SLR Contribution to the ITRF

- Review of the SLR Contribution to the ITRF
- TRF Implementation: Theoretical Aspects
- Reality of the Current SLR Network
- Reality of the Current Collocation Sites
- Some Analysis of the ILRS Pilot Project Solutions
- Conclusion

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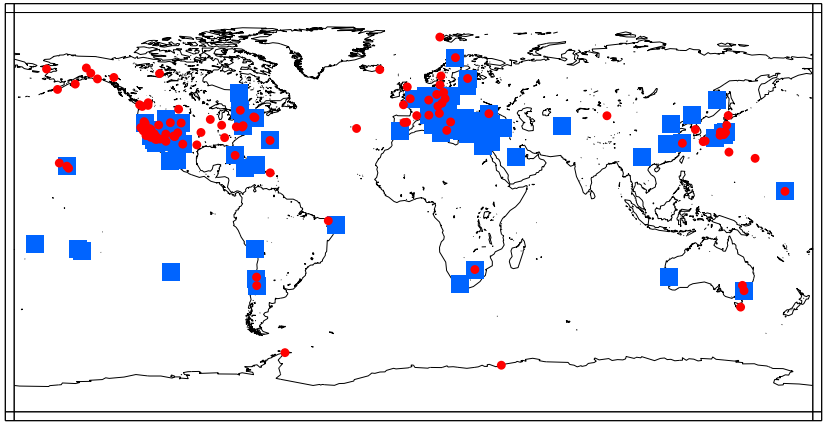
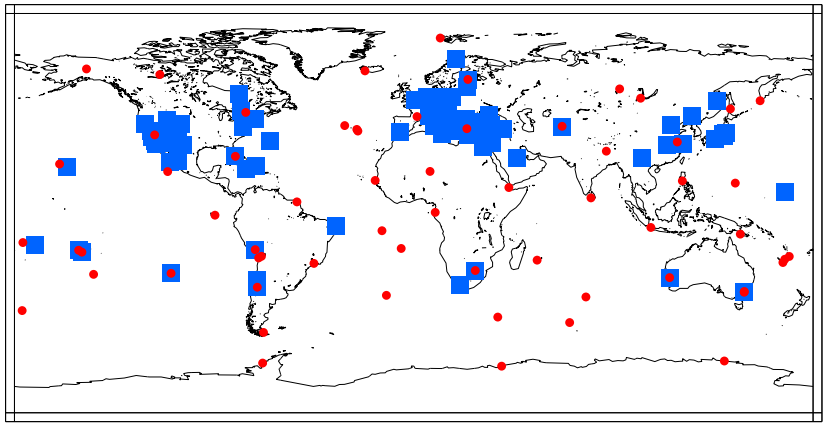
SLR Contribution to ITRF

- Origin: Center of Mass
- Scale: Together with VLBI
- Unconstrained Solutions for ITRF2000

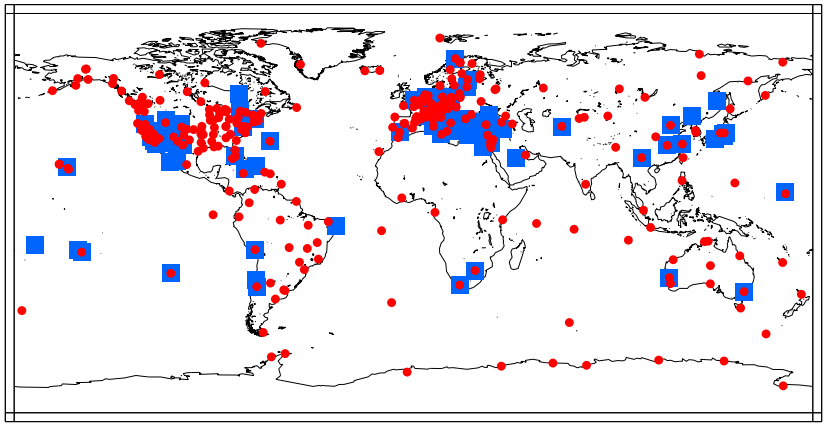
ITRF2000 Network (SLR/VLBI Colocations)



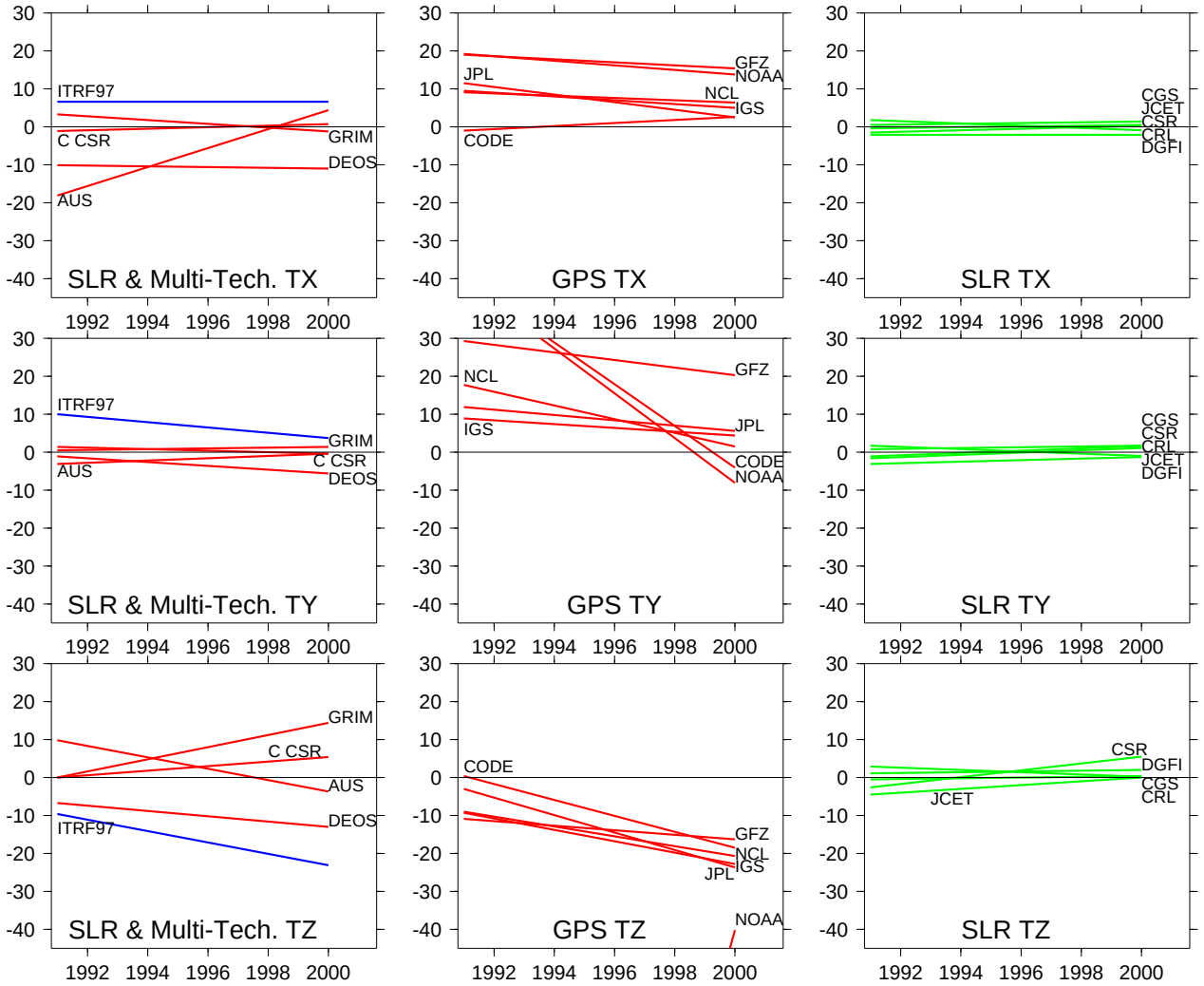
ITRF2000 Network (SLR/DORIS Colocations)



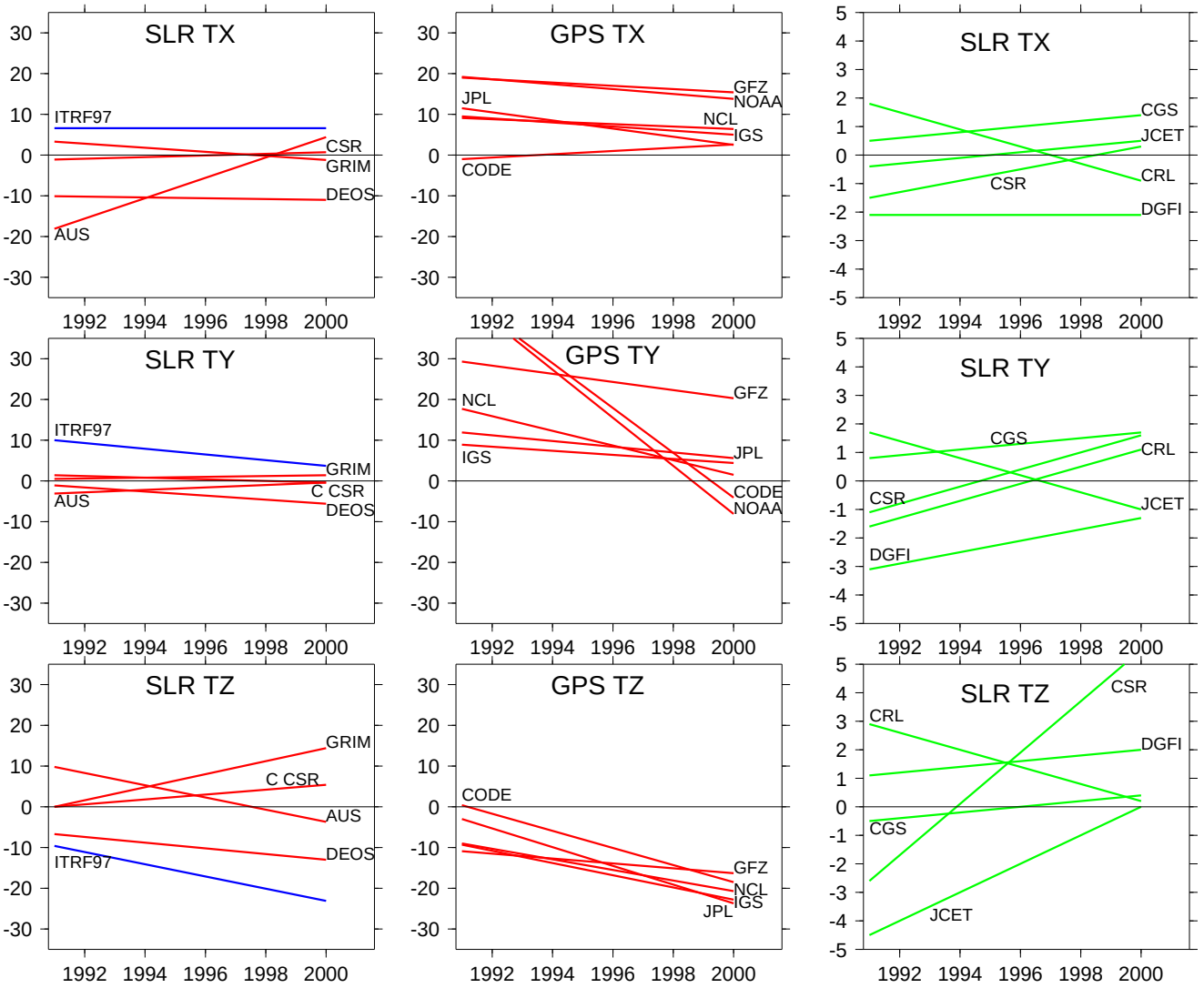
ITRF2000 Network (SLR/GPS Colocations)



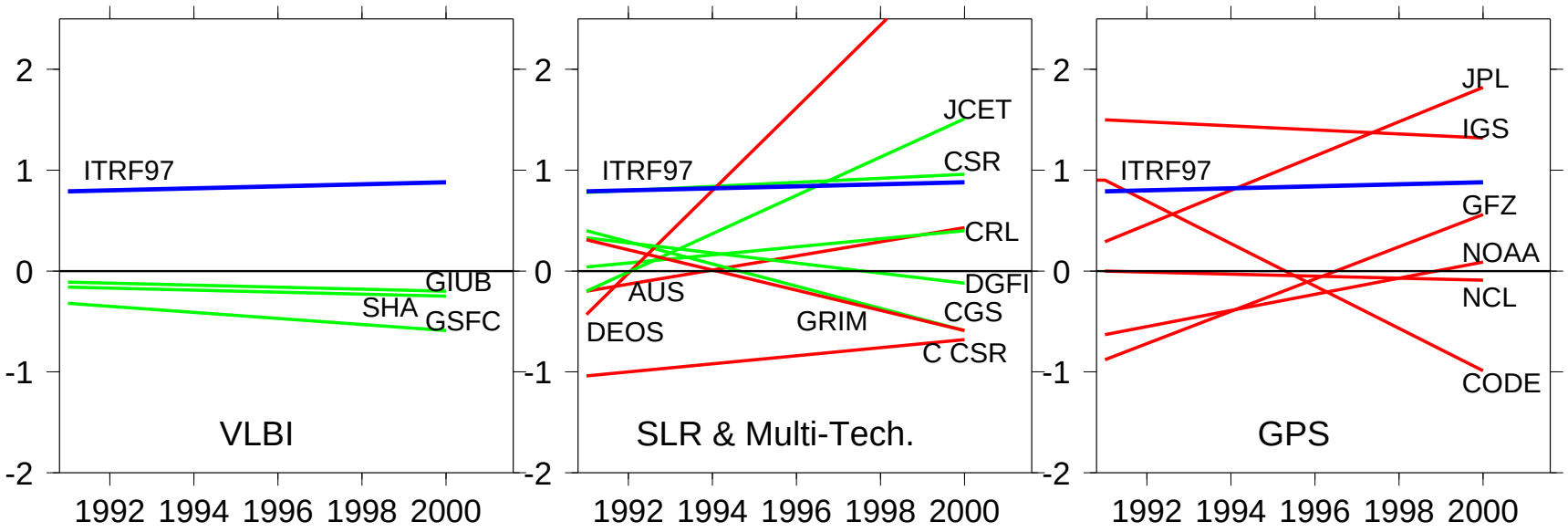
ITRF2000: Translation Variations (mm)



ITRF2000: Translation Variations (mm)



ITRF2000: Scale Variations (ppb = 10^{-9})



(1 ppb $\approx$ 6 mm shift in station heights)

TRS & TRF in Space Geodesy

TRS: Mathematical model of the physical world:
No physical existence
Observations provide "Network Shape"

- **TRF & Space Geodesy techniques:**
  - **Origin: Dynamical techniques provide the CoM**
  - **Scale: Same for all techniques ?**
  - **Orientation: Unobservable by any technique**
- **Specific constraints are needed to complete the TRF datum definition, leaving the shape undistorted**
- **Rank Deficiency in terms of Normal Equation System**
- **Separate the variance (noise) of the observations (having a stochastic character) & the (deterministic) frame parameters: Use of minimum constraints**

Datum Definition: ITRF combination

$$X_2 = X_1 + T + DX_1 + R.X_1 \tag{1}$$

Assuming constant velocities and differentiating Eq. (1) w.r.t time:

$$\dot{X}_2 = \dot{X}_1 + \dot{T} + \overbrace{D\dot{X}_1}^{\approx 0} + \dot{D}X_1 + \overbrace{R\dot{X}_1}^{\approx 0} + \dot{R}X_1 \tag{2}$$

$$\dot{T} = \begin{pmatrix} \dot{T}1 \\ \dot{T}2 \\ \dot{T}3 \end{pmatrix}, \qquad \dot{R} = \begin{pmatrix} 0 & -\dot{R}3 & \dot{R}2 \\ \dot{R}3 & 0 & -\dot{R}1 \\ -\dot{R}2 & \dot{R}1 & 0 \end{pmatrix}$$

=> 14 degrees of freedom to define a TRF.

Datum Definition / Minimum Constraints (1/4)

Application of Minimum Constraints (MC) approach based on theoretical works by many authors, since the 70's on, e.g.:

- **Free Network Adjustment**
- **S-transformation**
- **Minimum/Inner Constraints**

Main Goal: The "best" TRF datum definition preserving both the actual quality of space geodesy observations and the "Network Shape"

Datum Definition / Minimum Constraints (2/4)

The starting point is the standard relation between two TRF's:

$$X_2 = X_1 + A\theta \tag{3}$$

$$\theta = (T1, T2, T3, D, R1, R2, R3, \dot{T}1, \dot{T}2, \dot{T}3, \dot{D}, \dot{R}1, \dot{R}2, \dot{R}3)^T$$

$$A = \begin{pmatrix}
. & . & . & . & . & . & . & . & . & . & . & . & . & . \\
1 & 0 & 0 & x_i^0 & 0 & z_i^0 & -y_i^0 & & & & & & & \\
0 & 1 & 0 & y_i^0 & -z_i^0 & 0 & x_i^0 & & & & \approx 0 & & & \\
0 & 0 & 1 & z_i^0 & y_i^0 & -x_i^0 & 0 & & & & & & & \\
 & & & & & & & 1 & 0 & 0 & x_i^0 & 0 & z_i^0 & -y_i^0 \\
 & & & \approx 0 & & & & 0 & 1 & 0 & y_i^0 & -z_i^0 & 0 & x_i^0 \\
 & & & & & & & 0 & 0 & 1 & z_i^0 & y_i^0 & -x_i^0 & 0 \\
. & . & . & . & . & . & . & . & . & . & . & . & . & .
\end{pmatrix}$$

Datum Definition / Minimum Constraints (3/4)

L.S. of eq. (3) yields: $\theta = \overbrace{(A^T A)^{-1} A^T}^{\mathbf{B}} (X_2 - X_1)$

Using $B = (A^T A)^{-1} A^T$, **containing all the necessary info. to define a TRF, a "datum definition" equation at** Σ_θ **level could be written as:**

$$B(X_2 - X_1) = 0 \qquad (\Sigma_\theta) \qquad (4)$$

and in terms of normal equation: $\boxed{B^T \Sigma_\theta^{-1} B(X_2 - X_1) = 0}$

Datum Definition / Minimum Constraints (4/4)

The initial NEQ system of space geodesy observations could be written as:

$$N_{unc}(\Delta X) = K \tag{5}$$

where $\Delta X = X - X_{apr}$ **(Linearized Unknowns)**

Selecting a Reference TRF (X_R), MC equation is:

$$B^T \Sigma_\theta^{-1} B(\Delta X) = B^T \Sigma_\theta^{-1} B(X_R - X_{apr}) \tag{6}$$

Cumulating (5) and (6) yields:

$$(N_{unc} + B^T \Sigma_\theta^{-1} B)(\Delta X) = K + B^T \Sigma_\theta^{-1} B(X_R - X_{apr})$$

TRF + EOP Simultaneous Combination

CATREF Software upgraded:

- **inclusion of EOP's**
- **implementation of minimum constraint equations**

$$
\begin{aligned}
x_s^p &= x^p + R2_k \\
y_s^p &= y^p + R1_k \\
UT_s &= UT - \tfrac{1}{f} R3_k \\
\dot{x}_s^p &= \dot{x}^p + \dot{R}2_k \\
\dot{y}_s^p &= \dot{y}^p + \dot{R}1_k \\
LOD_s &= LOD + \tfrac{\Lambda_0}{f} \dot{R}3_k
\end{aligned}
$$

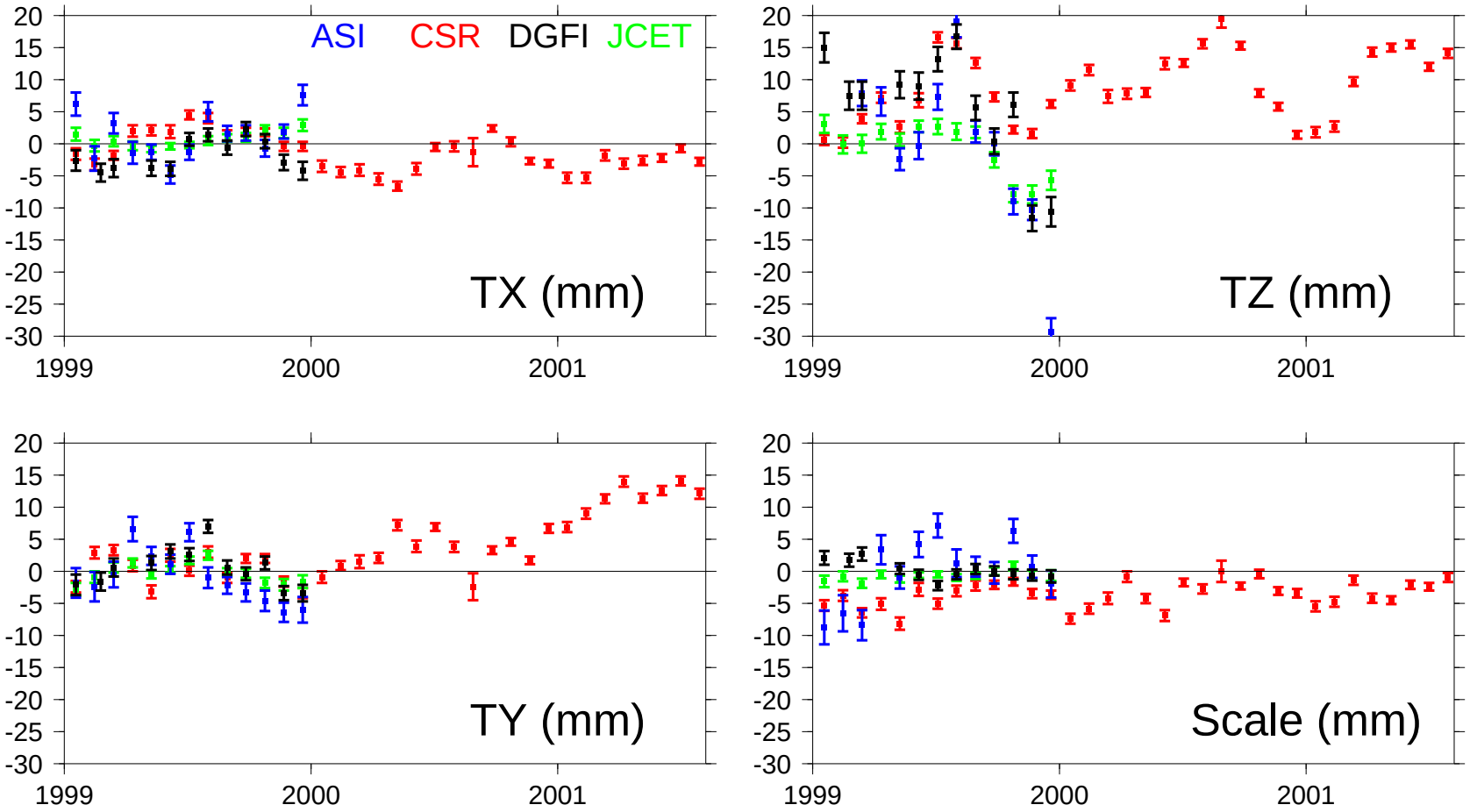
Some Analysis Tests follow using:

- **SLR: ILRS Pilot Project Monthly Solutions**
- **GPS: IGS weekly combined solutions**
- **VLBI: GSFC session sinex files**
- **DORIS: IGN monthly solutions**

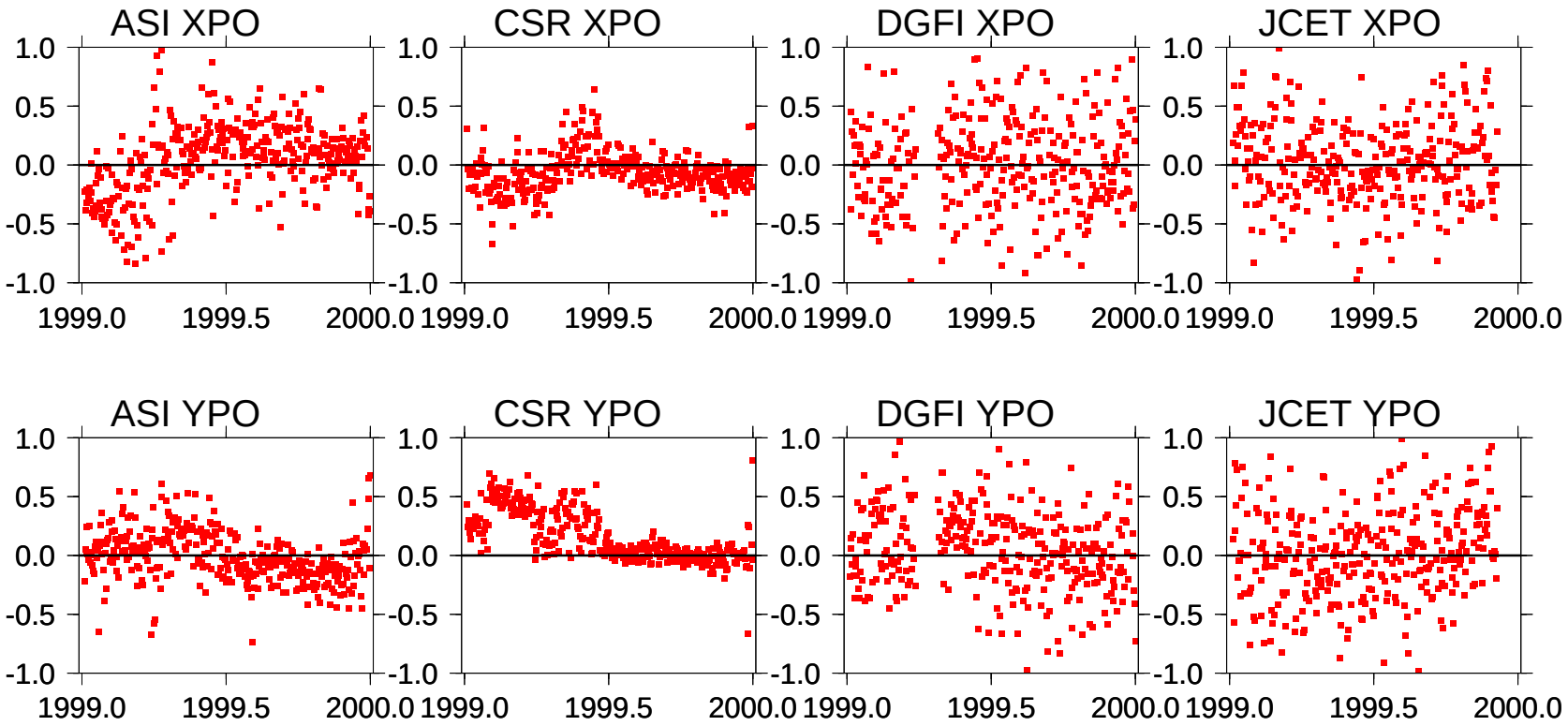
Focus on Origin, Scale, EOP consistency

ILRS Origin and Scale Consistency

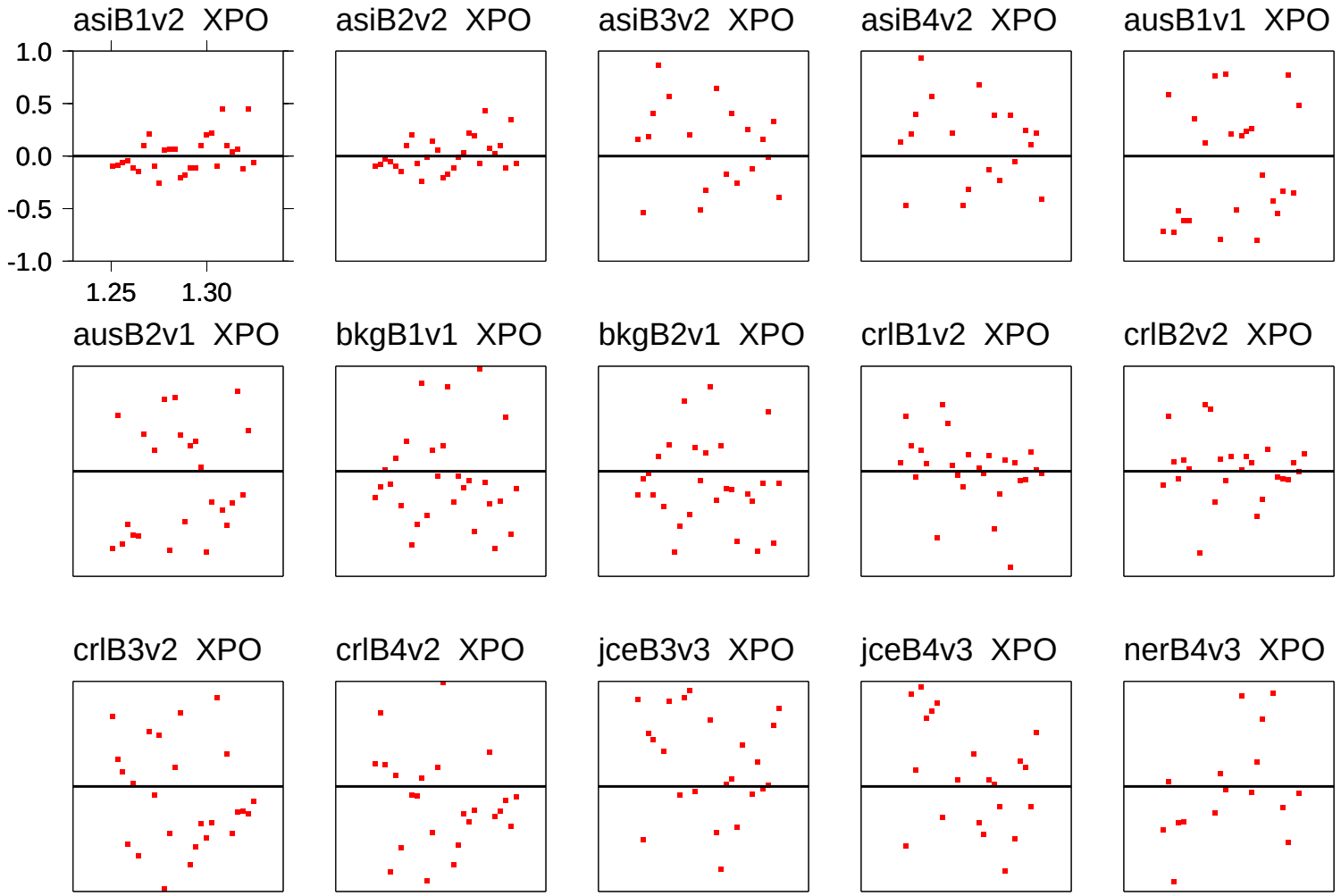
Data: Monthly solutions over 1-3 years: A-type Solutions



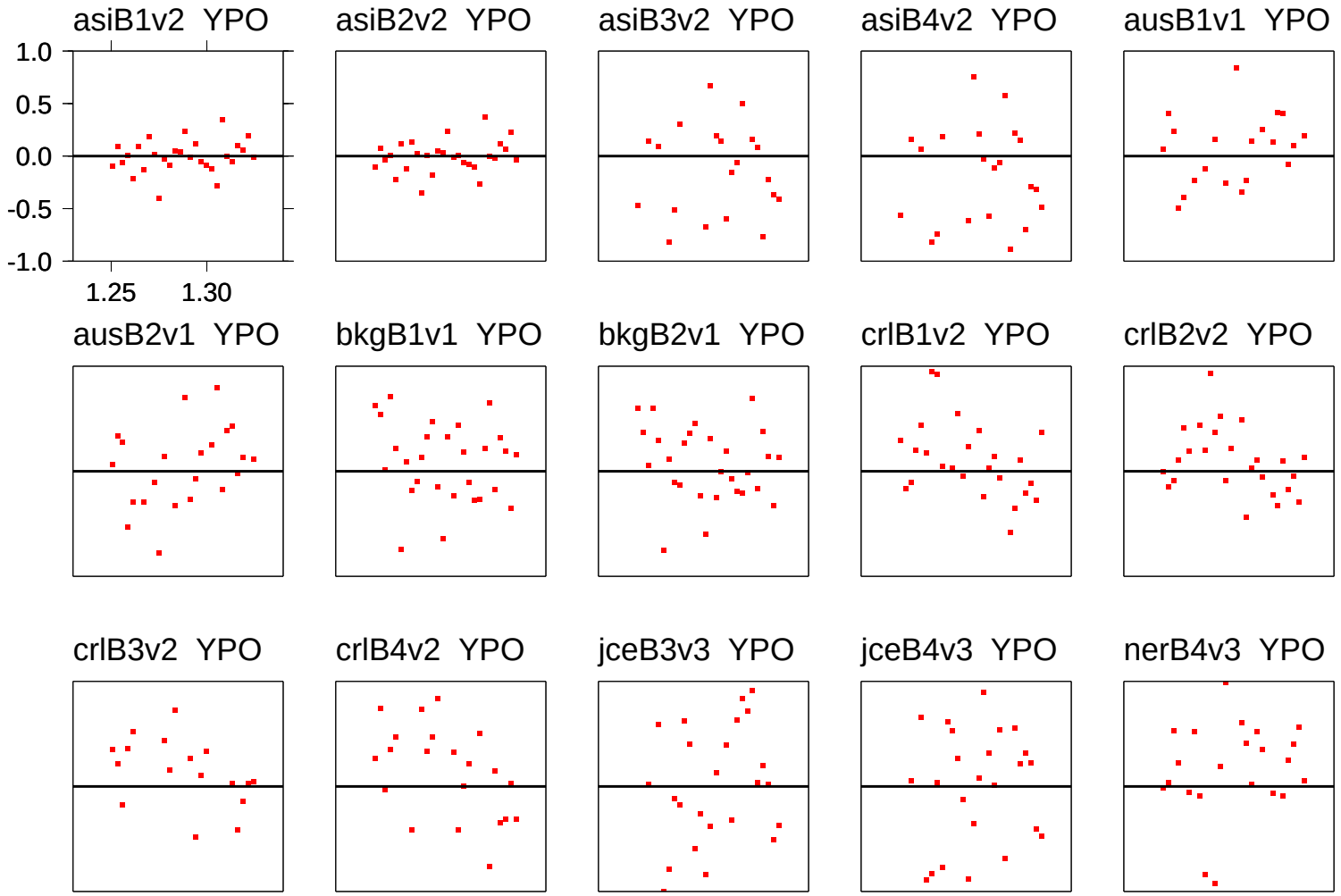
ILRS EOP Consistency: One year solutions (1999) X & Y_Pole Residuals



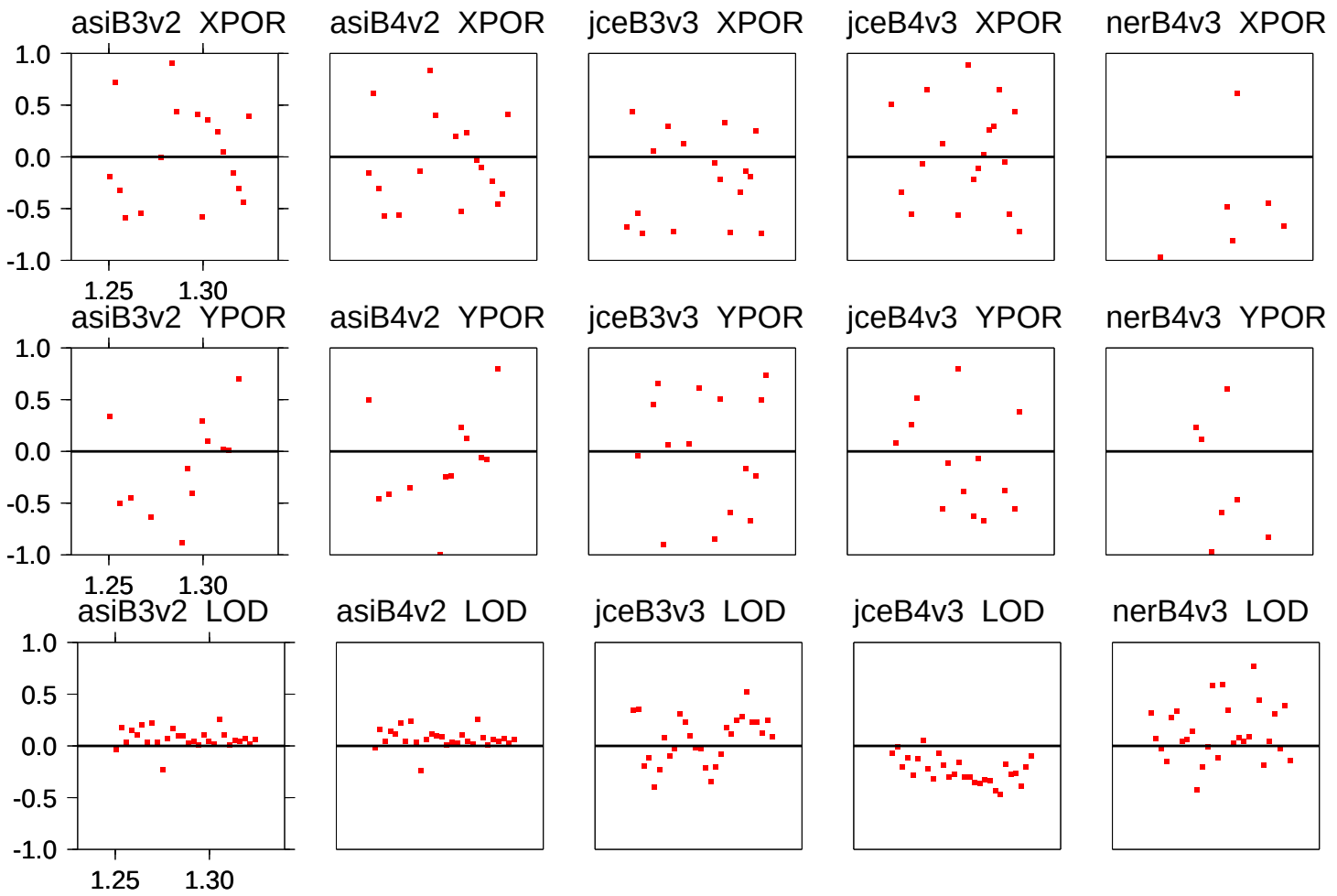
ILRS EOP Consistency: One month solutions: (APR 2001) X-Pole Residuals



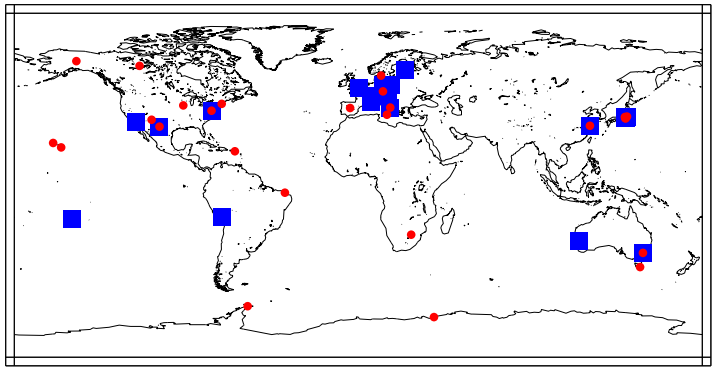
ILRS EOP Consistency: One month solutions: (APR 2001) Y-Pole Residuals



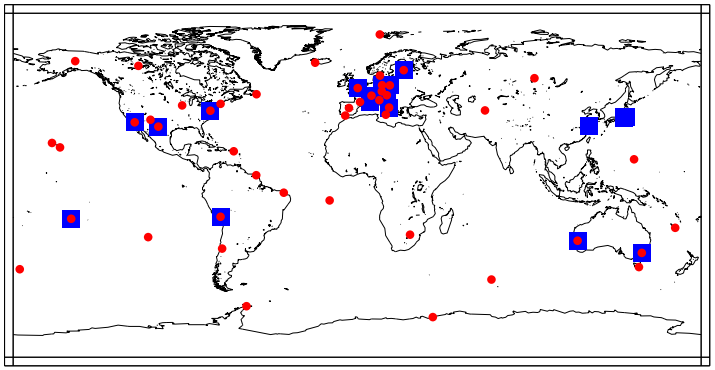
ILRS EOP Consistency: One month solutions: X-Pole, Y-Pole Rates and LOD Residuals



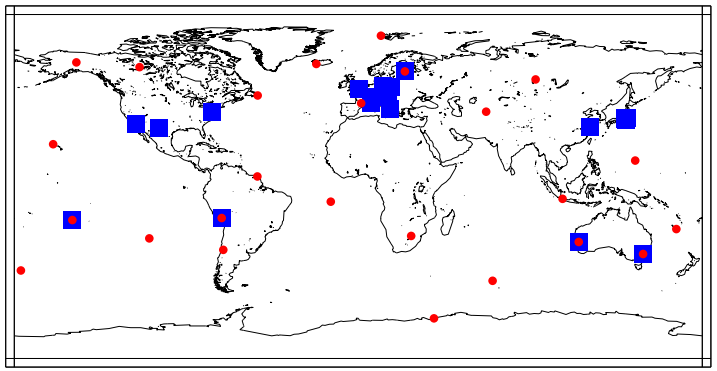
Current SLR-VLBI Collocations



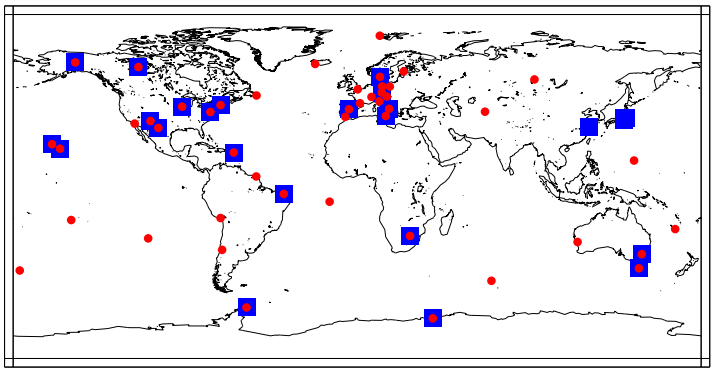
Current SLR-GPS Collocations



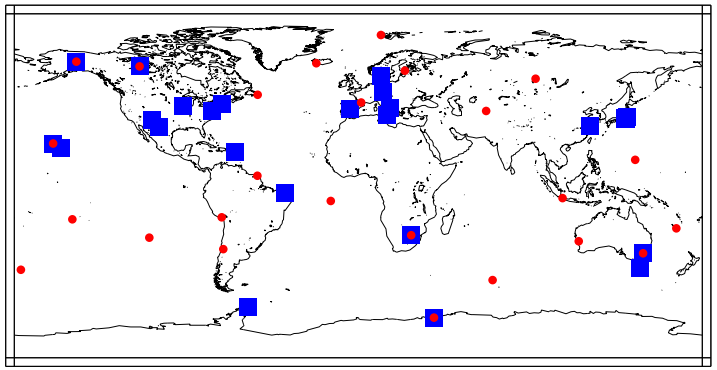
Current SLR-DORIS Collocations



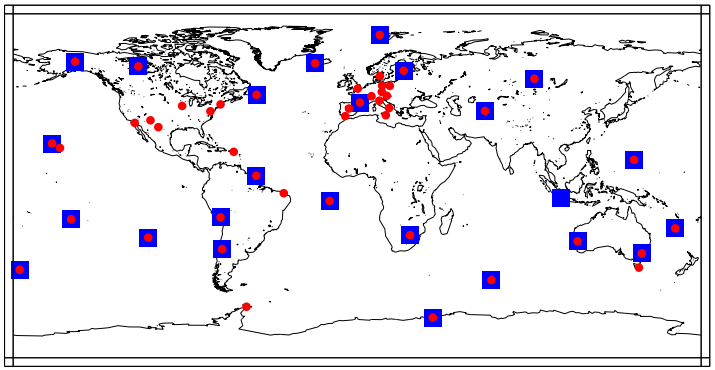
Current VLBI-GPS Collocations



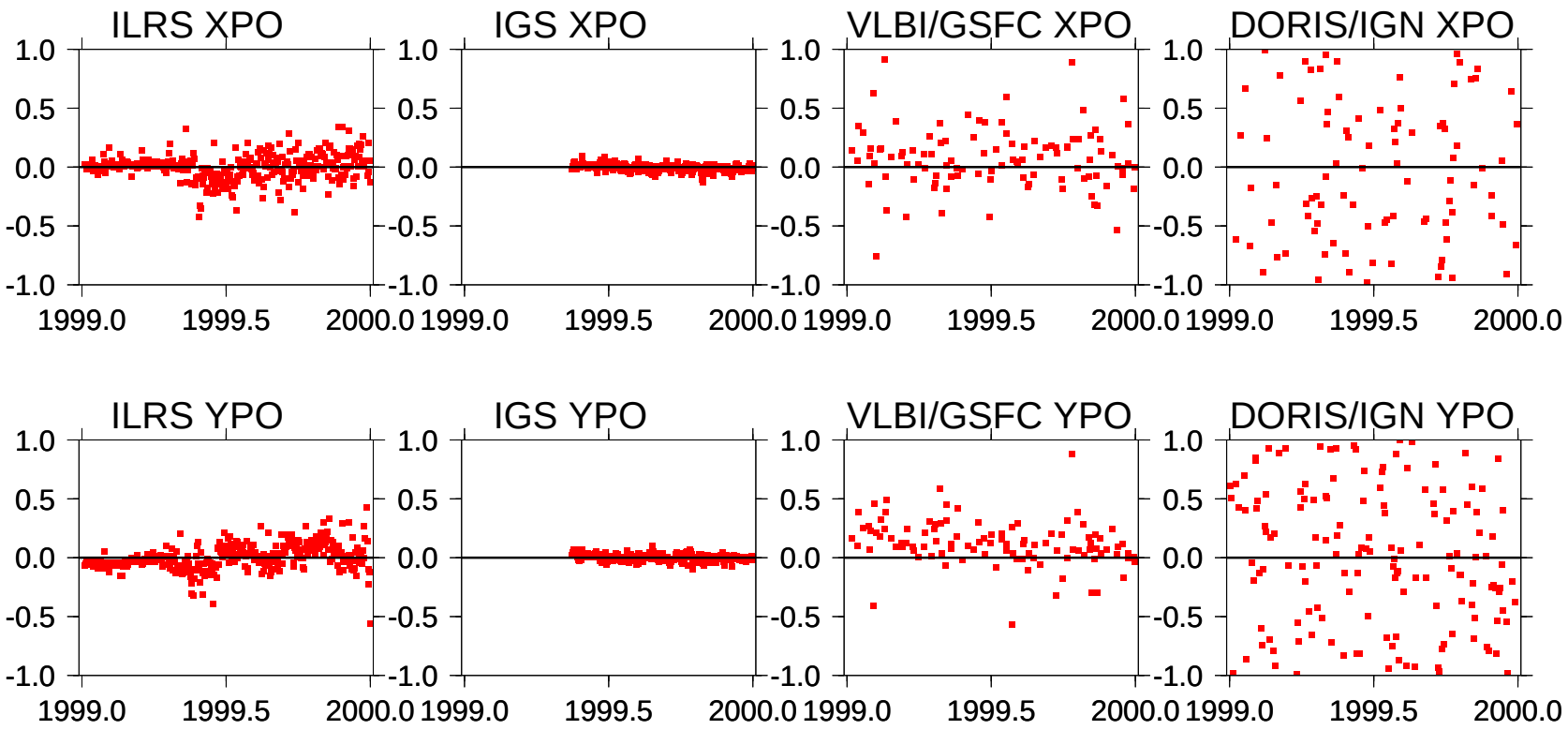
Current VLBI-DORIS Collocations



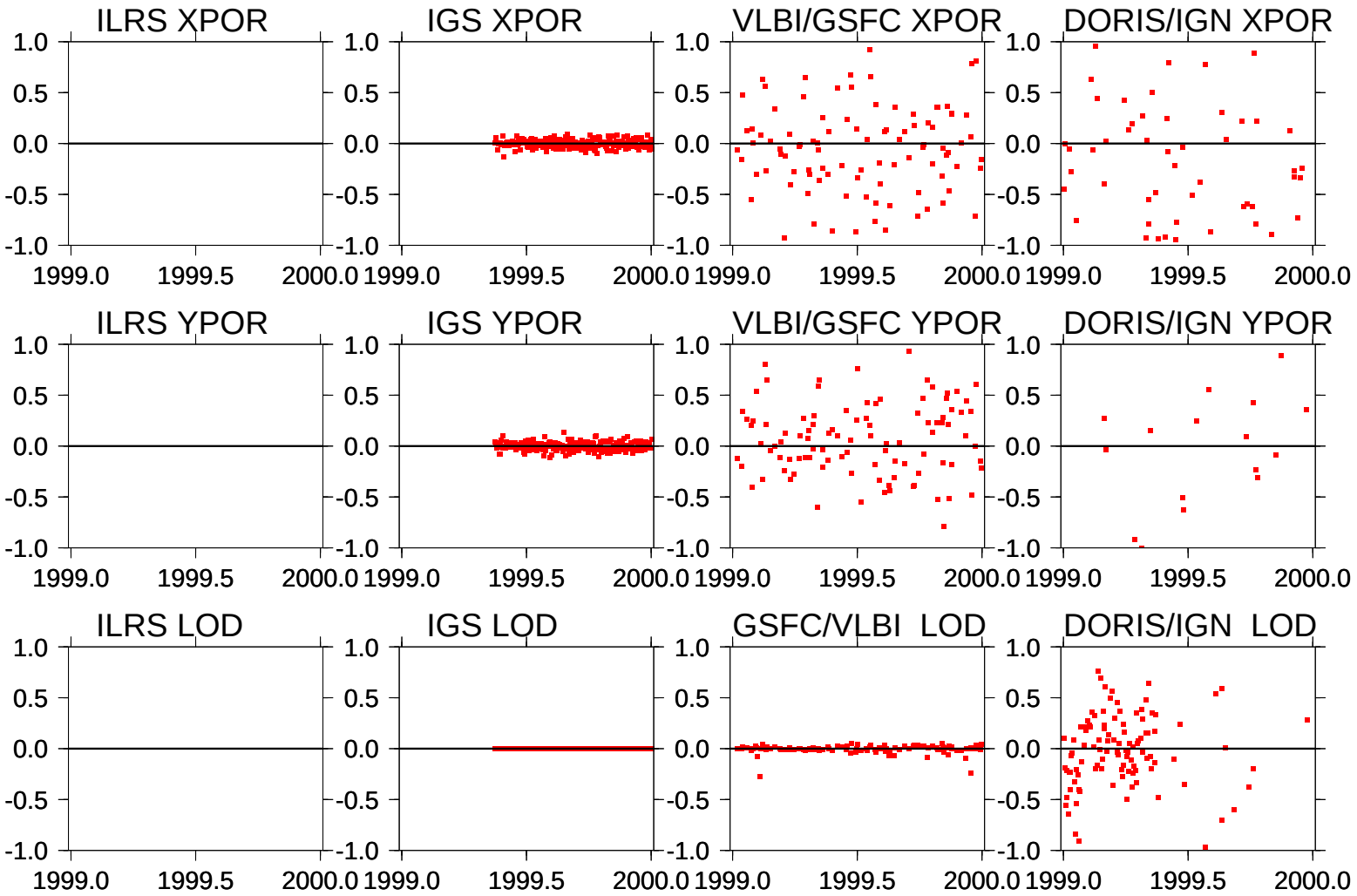
Current DORIS-GPS Collocations



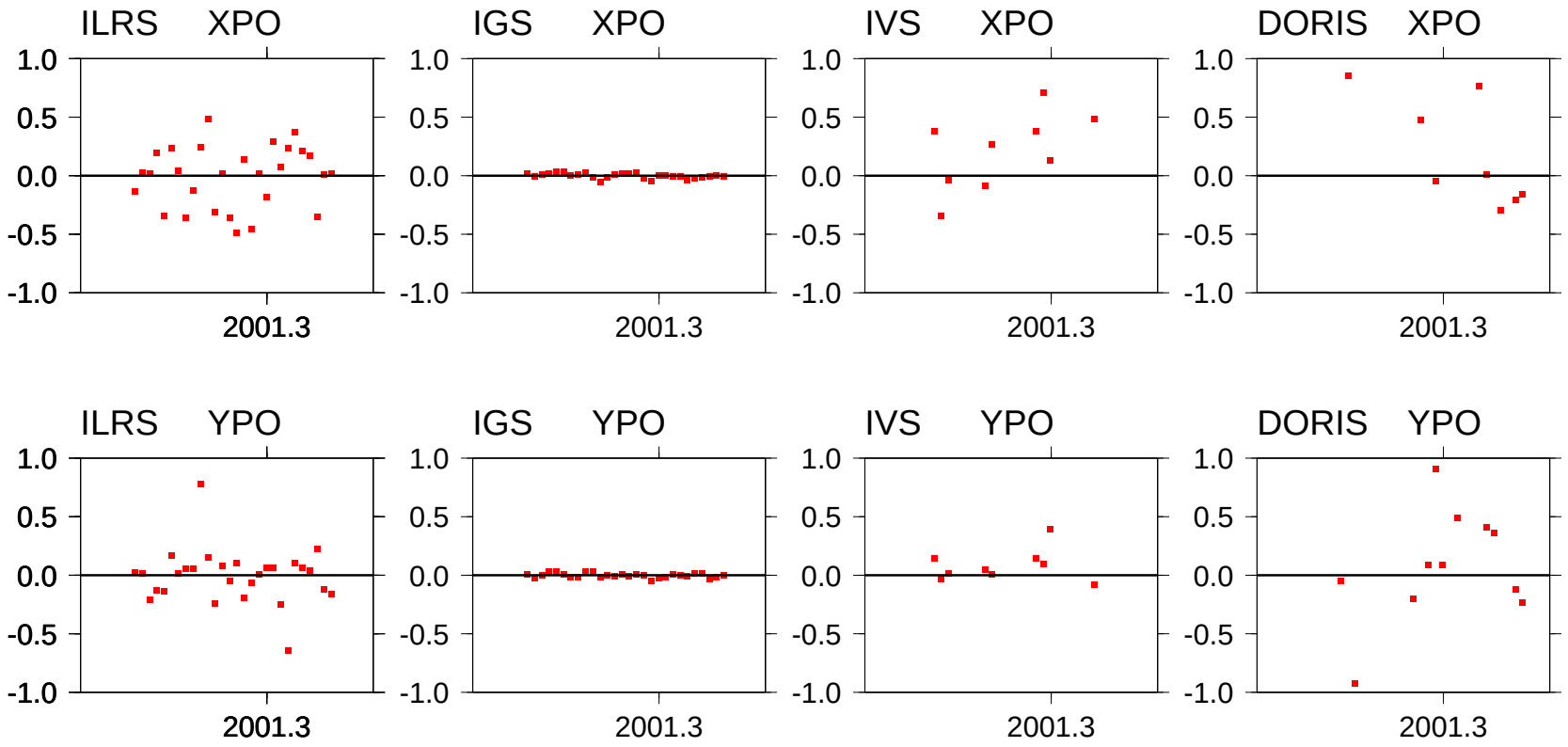
One year multi-technique combination
X & Y_pole Residuals



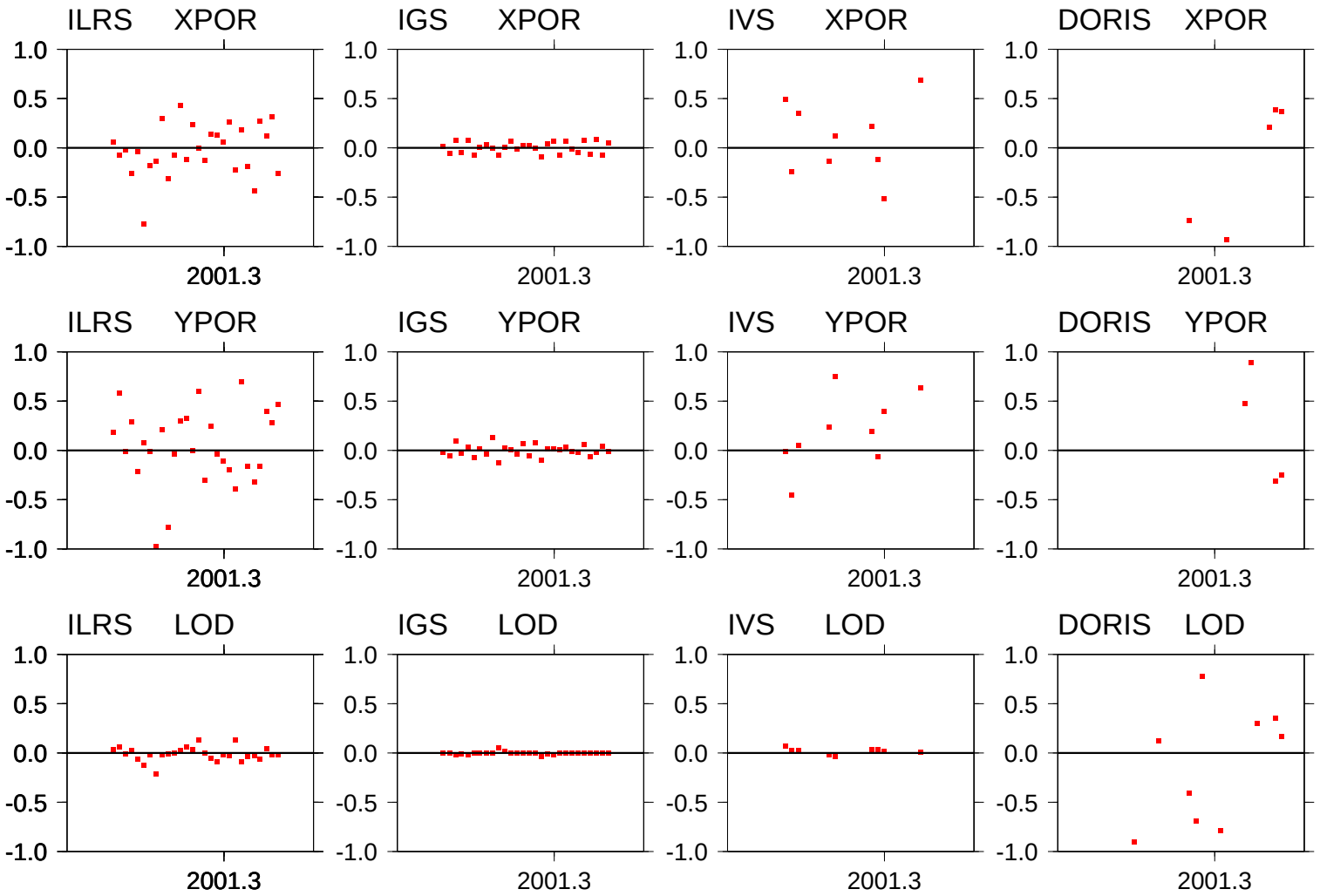
One year multi-technique combination
X & Y_pole Rate & LOD Residuals



One Month multi-technique combination: X & Y_pole Residuals



One Month multi-technique combination: X & Y_pole Rate & LOD Residuals



Conclusion

- **ILRS TRF Origin and Scale:**
  - **Some small Tz and Scale differences exist between AC's**
  - **More refinement is needed for the TRF origin and Scale maintenance ?**
  - **SLR and VLBI current networks/collocations are very poor: Scale Comparaison ?**
- **ILRS EOP:**
  - **Good estimates of X-pole, Y-pole and LOD**
  - **The rate estimates of X-pole & Y-pole degrade the overall results**
  - **Good agreement of X-pole, Y-pole and LOD with other techniques**