Retroreflector Array Transfer Functions
by
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1. “Toward Millimeter Accuracy”

2. Diffraction patterns of single cube corners.
  A. Coated circular cube corner.
  B. Uncoated circular cube corner.

3. Basic principles of retroreflector array design.

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  A. Cross section and range correction at a single orientation
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  C. Spinning satellite.
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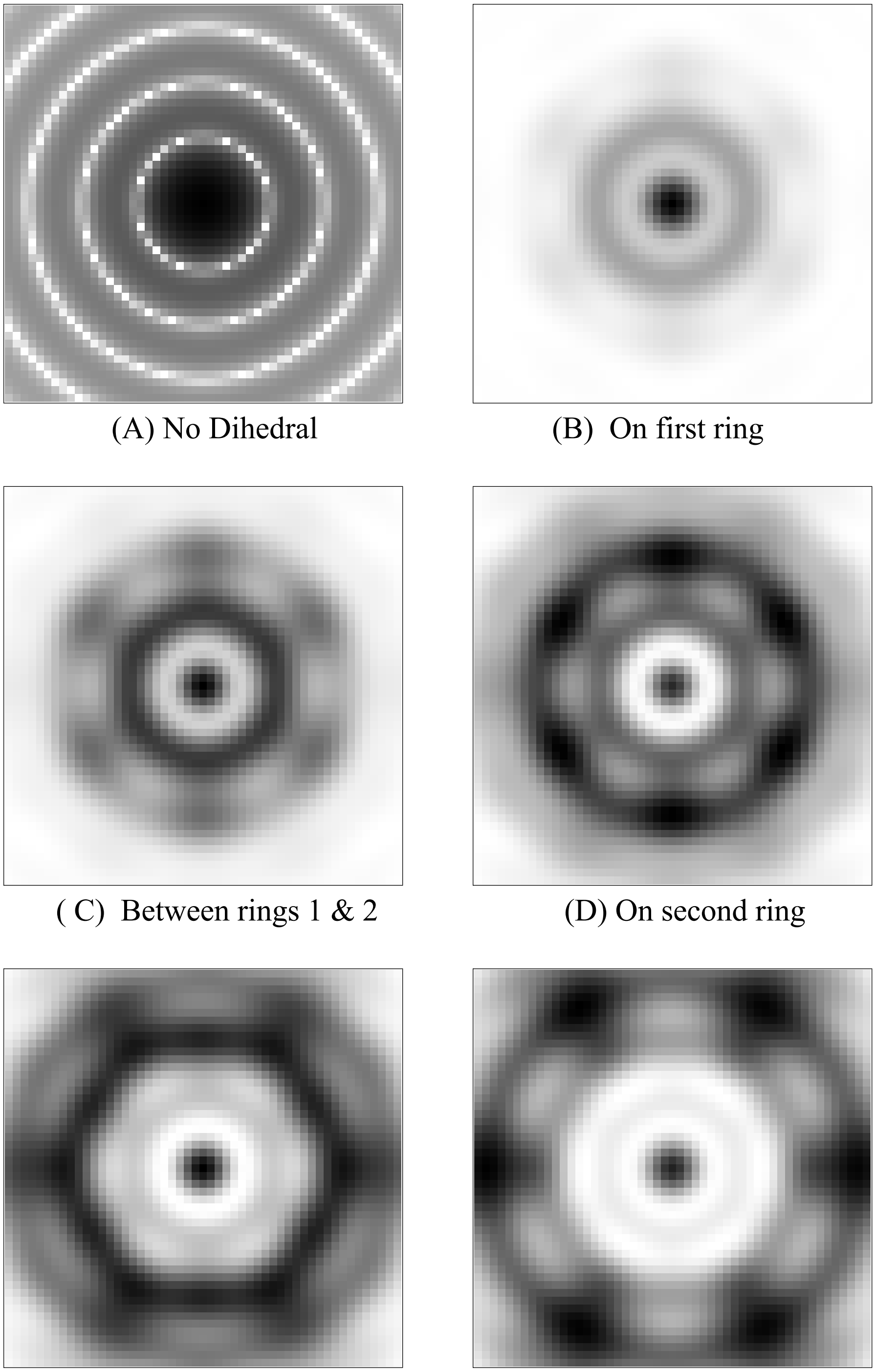
5. Transfer function of the TOPEX retroreflector array.

6. Transfer function of the WESTPAC retroreflector array



(A) No Dihedral

(B) On first ring

(C) Between rings 1 & 2

(D) On second ring

(E) Between rings 2 & 3

(F) On third ring

Figure 1. Coated 1.5 inch cube corner with various dihedral angle offsets.

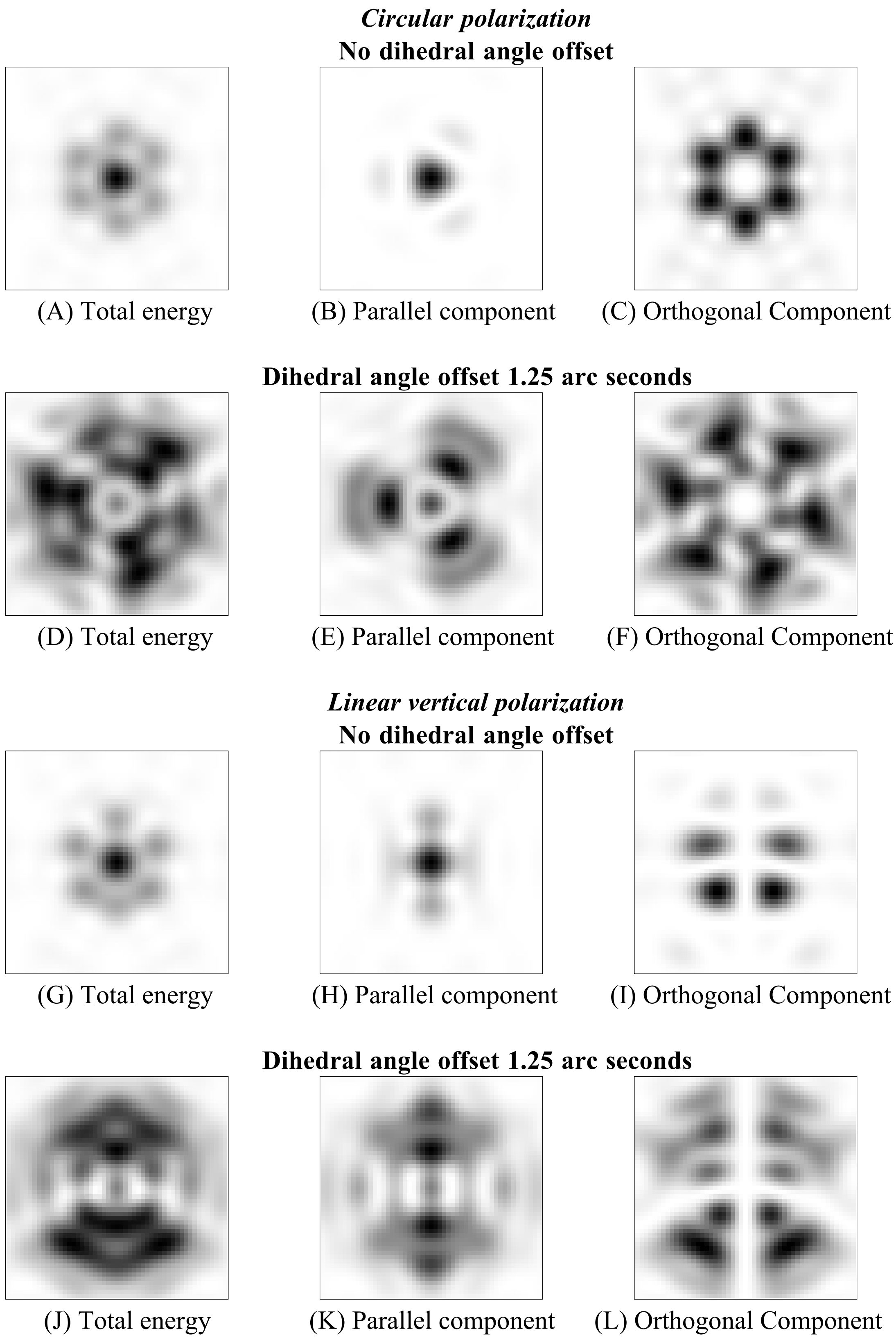


Figure 2. Diffraction pattern of an uncoated 1.5 inch cube corner with, and without, a dihedral angle offset, for circular and linear input polarization.

Principles of Retroreflector Array Design

A. Single cube gives constant range correction

B. Small range depth minimizes range uncertainties

C. Smooth diffraction pattern minimizes range variations

D. Small size and uncoated faces minimize thermal distortions

E. Dihedral angles need to be known to model transfer function

F. Coated vs uncoated cubes

- **a**. Loss of total internal reflection in uncoated cubes minimizes range depth
- **b**. Two polarization states reduce coherent variations in uncoated cubes
- **c**. Uncoated cubes have higher reflectivity at normal incidence
- **d**. Uncoated cubes have no metal surfaces to absorb solar radiation
- **e**. Uncoated cubes have no coatings that can peel or degrade with time
- **f**. Cutoff angle for total internal reflection varies with orientation of cube
- **g**. Linear polarization causes asymmetry in uncoated cubes with dihedral angle offset

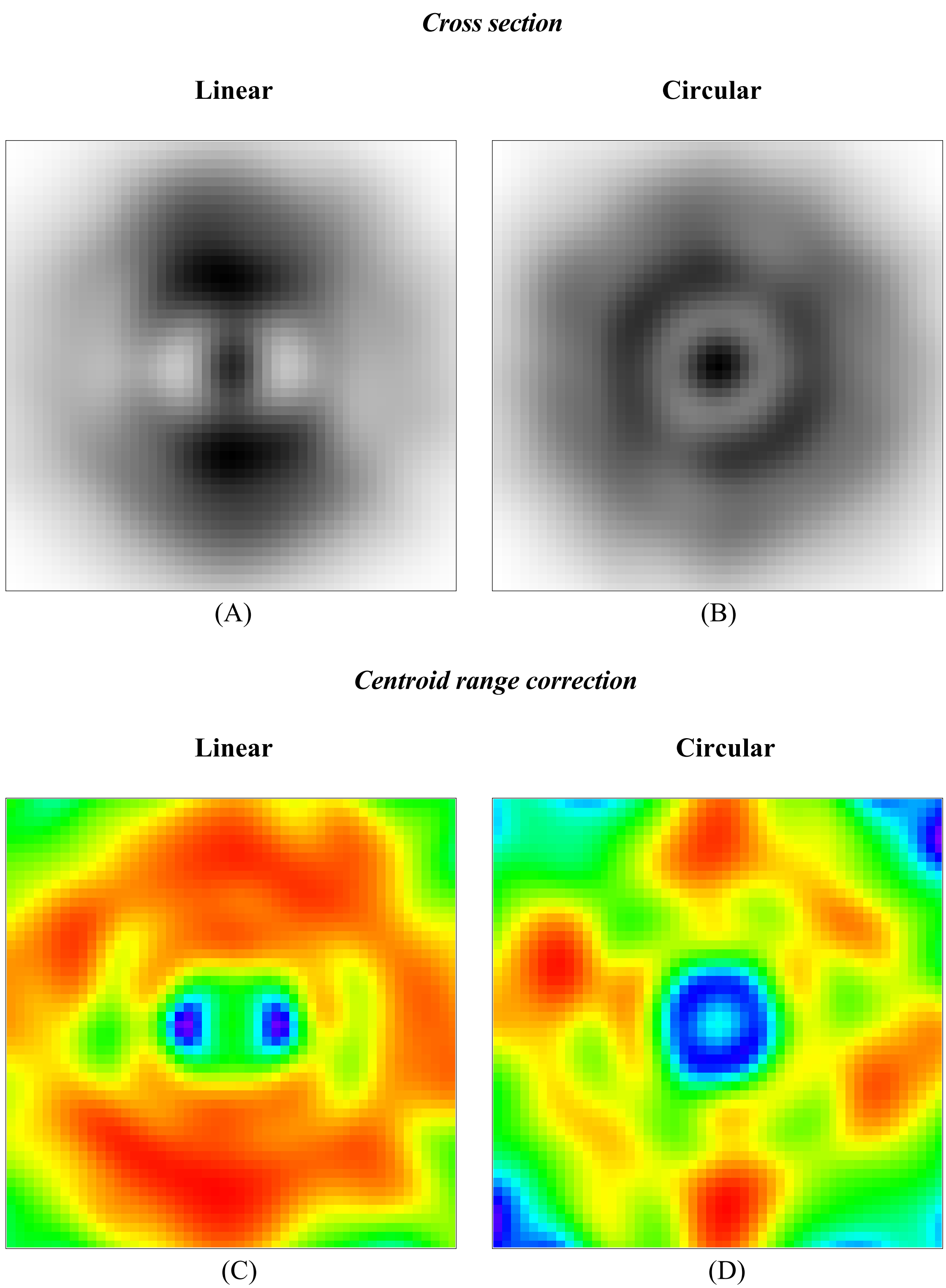


Figure 3. Cross section and range correction for linear vertical polarization and circular polarization. The satellite orientation angle is θ = 20 deg, and ϕ = 150 degrees. The dihedral angle offset is 1.25 arc seconds. The wavelength is 532 nanometers.

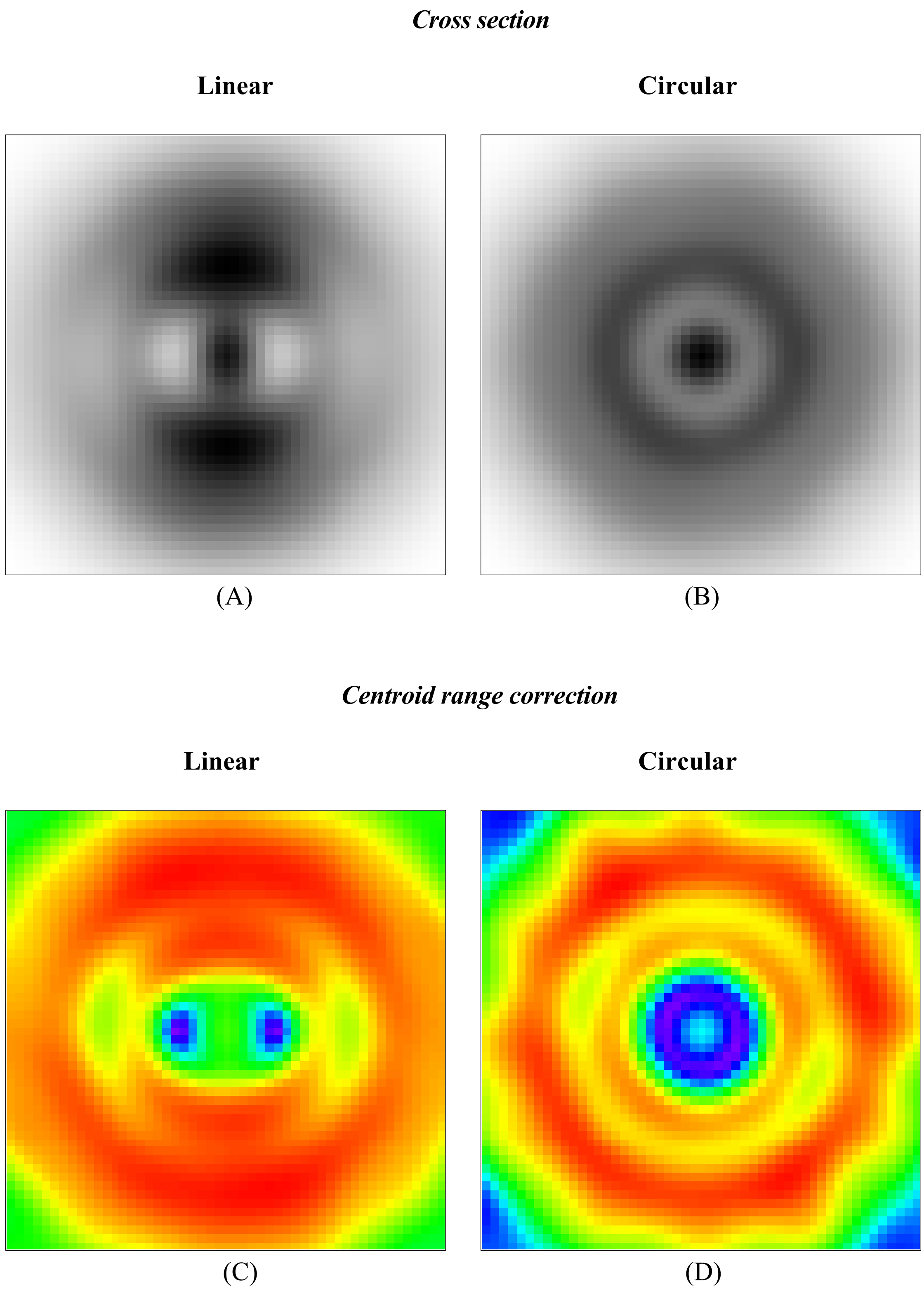
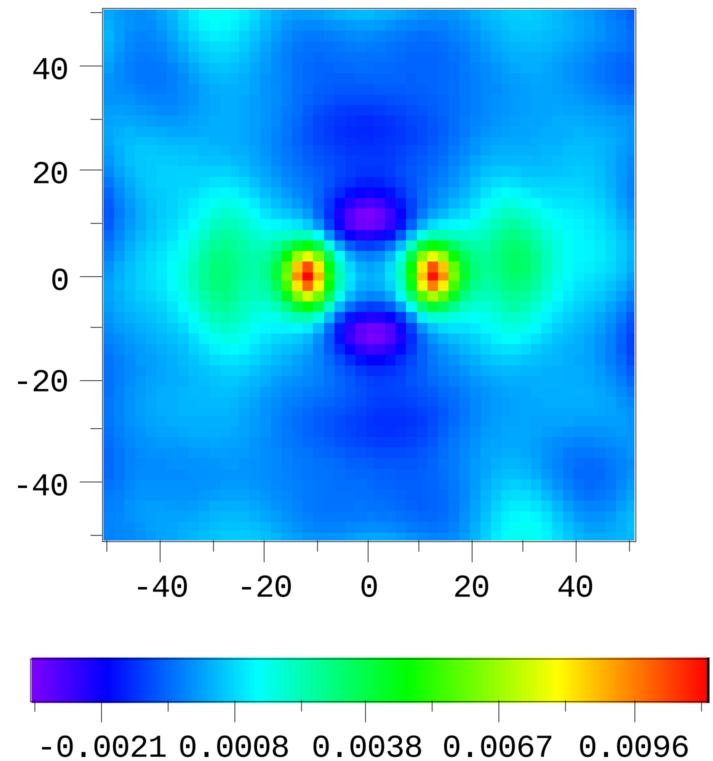


Figure 4. Cross section and range correction for linear vertical polarization and circular polarization. The cross section and range correction are averaged over 16 orientations. The dihedral angle offset is 1.25 arc seconds. The wavelength is 532 nanometers.



Microrad	Minimum	Average	Maximum	Max - Min
0.0	0.0002900	0.0002900	0.0002900	0.0000000
2.0	0.0001300	0.0002844	0.0004400	0.0003100
4.0	-0.0003500	0.0003055	0.0010100	0.0013600
6.0	-0.0012600	0.0004070	0.0023200	0.0035800
8.0	-0.0025000	0.0007455	0.0049800	0.0074800
10.0	-0.0035400	0.0013697	0.0088700	0.0124100
12.0	-0.0036900	0.0017901	0.0112100	0.0149000
14.0	-0.0030300	0.0015349	0.0093900	0.0124200
16.0	-0.0022200	0.0010137	0.0062200	0.0084400
18.0	-0.0016600	0.0006380	0.0040457	0.0057057
20.0	-0.0013800	0.0004527	0.0030525	0.0044325
22.0	-0.0013300	0.0003919	0.0027589	0.0040889
24.0	-0.0013900	0.0004026	0.0028730	0.0042630
26.0	-0.0014900	0.0004448	0.0031460	0.0046360
28.0	-0.0015300	0.0004764	0.0033378	0.0048678
30.0	-0.0014600	0.0004607	0.0032251	0.0046851
32.0	-0.0012962	0.0003913	0.0028010	0.0040972
34.0	-0.0011050	0.0003009	0.0022466	0.0033516
36.0	-0.0009526	0.0002213	0.0017655	0.0027181
38.0	-0.0008383	0.0001650	0.0014557	0.0022940
40.0	-0.0007818	0.0001302	0.0012738	0.0020556
42.0	-0.0007613	0.0001063	0.0011648	0.0019262
44.0	-0.0007415	0.0000879	0.0010549	0.0017964
46.0	-0.0007180	0.0000698	0.0008940	0.0016120
48.0	-0.0006269	0.0000507	0.0006611	0.0012880
50.0	-0.0008095	0.0000319	0.0007943	0.0016039

Figure 5. Centroid with circular polarization minus centroid with linear polarization averaged over 16 orientations.

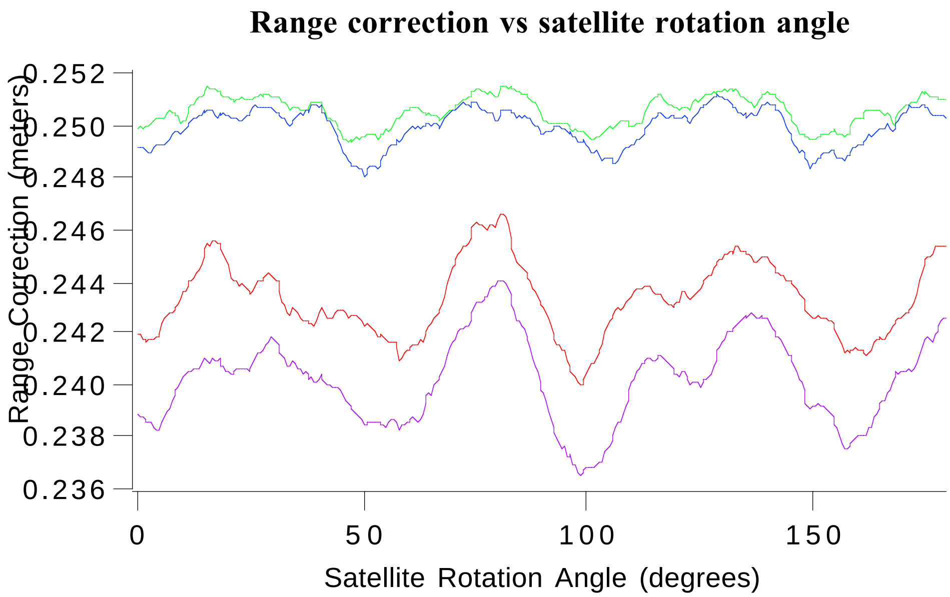


Figure 6. Centroid and half-max range correction vs satellite rotation angle at velocity aberration (0,35) and (35,0) µrad with linear vertical polarization (y-axis).

A. Velocity aberration x = 0 µrad, y = 35 µrad.

red = Centroid

green = Half-max

B. Velocity aberration x = 35 µrad, y = 0 µrad

Purple = Centroid

Blue = Half-max

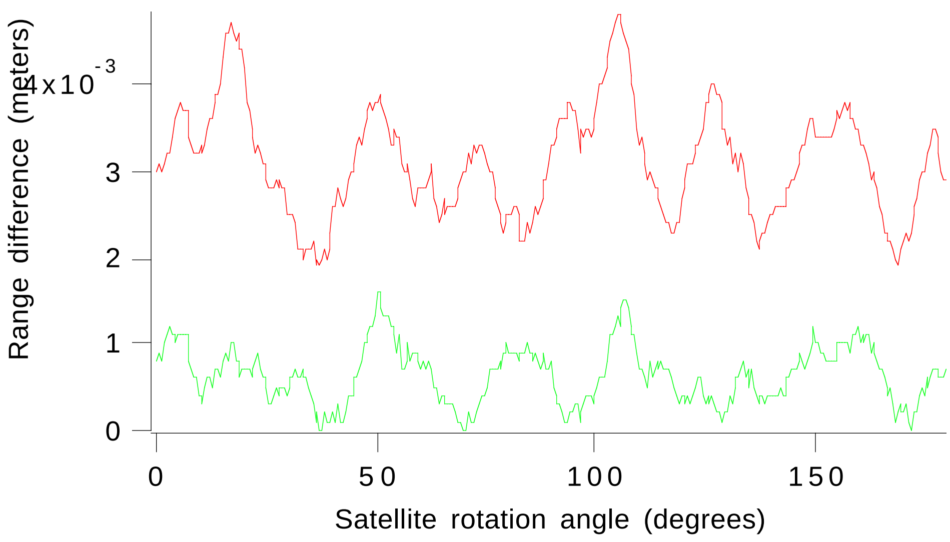


Figure 7. Range correction at (0,35) minus range correction at (35,0) µrad.

Red = centroid(0,35) - centroid(35,0)

Green = half-max(0,35) - half-max(35,0)

Coherent variations of the range correction

Δ	σ	σ_m	Δ/σ_m
Centroid, Equal Weighting			
-2.16	8.67	0.43	-4.98
Centroid, Weighted by Signal Strength			
0.04	7.08	0.35	0.11
Half-Area, Equal Weighting			
-2.55	10.07	0.50	-5.06
Half-Area Weighted by Signal Strength			
-.82	7.72	0.39	-2.12
Half-Maximum, Equal Weighting			
-3.47	9.84	0.49	-7.06
Half-Maximum, Weighted by Signal Strength			
-2.54	6.93	0.35	-7.33

Δ = difference between incoherent range and the average of a set of 400 coherent range measurements
σ = standard deviation of a single range measurement
$\sigma_m = \sigma/20$ = standard deviation of the mean
Pulse length = 200 nsec
Δ, σ, and σ_m are in millimeters.

Range correction vs average number of photoelectrons

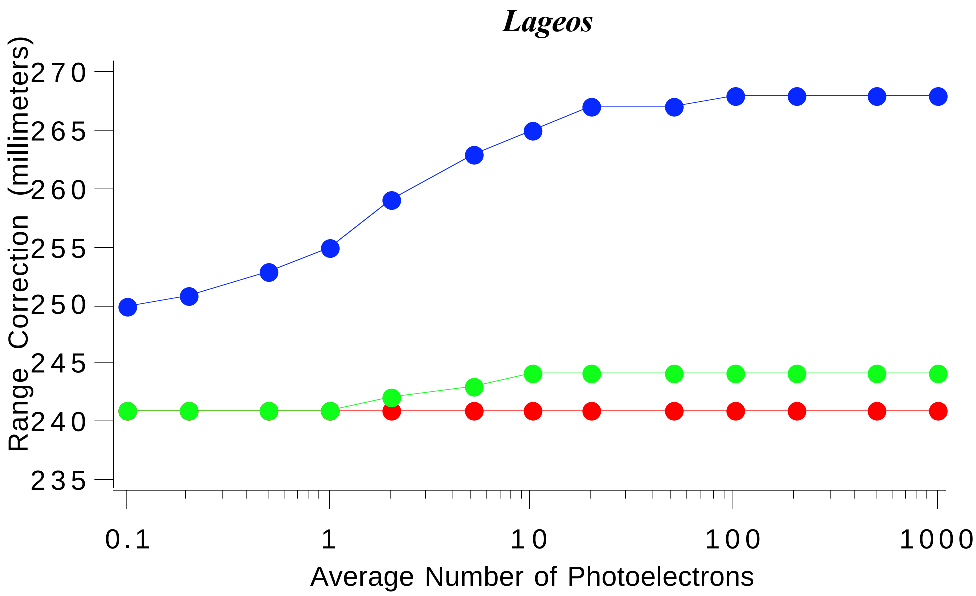


Figure 8. Range correction for Lageos vs number of photoelectrons.

Blue = Half-Max

Green = Half Area

Red = Centroid

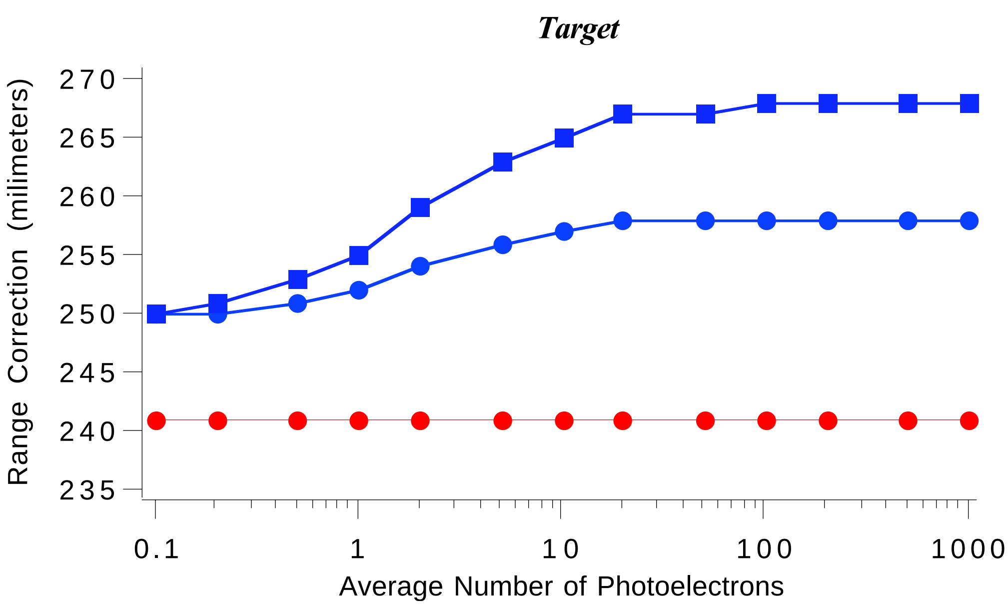


Figure 9. Range correction for target calibration compared to Lageos Half-Max

Square = Lageos half-max

Circle = Target half-max

Red = Centroid and Half Area

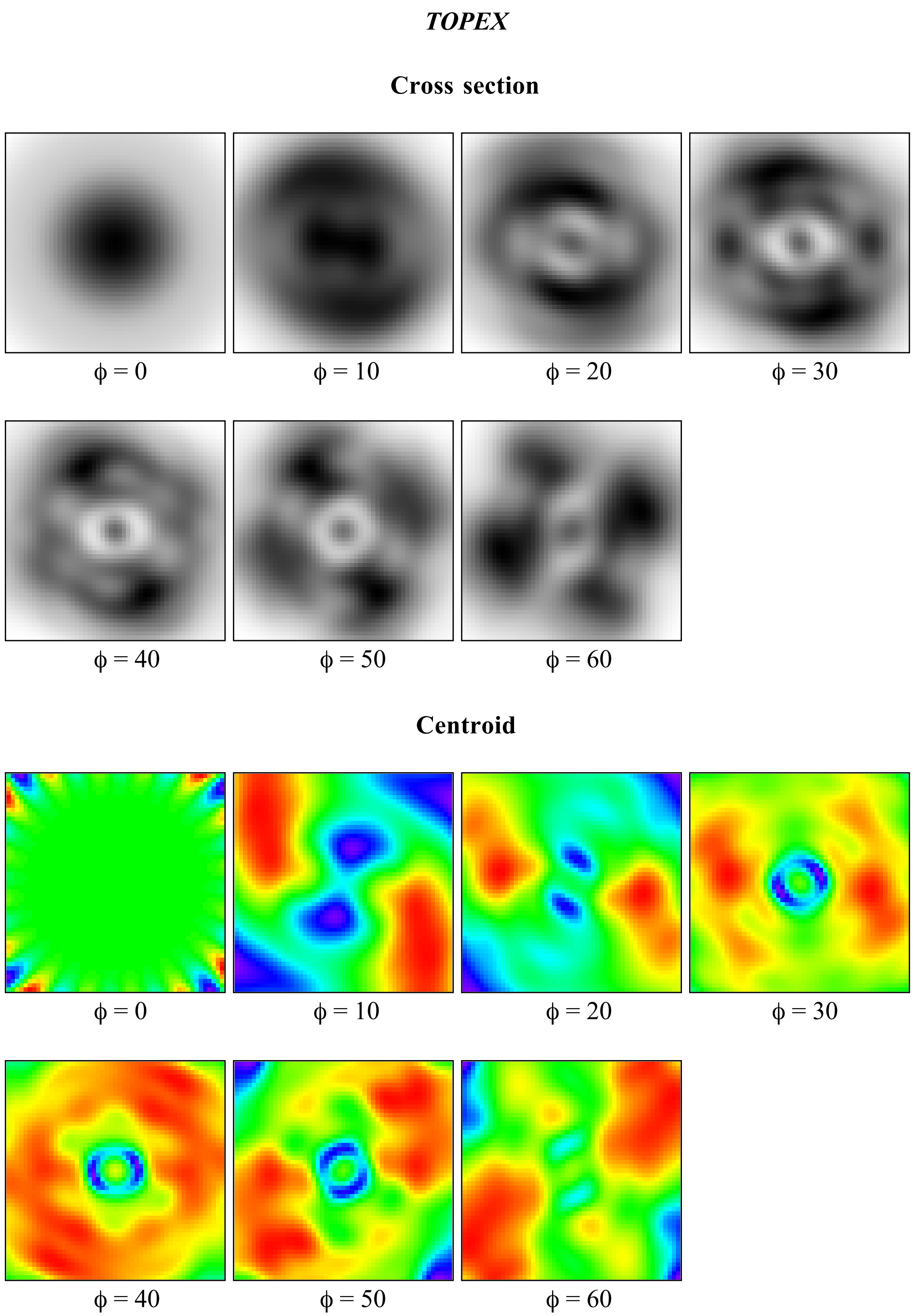
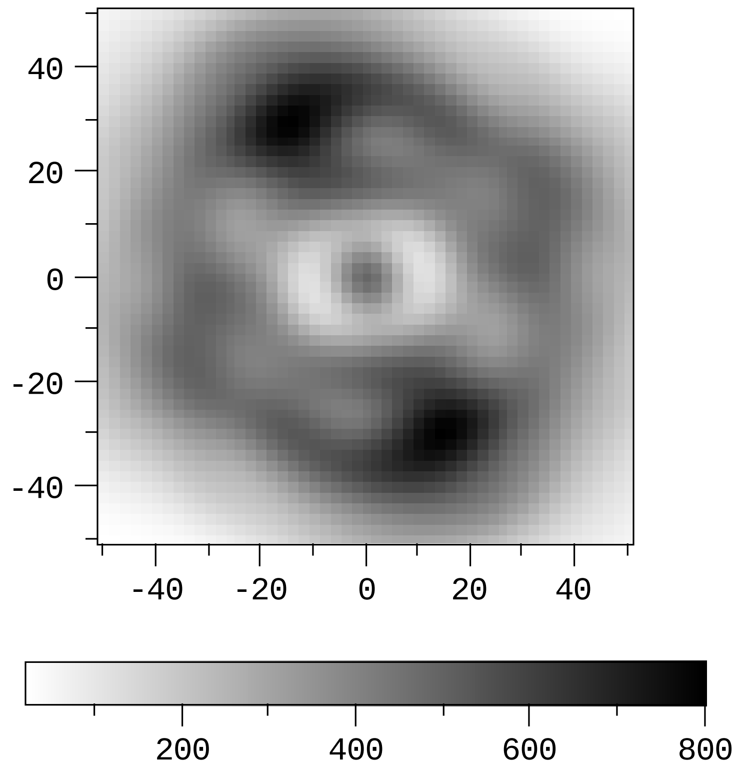
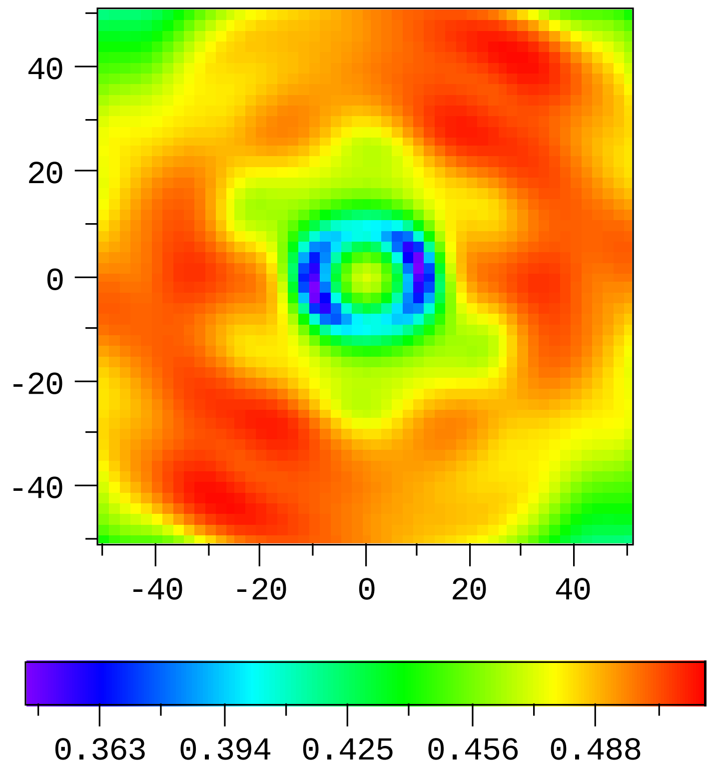


Figure 10. Topex cross section and centroid range correction vs incidence angle.



Microrad	Minimum	Average	Maximum	Max - Min
0.0	489.4849998	489.4849998	489.4849998	0.0000000
2.0	445.3091673	451.7482330	460.2240226	14.9148554
4.0	364.3271325	376.2008227	389.4566931	25.1295606
6.0	261.1307461	285.4701279	311.9223233	50.7915771
8.0	170.8840018	214.4467675	261.0669668	90.1829651
10.0	119.0825894	187.6761896	261.7588842	142.6762948
12.0	118.1161724	209.9228314	314.8369044	196.7207319
14.0	156.4614158	267.2486554	396.6649886	240.2035728
16.0	215.8758010	335.2755717	475.2044183	259.3286173
18.0	265.4050812	391.6353015	528.6968374	263.2917561
20.0	298.2786780	425.6962326	563.5087252	265.2300472
22.0	309.8627082	440.2211573	591.8992403	282.0365321
24.0	312.0507327	446.9562308	620.4357771	308.3850444
26.0	316.6035096	458.3720343	659.1760814	342.5725718
28.0	326.8786992	479.8423479	711.7925307	384.9138315
30.0	349.3779229	508.0959935	758.2196375	408.8417146
32.0	377.0046497	533.4914398	791.0682353	414.0635856
34.0	402.6764310	545.3260540	794.3722310	391.6958000
36.0	417.2127329	536.9520984	763.7709530	346.5582201
38.0	410.5974142	509.5386763	710.1096586	299.5122444
40.0	355.1654490	468.9192973	645.2657644	290.1003153
42.0	299.9186935	422.8592245	580.0306474	280.1119540
44.0	254.0910076	377.9353615	519.2259367	265.1349291
46.0	224.7911562	337.4302900	470.3656472	245.5744910
48.0	197.7001026	300.7906508	425.8873839	228.1872813
50.0	171.0913676	266.1335965	383.7737122	212.6823446

Figure 11. Topex cross section for ϕ = 40 degrees incidence angle



Microrad	Minimum	Average	Maximum	Max - Min
0.0	0.4724049	0.4724049	0.4724049	0.0000000
2.0	0.4666691	0.4673081	0.4687676	0.0020985
4.0	0.4523631	0.4539938	0.4566284	0.0042652
6.0	0.4257099	0.4293210	0.4320979	0.0063880
8.0	0.3839133	0.3970060	0.4074840	0.0235707
10.0	0.3470787	0.3792045	0.4040112	0.0569325
12.0	0.3677682	0.3979261	0.4228421	0.0550739
14.0	0.4164559	0.4323093	0.4444229	0.0279670
16.0	0.4441747	0.4568523	0.4678594	0.0236847
18.0	0.4573846	0.4696824	0.4867587	0.0293742
20.0	0.4629551	0.4753994	0.4949082	0.0319531
22.0	0.4642413	0.4777824	0.4982947	0.0340534
24.0	0.4640330	0.4796338	0.5003336	0.0363006
26.0	0.4658928	0.4828465	0.5022588	0.0363660
28.0	0.4696983	0.4875240	0.5045417	0.0348434
30.0	0.4761811	0.4926806	0.5068545	0.0306734
32.0	0.4824622	0.4969321	0.5110175	0.0285553
34.0	0.4868215	0.4995607	0.5122519	0.0254304
36.0	0.4884893	0.5004261	0.5110413	0.0225520
38.0	0.4872767	0.4999017	0.5084432	0.0211665
40.0	0.4848260	0.4985097	0.5070389	0.0222129
42.0	0.4823142	0.4968927	0.5052074	0.0228932
44.0	0.4808566	0.4956154	0.5047393	0.0238827
46.0	0.4804566	0.4948373	0.5079527	0.0274962
48.0	0.4791453	0.4941571	0.5119505	0.0328051
50.0	0.4766798	0.4930398	0.5144387	0.0377589

Figure 12. Topex centroid range correction (meters) at $\phi = 40$ degrees incidence angle.

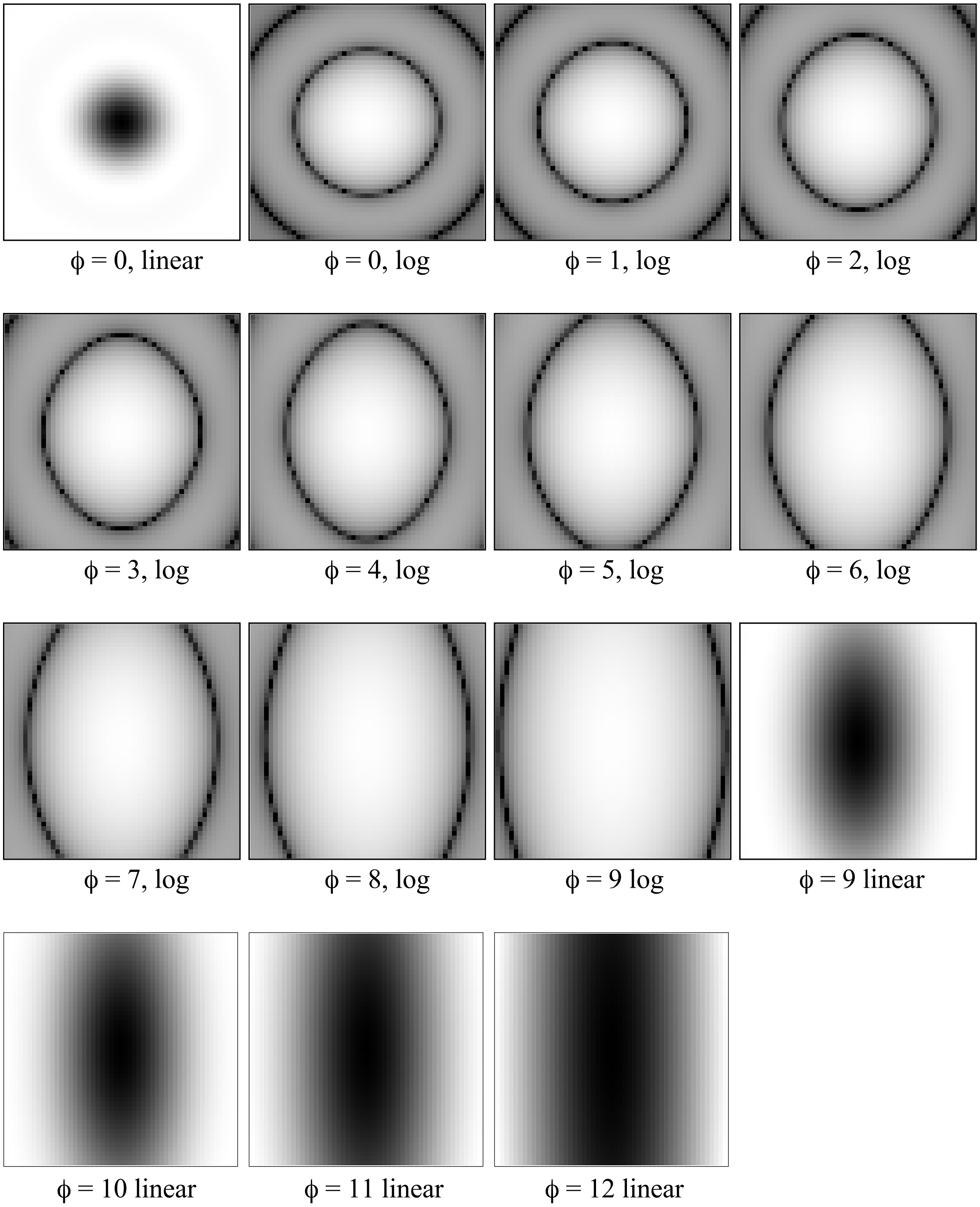


Figure 13. Cross section of a Westpac cube corner vs incidence angle in degrees. The cutoff angle is 13 degrees.

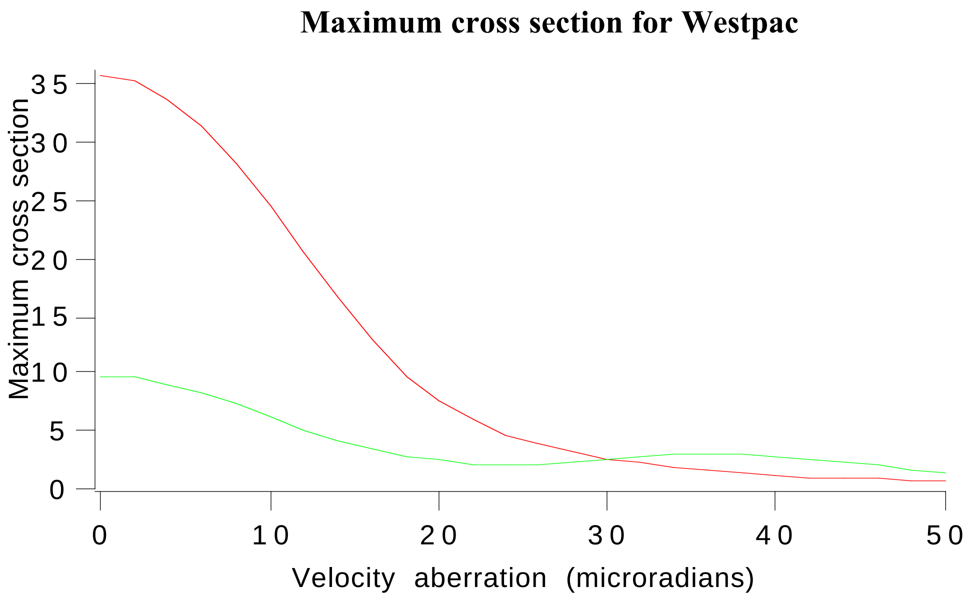


Figure 14. Maximum cross section vs velocity aberration for a Westpac cube corner.
Red = No dihedral angle offset
Green = Dihedral angle 1.75 arc seconds

Microradians	Cross-section	angle(deg)	Cross Section sq m)
0.0	35.8402293	0.0	2418805.6
2.0	35.3181832	0.0	2383573.5
4.0	33.7898797	0.0	2280430.5
6.0	31.3649939	0.0	2116778.4
8.0	28.2140207	0.0	1904123.7
10.0	24.5513900	0.0	1656938.1
12.0	20.6147328	0.0	1391258.7
14.0	16.6427276	0.0	1123193.8
16.0	12.8702210	0.0	868592.7
18.0	9.6913873	1.0	654057.8
20.0	7.4623938	2.0	503626.2
22.0	5.8602015	3.0	395496.6
24.0	4.6632240	4.0	314714.3
26.0	3.7767337	4.0	254886.3
28.0	3.1200028	5.0	210564.5
30.0	2.5685487	5.0	173347.7
32.0	2.1777955	6.0	146976.3
34.0	1.8420879	6.0	124319.9
36.0	1.5592573	7.0	105232.0
38.0	1.3636733	7.0	92032.4
40.0	1.1796034	7.0	79609.7
42.0	1.0083017	7.0	68048.8
44.0	0.8980635	8.0	60609.0
46.0	0.8019726	8.0	54124.0
48.0	0.7107092	8.0	47964.7
50.0	0.6246997	8.0	42160.1

Table 2. Maximum cross section vs velocity aberration with no dihedral angle offset. Column 1 is the velocity aberration, column 2 is the cross section in units of 4π x 10,000 sq meters, column 3 is the incidence angle where the maximum occurs, and column 4 is the cross section in sq meters.

Microrad	Minimum	Average	Maximum	Max - Min
26.0	0.7717976	1.5842181	2.6464027	1.8746051
28.0	0.5466180	1.3367811	2.4216995	1.8750815
30.0	0.3655866	1.1131764	2.1982273	1.8326407
32.0	0.2268892	0.9147052	1.9784917	1.7516025
34.0	0.1268098	0.7419771	1.7648064	1.6379966
36.0	0.0602277	0.5954080	1.5592573	1.4990296
38.0	0.0211537	0.4723924	1.3636733	1.3425196
40.0	0.0032599	0.3711284	1.1796034	1.1763435
42.0	0.0003617	0.2896156	1.0083017	1.0079400
44.0	0.0005316	0.2254193	0.8507194	0.8501879
46.0	0.0008714	0.1759270	0.7075042	0.7066328
48.0	0.0017305	0.1378011	0.5790063	0.5772758
50.0	0.0020692	0.1094539	0.4652917	0.4632225

Table 3. Cross section for Westpac at 7 deg incidence angle with no dihedral angle offset . The maximum values for velocity aberration 36, 38, 40, and 42 microradians are the values shown in column 2 of table 2.

Microradians	Cross-section	angle(deg)	Cross Section(sq m)
0.0	9.7185625	0.0	655891.8
2.0	9.5451555	0.0	644188.8
4.0	9.0422200	0.0	610246.4
6.0	8.2595189	0.0	557423.1
8.0	7.2731369	0.0	490853.6
10.0	6.1761959	0.0	416822.6
12.0	5.0703650	0.0	342191.7
14.0	4.1527559	1.0	280263.5
16.0	3.3936579	2.0	229033.1
18.0	2.8364401	2.0	191427.3
20.0	2.4167251	2.0	163101.3
22.0	2.1403943	2.0	144452.1
24.0	1.9977891	2.0	134827.9
26.0	2.0994745	0.0	141690.5
28.0	2.3420730	0.0	158063.1
30.0	2.6136478	0.0	176391.3
32.0	2.8498785	0.0	192334.2
34.0	3.0019057	0.0	202594.3
36.0	3.0466102	0.0	205611.3
38.0	2.9844830	0.0	201418.5
40.0	2.8160866	0.0	190053.6
42.0	2.5648466	0.0	173097.8
44.0	2.2631820	0.0	152738.9
46.0	1.9465926	0.0	131372.7
48.0	1.6624684	1.0	112197.6
50.0	1.4361345	1.0	96922.7

Table 4. Maximum cross section vs velocity aberration with a dihedral angle offset of 1.75 arc seconds.

The Westpac cube corners have a reflection loss of about a factor of 2. The values in column 2 of tables 2 and 4 include a loss of .931 for reflection losses at the front face. In column 4 of these tables the cross section from column 2 has been divided by .931 to remove the loss at the front face and multiplied by .5 to give a total reflection loss of 50%. The result is then multiplied by 4π x 10,000 to convert to standard units.