -\cos g \\ \cos g & -\sin g \end{vmatrix}$$

STAR CALIBRATIONS

Overall system focus is maintained over a wide ambient temperature range by a computer-controlled three-power beam expander located in the common transmit/receive path. The quality of the system focus is determined and controlled by the sharpness of star images in the CCD camera used for star calibrations. Periodic image checks ensure not only the sharpness of the star image for calibration but also the collimation of incoming and outgoing beams in other legs of the transceiver and the stability of the system focus at the variable aperture field stop (spatial filter).

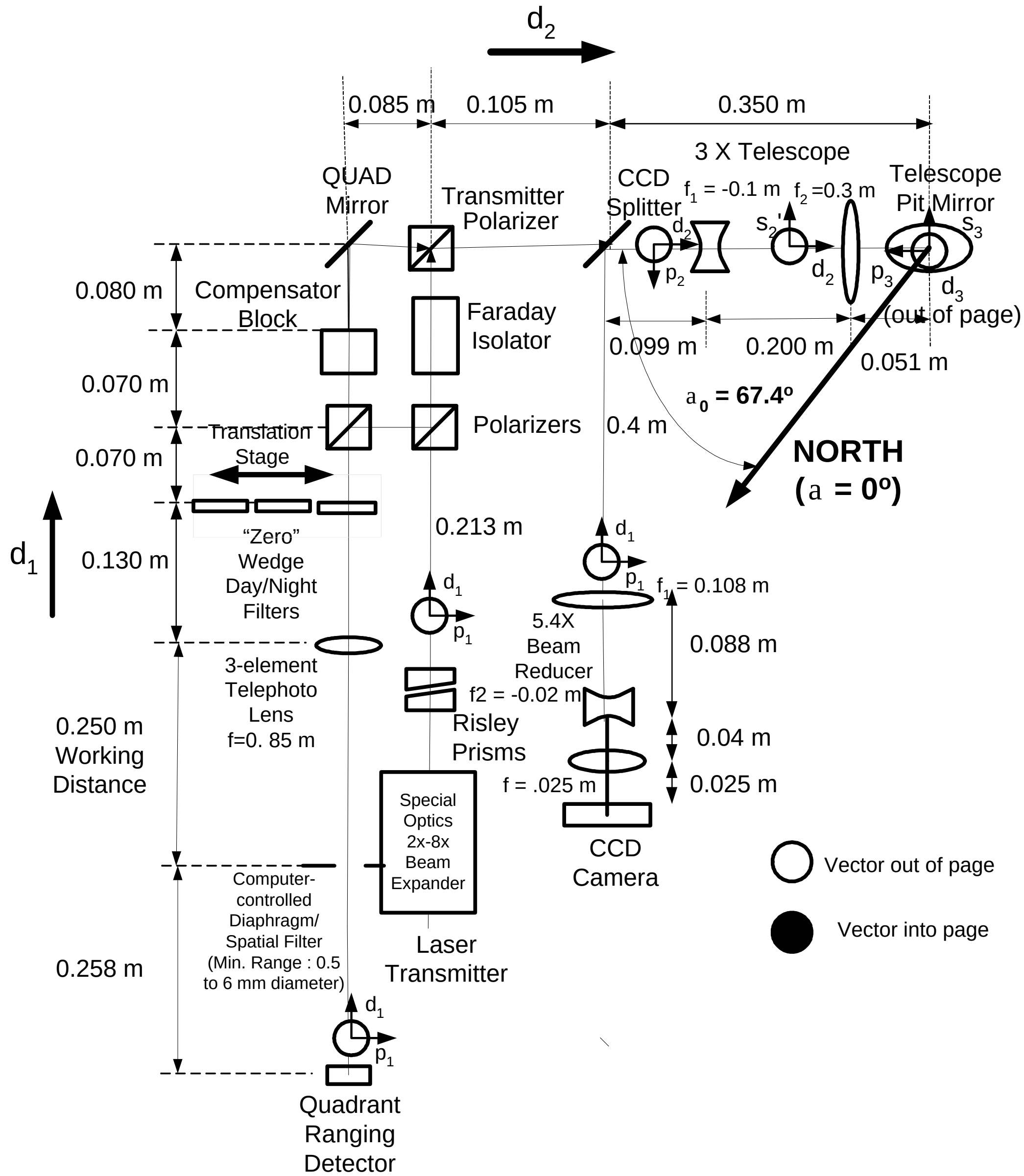

Figure 1: Block diagram of the upgraded SLR2000 optical transceiver.

For star calibrations, the ray model provides the information needed to automatically drive an off-axis star to the telescope optical axis. If n_p and n_s respectively denote the column and row number of the pixel containing the star image in the CCD array as viewed from behind the detector, the offset of the star image in azimuth and elevation space is given by the matrix equation

$$\begin{vmatrix} \Delta a \\ \Delta e \end{vmatrix} = \frac{0.5 arc\sec}{pixel} \begin{vmatrix} \sec e \sin g & -\sec e \cos g \\ \cos g & \sin g \end{vmatrix} \begin{vmatrix} n_p \\ n_s \end{vmatrix}$$

In particular, it provides the scale factor (0.5 arcsec/pixel) relating the magnitude of the position offset of the star image in the CCD camera to the magnitude of the angular offset in azimuth-elevation space. The dependence on g and e provides the star (or satellite) image orientation in the CCD camera as a function of the instantaneous mount azimuth-elevation angle.

RECEIVER POINTING ERROR CORRECTION

In high power manned systems, the operator can manually make two-axis corrections in the pointing angle in an attempt to peak the signal strengths off the satellite. This is not possible in a photon-counting system where the mean signal per pulse is normally much less than one photoelectron. Thus, SLR2000 uses a quadrant ranging detector, which, in addition to providing precise timing on single photoelectron returns, informs the system computer of the quadrant that detected it. This information is accumulated over many laser fires (e.g. a frame interval) by a Correlation Range Receiver (CRR), which extracts the signal counts and discards the vast majority of background noise counts. The remaining counts (mostly signal) are then tallied by quadrant of occurrence, and the differences between quadrant counts are then used to compute a centroid for the count distribution during that frame [Degnan and McGarry, 1997]. The ray model provides the following equation

$$\begin{vmatrix}\Delta a\\ \Delta e\end{vmatrix} = \frac{10.5\,arc\sec}{mm}\begin{vmatrix}\sec e\sin g & -\sec e\cos g\\ \cos g & \sin g\end{vmatrix}\begin{vmatrix}p_c\\ s_c\end{vmatrix}$$

for converting the centroid position (p_c,s_c) on the quadrant detector into azimuth and elevation pointing corrections, Da and De. From the equation, we also see that a 1 mm displacement of the centroid from the optic axis corresponds to a 10.5 arcsec pointing error and the orientation is again determined by the dependence on g and e.

TRANSMITTER POINT AHEAD

Balancing the count distribution among the detector quadrants orients the receiver/telescope optical axis toward the "apparent" position of the satellite, i.e. where it was located one light transit time earlier when photons from the previous pulse were reflected from the satellite retroreflector array. However, for the tighter transmitter divergences, the point-ahead can sometimes exceed the beam divergence so that the future position of the satellite may fall outside the transmitted laser beam if left uncorrected for point-ahead. Thus, to achieve maximum illumination of the satellite and the highest count rate on subsequent pulses, we must offset the transmitter axis from the "apparent" receiver axis by the angular travel accumulated by the satellite during the time it takes a light pulse to travel to and from the satellite. In SLR2000, the point-ahead correction is accomplished via two Risley prisms in the transmit leg of the transceiver (see Fig. 1). The ray model allows us to compute the proper orientation of the two Risleys in real time to produce the appropriate point-ahead as a function of the instantaneous azimuth and elevation rates, $\dot{a}$ and $\dot{e}$.

Let us denote the magnitudes of the angular deflections of the first and second wedges by d_1 and d_2 and the orientations of the individual prism deflections by the angles x_1 and x_2, as measured from the positive p-axis as defined previously. Ideally, $d_1 = d_2 = d$ permits total cancellation of the deflections when the wedges are oppositely aligned, but we are allowing for some manufacturing error in the prisms. The beam deflection is in the direction of the thickest part of the prism and has a magnitude δ = (n-1)θ where n is the glass index of refraction and θ is the wedge angle. The ray matrix formulation provides the following relationship between the ray angle out of the Risley prisms (left-most vector) and the transmitter point ahead in azimuth and elevation space (right-most vector)

$$\begin{vmatrix} ap_{rp} \\ as_{rp} \end{vmatrix} \equiv \begin{vmatrix} d_1 \cos x_1 + d_2 \cos x_2 \\ d_1 \sin x_1 + d_2 \sin x_2 \end{vmatrix} = m_T \begin{vmatrix} -\sin g \cos e & -\cos g \\ \cos g \cos e & -\sin g \end{vmatrix} \begin{vmatrix} \dot{a} t_r \\ \dot{e} t_r \end{vmatrix}$$

where m_T = 30.48 is the total post-Risley magnification of the transmitter beam and t_r is the roundtrip time of flight to the satellite. The latter provides two equations for the two unknowns, x_1 and x_2, from which we can obtain the following sequential solution

$$\Delta x \equiv x_2 - x_1 = \cos^{-1}\left\{ \frac{m_T^{\,2}\left[(\dot{a} t_r \cos e)^2 + (\dot{e} t_r)^2\right] - \left(d_1^2 + d_2^2\right)}{2 d_1 d_2} \right\}$$

$$\cos(x_1) = \frac{-m_T\left[(\dot{a} t_r \cos e)(d_1 \sin g + d_2 \sin(g - \Delta x)) + (\dot{e} t_r)(d_1 \cos g + d_2 \cos(g - \Delta x))\right]}{d_1^2 + d_2^2 + 2 d_1 d_2 \cos(\Delta x)}$$

$$\sin(x_1) = \frac{m_{\cdot T}\left[(\dot{a} t_r \cos e)(d_1 \cos g + d_2 \cos(g - \Delta x)) - (\dot{e} t_r)(d_1 \sin g + d_2 \sin(g - \Delta x))\right]}{d_1^2 + d_2^2 + 2 d_1 d_2 \cos(\Delta x)}$$

$$x_2 = x_1 + \Delta x$$

where the second and third equations unambiguously define the orientation of the first prism.

CONTROLLING TRANSMITTER BEAM DIVERGENCE

Signal count rates and orbital time bias estimates vary widely over the range of satellite altitudes. In order to obtain an acceptable photon count rate for the higher satellites (e.g. LAGEOS, ETALON, GPS) while still meeting eye safety requirements at the telescope exit aperture, we must tightly control the SLR2000 transmit beam diameter (w = 36 cm) and divergence half angle (q_t = 4.3 arcsec). For lower satellites, the angular uncertainty of the satellite position and the signal count rates are both relatively high. Therefore, although it may be helpful to relax the beam divergence half angle to as much as 17 arcsec, the transmitter spot radius must be kept relatively constant at about $w = a/1.12$ = 18 cm where a = 20 cm is the telescope primary radius [Klein and Degnan, TBD] to maintain eye safety and control vignetting losses. Transmitter beam size and divergence at the telescope exit window are controlled in SLR2000 by a commercial Special Optics beam expander in the transmitter path as in Fig. 1.

For real time beam control, the ray model relates the desired quasi-Gaussian transmitter beam parameters at the telescope exit aperture to the corresponding parameters at the output lens of the beam expander. We can represent the Gaussian beam at any point z in the propagation path by the complex "q-parameter" defined by

$$\frac{1}{q(z)} = \frac{1}{R(z)} - j\frac{l}{pw^2(z)}$$

where l = 532 nm is the laser wavelength and $R(z)$ and $w(z)$ are the phasefront radius of curvature and the Gaussian beam radius respectively. At any subsequent point in the path, the q-parameter is given by

$$\frac{1}{q(z)} = \frac{C + \dfrac{D}{q(z_0)}}{A + \dfrac{B}{q(z_0)}} \qquad\qquad M(z, z_0) = \begin{vmatrix} A & B \\ C & D \end{vmatrix}$$

where $M(z, z_0)$ is the ray matrix which propagates the rays between the two points in the propagation path. The ray model provides the following two expressions for the beam diameter in the telescope window

$$w = m_T w_0 \sqrt{1 + \frac{2d_T}{m_T R_0} + \left(\frac{d_T}{m_T R_0}\right)^2 \left[1 + \left(\frac{l R_0}{p w_0^2}\right)^2\right]} \cong m_T w_0 \left(1 + \frac{d_T}{m_T R_0}\right)$$

and the far-field beam divergence half angle

$$q_t = \frac{l}{p m_T w_0} \sqrt{1 + \left(\frac{p w_0^2}{l R_0}\right)^2} = q_{\min} \sqrt{1 + \left(\frac{p w_0^2}{l R_0}\right)^2} \cong \frac{w_0}{m_T R_0}$$

where $m_T = 30.48$ is again the total post-Risley magnification of the transmitter beam, d_T = 0.033 m, and $R_0 = R(z_0)$ and $w_0 = w(z_0)$ are the phase front curvature and beam radius at the output of the transmitter beam expander. The approximations hold for $p w_0^2 / l R_0 >> 1$ where the beam divergence is purposefully set well beyond the diffraction limit of the large exit beam, i.e. q_{min} = 0.2 arcsec. The latter two approximate equations can be solved for the required Gaussian beam properties out of the transmitter magnifier, i.e

$$w_0 = \frac{w - m_T d_T q_t}{m_T} \cong \frac{w}{m_T} = 5.9mm$$

and

$$R_0 = \frac{w - m_T d_T q_t}{m_T^2 q_t} \cong \frac{w}{m_T^2 q_t} = \frac{1.92 \times 10^{-4} m}{q_t}$$

where, for a divergence half-angle range of 20 µrad < q_t < 80 µrad, 2.4 m < R_0 < 9.6 m. The latter values for w_0 and R_0 are produced by manipulating two lens positions in the magnifier.

RECEIVER SPATIAL FIELD OF VIEW

A changing beam divergence must be accompanied by a corresponding change in the receiver FOV to avoid missing possible satellite returns. The ray model allows us to compute the diameter of the spatial filter pinhole as a function of the full receiver FOV. The aperture diameter, D_a, is adjusted by means of a computer-controlled stepper motor to match or slightly exceed the transmitter beam divergence discussed previously according to the equation

$$D_a \geq \frac{0.25mm}{arc\,\sec} q_t$$

SPECTRAL BANDWIDTH

The spectral bandwidth is controlled by a translation stage perpendicular to the optical path, which presently inserts one of the following into the receiver path: an open aperture (night), a 1 nm filter (twilight), or a 0.2 nm (daylight) filter. The wedge in these filters must be compensated at the arcsecond level to avoid introducing angular biases and severe vignetting at the spatial field stop.

SUMMARY

To summarize, the 2-D ray matrix approach provides us with the mathematical tools to calculate in real time:

- The scale factor and angular rotation for converting star image offsets from the CCD camera center to azimuth and elevation biases
- The scale factor and angular rotation for converting quadrant centroid position to satellite pointing correction in azimuth-elevation space
- Transmitter point ahead as a function of round trip time-of-flight and the instantaneous azimuthal and elevation angular rates
- Iris diameter (spatial filter) setting for a given receiver FOV
- Transmitter beam size and divergence at the telescope exit aperture as a function of transmit telescope lens spacings

REFERENCES

Degnan, J. J., “Ray Matrix Analysis of the SLR2000 Optical System”, Internal Report, Sigma Space Corporation, to be published, 2004.

Degnan, J.J. and J. F. McGarry, “SLR2000: Eyesafe and autonomous single photoelectron satellite laser ranging at kilohertz rates”, in *Laser Radar Ranging and Atmospheric Lidar Techniques*, *SPIE Vol. 3218*, pp. 63-77, 1997.

Klein, B. J. and J.J. Degnan, "Optical Antenna Gain. I. Transmitting Antennas", *Applied Optics*, 13, pp. 2134-2141, 1974.

Kogelnik, H. and T. Li, "Laser beams and resonators", *Applied Optics* , Vol. *5*, pp.1550-1569, 1966.