

Mechanical Measurement of Laser Pulse Duration

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Abstract:

An original interferometric device has been developed, which allows the measurement of the duration of laser pulses in the picosecond domain, with a precision of a few percent. This could be an affordable and convenient alternative to the streak-camera methods available today.

Review of actual methods for pulse duration measurements:

There are several methods available, but each has its advantages and inconveniences, and we will try to put them on balance.

1. **Streak Camera:** today, this is perhaps the first method which comes to mind, at least in the picosecond domain in which almost all current SLR stations work. It has a lot of qualities:
 - a) Its precision is quite good.
 - b) It gives not only the width of the pulse but also the details of the temporal profile, without any ambiguity.
 - c) It can be equipped with a Micro-Channel Plate (MCP) in order to reach extremely low light levels (photon counting mode).

Nevertheless, some drawbacks can attenuate our enthusiasm:

- a) The temporal resolution is limited by the velocity dispersion of the photoelectrons during the optoelectronic conversion at the photocathode.
 - b) This is a very delicate instrument, especially if it is equipped with a MCP stage.
 - c) The need for an optical delay line of several tens of meters with a rather good stability requires either a lot of space or a single-mode optical fiber.
 - d) This is a very expensive instrument.
2. **Autocorrelator:** this is the first choice for the sub-picosecond domain. Since most designs are based on Fourier analysis of the power spectrum, it is very difficult to have enough spectral resolution for use with picosecond or longer pulses. Beside being less expensive than a Streak Camera (even if it is not really cheap), it has almost only drawbacks:
 - a) It needs a high power density for operation, since it involves a multiplicative (i.e. non-linear) process.
 - b) Most of the designs, in particular for rather long pulses, are statistical and hence need a lot of pulses and a good stability of the studied source.
 - c) In some cases, the temporal profiles of the pulses can be retrieved, but since the autocorrelation is symmetric, there are ambiguities about the shape, especially if they are asymmetric.

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3. **Interferometry:** the coherence length of short laser pulses is shorter or equal to the distance corresponding to the pulse duration. An experiment related in a paper by Morley S. Lipsett and Dr. L. Mandel³ shows that in the case of ruby, for pulse duration of about $0.5\mu\text{s}$, the coherence length is effectively of that order: it is reasonable to assume that with much shorter pulses, that should be even more confirmed. Under such rather loose restrictions, the contrast of the fringes of an interferometer depends on the ratio between the pulse width and the path difference. The advantages of such a device are:
- a) It can easily operate at low light levels, or even in photon-counting mode with the use of an intensified CCD camera, since interferometry is an additive (i.e. linear) phenomena.
 - b) Standalone operation is possible, using the eye as detector, even if a more sophisticated configuration, including a CCD camera and a computer for image processing, should allow to achieve a better precision.
 - c) Single pulse operation is possible, at least in a "pass/fail" mode, i.e. to determine, when tuning a laser, if pulse lengthens or shortens.
 - d) A statistical mode, rather similar to the statistical mode of an autocorrelator, can give partial pulse shape information, with similar ambiguities problems since, like the autocorrelation, the interference contrast is a symmetric function of the path difference.
 - e) The precision expected, based on personal experience in the stellar interferometry field, should be of about ten percent in standalone mode after some training of the operator, and of few percent in computer assisted mode.
 - f) The design is very cheap, being basically an interferometer with a variable delay.

Principle of Operation:

Note: for all the following discussion, we will suppose that the temporal profiles of the laser pulses are gaussian. This is mostly for convenience, but similar results could be derived for other pulse shapes, with almost no changes except in the numerical values of parameters.

If we consider a gaussian pulse of standard deviation σ going through an interferometer with a path difference $x_0 = 2 \cdot \delta$, we obtain two output pulses separated by a delay $\tau = 2 \cdot \delta / c$. Since the contribution to interference fringes can only come from the intersecting part of the pulses, the light from the remaining parts contributes only to decrease the contrast by increasing the "background" level.

Since it is very difficult to estimate absolute value of contrast, in particular using the human eye, the design includes a small auxiliary interferometer, with a null path difference, but in which the contrast of the fringes is preset to a known value by insertion of density in one of its arms. This give a contrast reference which can be chosen equal to a convenient value, and so it is only necessary to detect the equality of the two contrasts, a task which is far easier.

The details of computation are given in Appendix.

³ LIPSETT, M. S. AND L. MANDEL: Coherence Time Measurements of Light from Ruby Optical Masers, *Nature* **198** (1963) 553-555

If we consider various implementations of interferometers, it can easily be seen that they are not equivalent.

The most important properties for this application are:

- a) A wide field, i.e. relative insensitivity to the alignment of the incident beam.
- b) A path difference easy to adjust over a wide span without needing realignment.
- c) The use of an optical delay line should be avoided since it could be a cause of instability.
- d) A near achromatic design: only necessary if operation with different laser types is desirable, without needing specific calibration.
- e) A good stability: it should not need realigning after transportation, etc.

Review of the various interferometers configurations:

- a) **Michelson Interferometer** (Figure 1, Figure 2): its advantages are its simplicity and an adjustable path difference without additional optics. However, it is delicate to align and it has a narrow field.
- b) **Sagnac Interferometer** (Figure 3, Figure 4): its elegant configuration and its extreme simplicity could speak in its favor but it is rather sensitive to misalignment and overall it has an always null path difference since all light paths are shared.
- c) **Mach-Zehnder Interferometer** (Figure 5, Figure 6, Figure 7, Figure 8): its wide field is very valuable, but it needs a variable delay line for adjusting the path difference and that can be a source of instability.
- d) **Modified Mach-Zehnder Interferometer (MMZI)** (Figure 9, Figure 10): if, into the previous design, the mirrors are replaced by corner cubes, a new configuration is obtained. Even if at a first glance it seems to have similarities with a Michelson, in reality the intrinsic properties of corner cubes confer to this design the characteristics of the Mach-Zehnder Interferometer, plus several interesting ones:
 - It is almost as simple as a Michelson.
 - It is auto-aligning.
 - It has a wide field.
 - It needs only one beam splitter instead of two in the Mach-Zehnder.
 - It does not need any additional optics to adjust its path difference.
 - Its stability is excellent.

Modified Mach-Zehnder Interferometer Prototype:

A prototype of the proposed device has been built. Several views (Figure 11, Figure 12, Figure 13, Figure 14) are shown. To calibrate the null path difference position, white light fringes (Figure 15) are used. Various examples of the fringes obtained with lasers are given:

- a) **Continuous Helium-Neon laser** (Figure 17) with their profiles (Figure 18).
- b) **Pulsed doubled Nd:YAG laser** at three different path differences, showing the decrease of the contrast as the path difference is increased:
 - Figure 19 and Figure 20 (profiles) for null path difference leading to ~100% contrast in the outer fringes, to be compared to the ~50% contrast reference at center.
 - Figure 21 and Figure 22 (profiles) for a path difference corresponding to near equal contrast in both reference and measure channels.
 - Figure 23 and Figure 24 (profiles) correspond to a larger path difference leading to lower contrast in the outer measure channel.

The public domain program *ImageJ* was used to produce the profile plots.

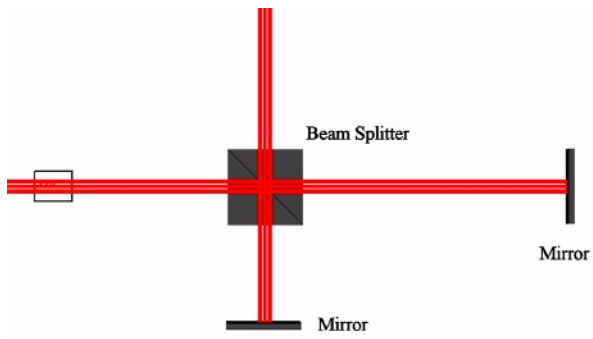


Figure 1: Michelson (unbalanced)

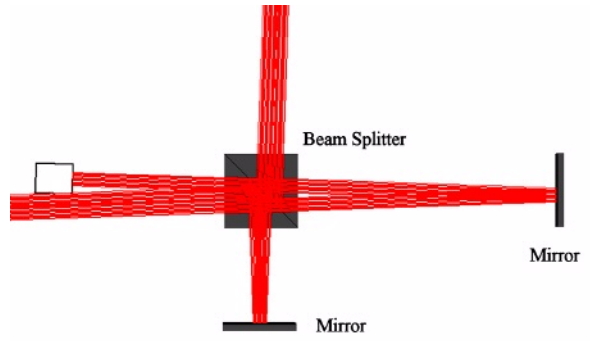


Figure 2: Michelson (unbalanced, off-axis)

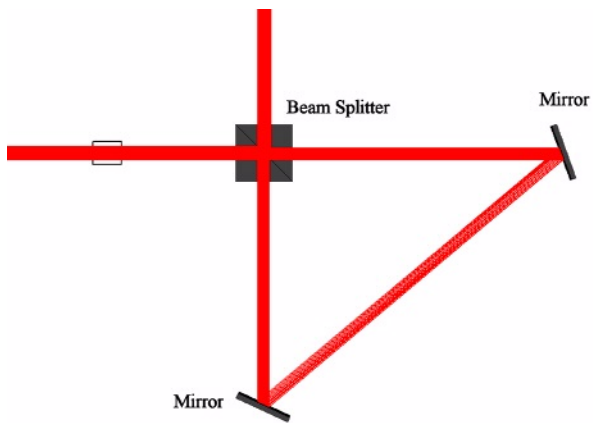


Figure 3: Sagnac

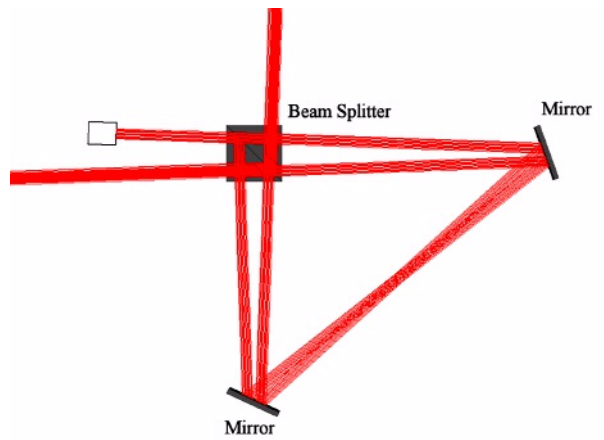


Figure 4: Sagnac (off-axis)

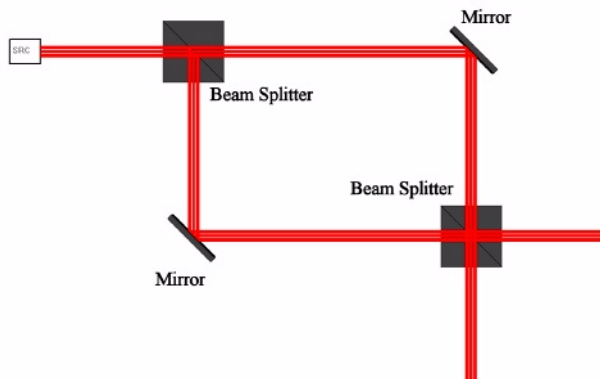


Figure 5: Mach-Zehnder (balanced)

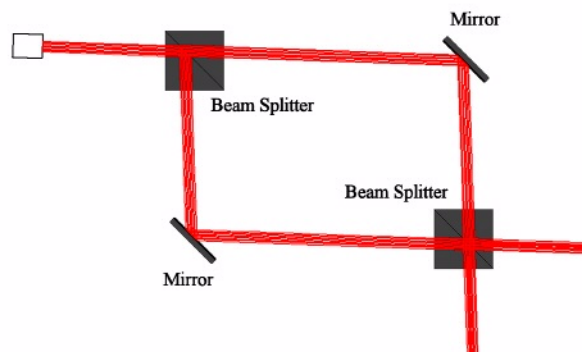


Figure 6: Mach-Zehnder (balanced, off-axis)

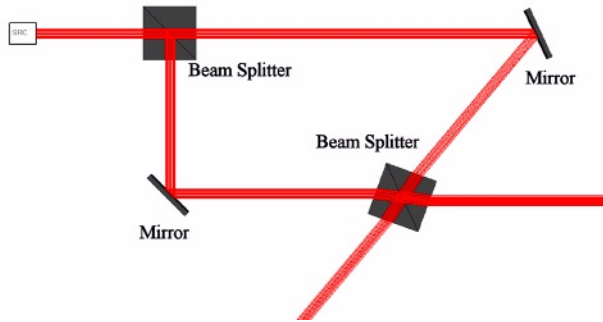


Figure 7: Mach-Zehnder (unbalanced, no delay line)

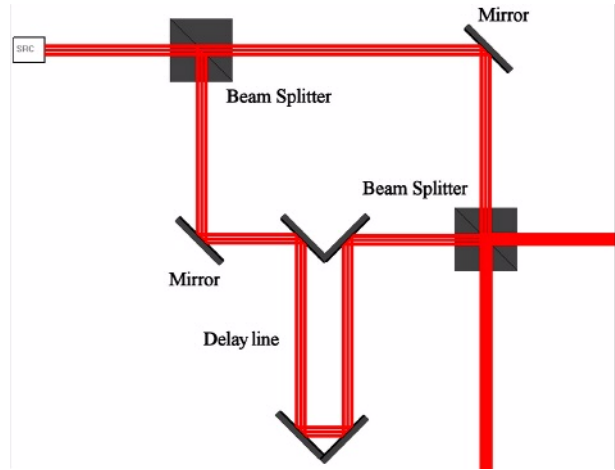


Figure 8: Mach-Zehnder (unbalanced, delay line)

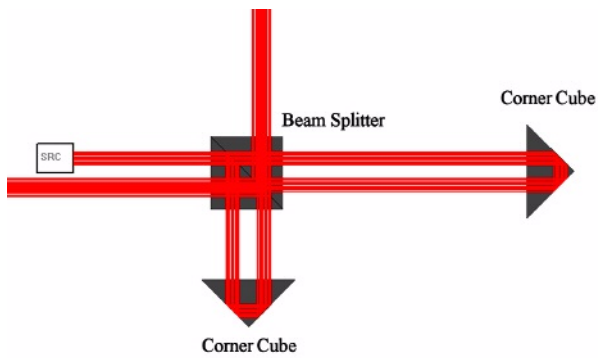


Figure 9: MMZI (unbalanced)

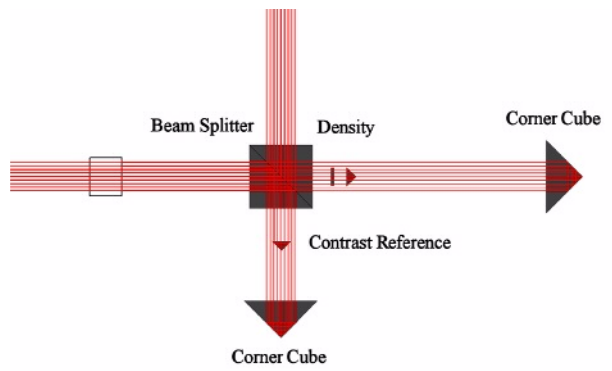


Figure 10: MMZI & contrast reference (unbalanced)

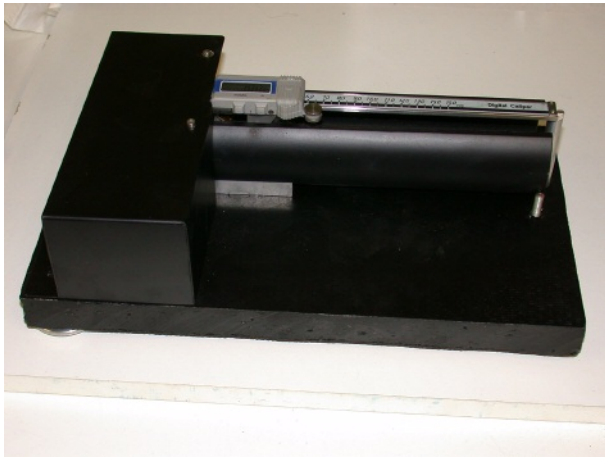


Figure 11: Prototype of the MMZI (1)



Figure 12: Prototype of the MMZI (2)

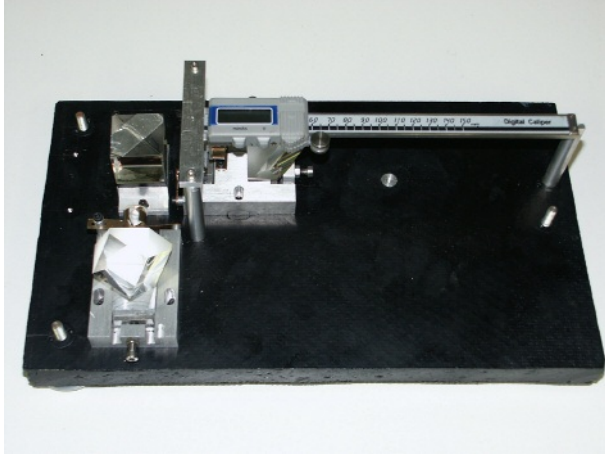


Figure 13: Prototype of the MMZI (3)

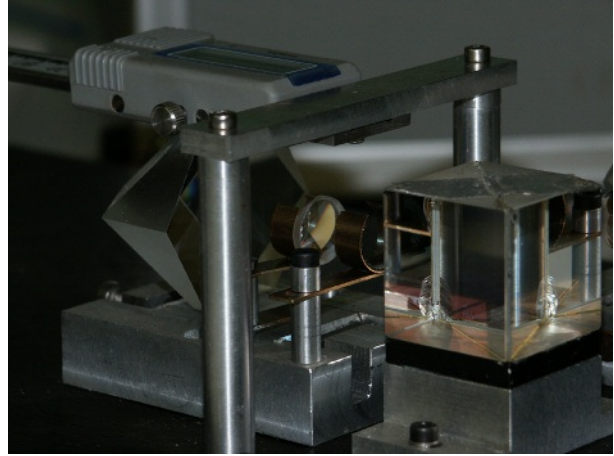


Figure 14: Detail of MMZI showing contrast reference with density

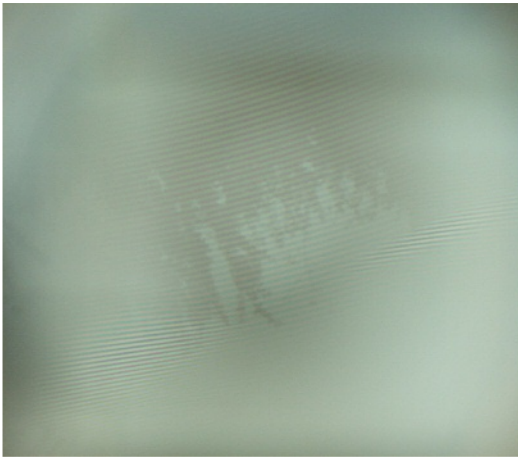


Figure 15: White light fringes: wider fringes at center are from reference

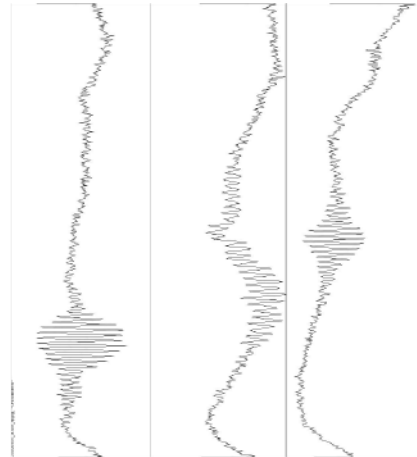


Figure 16: Profiles of white fringes. wider fringes at center are from reference

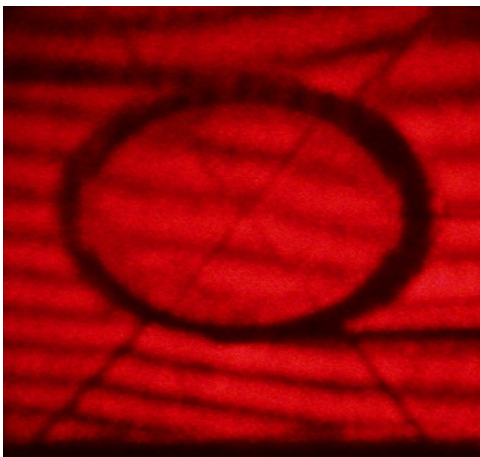


Figure 17: Continuous He-Ne laser fringes

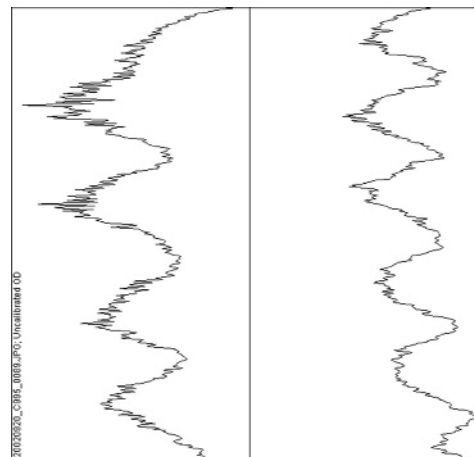


Figure 18: Profiles of Continuous He-Ne laser fringes

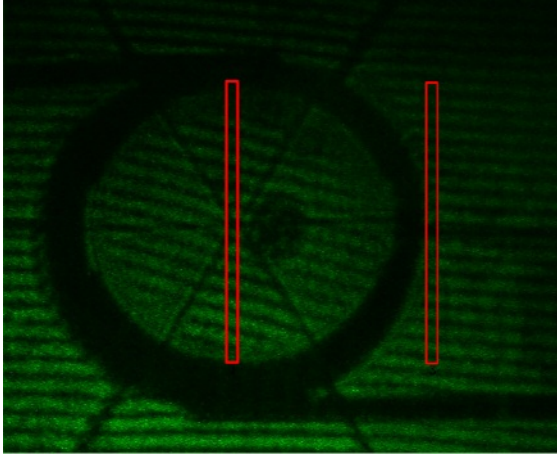


Figure 19: Doubled Nd:YAG laser fringes, null path difference

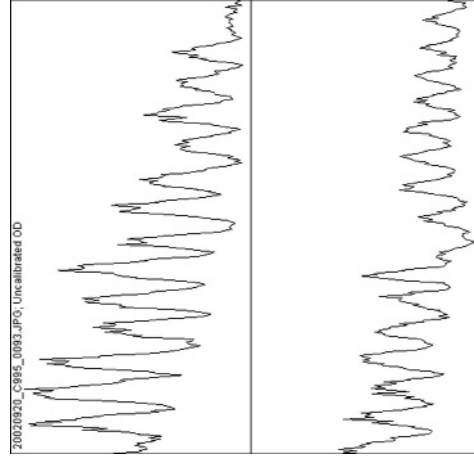


Figure 20: Profiles of doubled Nd:YAG laser fringes, null path difference

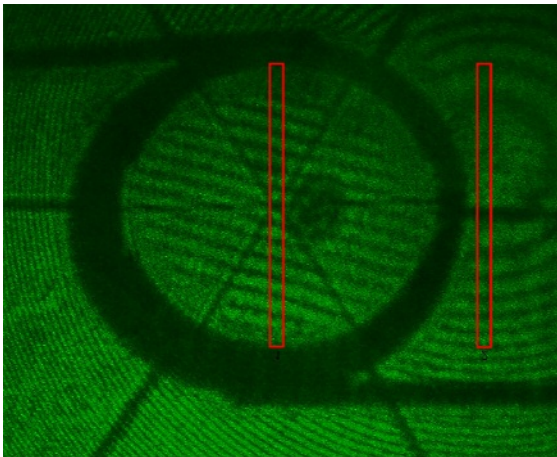


Figure 21: Doubled Nd:YAG: difference path set for equal 50% contrast

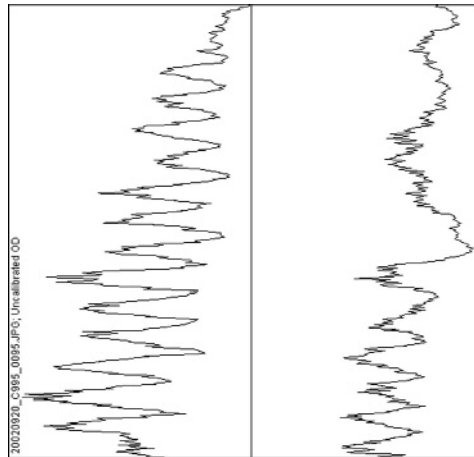


Figure 22: Profiles of doubled Nd:YAG (equal 50% contrast)

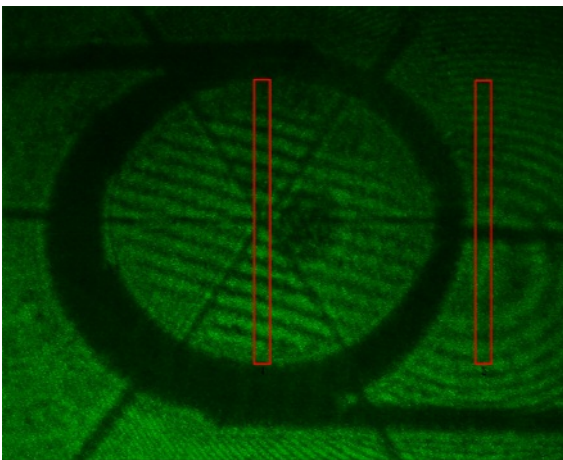


Figure 23: Doubled Nd:YAG; larger path difference, lower contrast

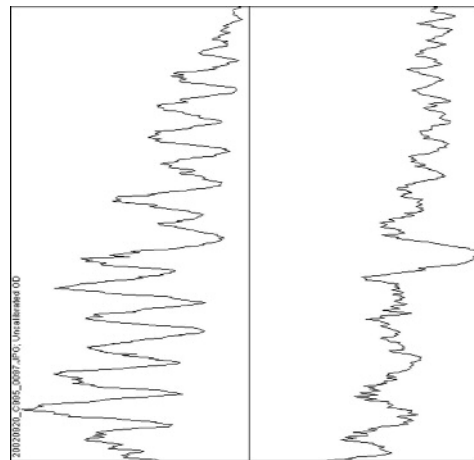


Figure 24: Profiles of doubled Nd:YAG; larger path difference, lower contrast

Appendix: Semi-Analytical fringe contrast derivation in a MMZI

Notes:

- The program MuPad Pro (version 2.0) was used for these symbolic calculations.
- In all this computation, we will assume that the time profiles of the laser pulses are gaussian; this is merely for convenience and a similar treatment could be applied to other pulse shapes as well.
- Since the gaussian curve is symmetric, only positive solutions need to be taken into account.

First, make a start clean:

- `reset();`

I - Fringe contrast of in an unbalanced interferometer for gaussian laser pulses:

Lets define a gaussian $g1$ with standard deviation σ , centered at origin, of unity amplitude:

- `assume(' σ', Type::Interval(0, infinity));`
- `g1 := exp(-x^2/(σ^2))`

$$e^{-\frac{x^2}{\sigma^2}}$$

Its Full Width at Height Middle (FWHM) is:

- `fwhm:= ' σ' -> simplify(2*solve((g1=1/2),x, PrincipalValue))[1];`
- `fwhm(' σ')`

$$2 \cdot \sigma \cdot \sqrt{\ln(2)}$$

or, in numerical form:

- `float(fwhm(' σ'))`

$$1.665109222 \cdot \sigma$$

Let now consider a delayed copy of the same gaussian as obtained at the output of an interferometer: since the light travels a round trip, a displacement δ of one corner cube leads to a path difference $x0$:

- `assume(' δ', Type::Positive);`
- `x0:=2*' δ'`

$$2 \cdot \delta$$

then the delayed gaussian $g2$ is defined by:

- `g2 := ' δ' -> exp(-(x-x0)^2/(σ^2));`
- `g2(' δ')`

$$e^{-\frac{(x-2\cdot\delta)^2}{\sigma^2}}$$

The contrast of the fringes obtained from the two gaussian g_1 et g_2 , is given by the ratio of their Intersection area over their Union area. We first need the abscissa of the Gaussians intersect:

- $g12eq := \text{solve}(g1 = g2(\delta), x, \text{PrincipalValue})[1]$

δ

the Intersection of the two gaussians is delimited by the curve:

- $\text{delete } \delta;$
- $\text{assume}(\delta, \text{Type}::\text{Interval}(0, \text{infinity})):$
- $g12i := \delta \rightarrow \text{piecewise}([x > g12eq, g1], [x \leq g12eq, g2(l)]):$
- $g12i(\delta);$

$$\begin{cases} \frac{x^2}{\sigma^2} & \text{if } \delta < x \\ e^{-\frac{(x-2\delta)^2}{\sigma^2}} & \text{if } x \leq \delta \end{cases} \quad [1]$$

while the Union of the two gaussians is delimited by the curve:

- $g12u := \delta \rightarrow \text{piecewise}([x > g12eq, g2(\delta)], [x \leq g12eq, g1]):$
- $g12u(\delta);$

$$\begin{cases} \frac{(x-2\delta)^2}{\sigma^2} & \text{if } \delta < x \\ \frac{x^2}{\sigma^2} & \text{if } x \leq \delta \end{cases} \quad [2]$$

Integration of [1] gives the area of the Intersection of the two gaussians:

- $g12i_a := \delta \rightarrow \text{factor}(\text{int}(g12i(\delta), x = -\text{infinity}.. \text{infinity})):$
- $g12i_a(\delta);$

$$-\sigma \cdot \sqrt{\pi} \cdot \left(\text{erf}\left(\frac{\delta}{\sigma}\right) - 1 \right)$$

while integration of [2] gives the area of their Union:

- $g12u_a := \delta \rightarrow \text{factor}(\text{int}(g12u(\delta), x = -\text{infinity}.. \text{infinity})):$
- $g12u_a(\delta);$

$$\sigma \cdot \sqrt{\pi} \cdot \left(\text{erf}\left(\frac{\delta}{\sigma}\right) + 1 \right)$$

Hence the fringe contrast is:

- $g12_c := \delta \rightarrow g12i_a(\delta) / g12u_a(\delta):$
- $g12_c(\delta);$

$$\frac{\text{erf}\left(\frac{\delta}{\sigma}\right) - 1}{\text{erf}\left(\frac{\delta}{\sigma}\right) + 1}$$

If the ratio δ/σ is noted by α , the contrast can be rewritten:

- `g12_ca := 'α' -> subsex(g12_c('δ'), 'δ'/σ', 'α');`
- `g12_ca('α')`

$$\frac{\operatorname{erf}(\alpha) - 1}{\operatorname{erf}(\alpha) + 1}$$

The slope (i.e. first derivative) of the fringe contrast is:

- `g12_ca_d1 := 'α' -> factor(diff(g12_ca('α'), 'α')):`
- `g12_ca_d1('α')`

$$\frac{4 \cdot e^{-\alpha^2}}{(\operatorname{erf}(\alpha) + 1)^2 \cdot \sqrt{\pi}}$$

The choice of the contrast reference must be done to maximize the variation of the contrast versus the path difference. As shown by the Graph 5, the contrast is a decreasing function of δ and its slope is maximum for a null path difference: that choice would correspond unfortunately to a zero sensitivity. On the other side, since the pulse width is proportional to the measured path difference, the accuracy will be better for large δ values, but then the contrast and also its slope tend to zero: this choice is also impracticable.

The contrast must also be kept as far as possible of extreme values which are difficult to discriminate either by the naked eye or by electronic means. This analysis seems in favor of a contrast value of 50%.

The contrast slope decreasing monotonically, we cannot use an extremum of the slope to induce a better value, but it is possible to build an function which can serve as a test: since the pulse width is proportional to α and the accuracy to the (absolute value of the) contrast slope, the product of this slope by the α ratio can be used:

- `opt_test := 'α' -> 2*g12_ca_d1('α')*α':`
- `opt_test('α')`

$$\frac{8 \cdot e^{-\alpha^2} \cdot \alpha}{(\operatorname{erf}(\alpha) + 1)^2 \cdot \sqrt{\pi}}$$

Let locate the extremum of this function by studying its derivative:

- `opt_test_d1 := 'α' -> factor(diff(opt_test('α'), 'α')):`
- `opt_test_d1('α')`

$$\frac{8 \cdot \left(-\pi - \pi \cdot \operatorname{erf}(\alpha) + 2 \cdot \pi \cdot \alpha^2 + 2 \cdot \pi \cdot \alpha^2 \cdot \operatorname{erf}(\alpha) + 4 \cdot \sqrt{\pi} \cdot \alpha \cdot e^{-\alpha^2} \right) \cdot e^{-\alpha^2}}{(\operatorname{erf}(\alpha) + 1)^3 \cdot \sqrt{\pi}^3}$$

The Graph 6 shows that this test function has a negative minimum for α close to 0.5; the exact value is given by the root of the derivative:

- `alpha_c_opt := numeric::realroot(opt_test_d1('α')=0, 'α'=0.01..0.99)`
0.4649571239

For this "optimal" contrast, the relation between δ and σ becomes:

- `delta_opt:=`σ` -> alpha_c_opt*`σ`:`
- `delta_opt(`σ`)`

`0.4649571239·σ`

which can be easily inverted:

- `sigma_opt := `δ` -> solve(delta_opt(`σ`)=`δ`,`σ`)[1]:`
- `sigma_opt(`δ`)`

`2.150735947·δ`

This delay corresponds to an "optimal contrast" of:

- `g12_c_opt:=float(subs(g12_ca(`α`,`α`=alpha_c_opt))`

`0.3430283747`

which is reasonably comfortable.

On the other hand, a contrast of 50% corresponds to α =alpha_c_50 such as:

- `alpha_c_50 := solve(g12_ca(`α`) = 0.5, `α`):`
- `alpha_c_50:=float(alpha_c_50)[1]`

`0.3045701942`

Then, for this 50% contrast, the relation between δ and σ becomes

- `delta_50:=`σ` -> alpha_c_50*`σ`:`
- `delta_50(`σ`)`

`0.3045701942·σ`

which can be easily inverted:

- `sigma_50:=`δ` -> solve(delta_50(`σ`)=`δ`,`σ`)[1]:`
- `sigma_50(`δ`)`

`3.283315371·δ`

Using the relation between σ and FWHM, the expression of δ can be explicitated for a 50% contrast:

- `fwhm_50:=`δ` -> float(subs(fwhm(`σ`,`σ`=sigma_50(`δ`)))):`
- `fwhm_50(`δ`)`

`5.467078704·δ`

while for an "optimal contrast", we get:

- `fwhm_opt:=`δ` -> float(subs(fwhm(`σ`,`σ`=sigma_opt(`δ`)))):`
- `fwhm_opt(`δ`)`

`3.58121026·δ`

In order to get pulse duration, we need to convert spatial length δ to time: taking in account the speed of light in the air:

- `delete `δ`,`c`,`n`:`
- `assume(`δ`,`Type::Interval(0,infinity)):`
- `assume(c,`Type::Interval(0,infinity)):`
- `assume(n,`Type::Positive`):`

The time delay τ (in ps) corresponding to a space delay δ (in mm) is given by:

- $\tau := \sigma \cdot n/c$:
- $\tau(\sigma)$

$$\frac{n \cdot \sigma}{c}$$

Using the definition of $\tau(\sigma)$, the FWHM (in mm) can be rewritten (in ps):

- $fw_{hm_ps1} := \sigma \rightarrow \text{subs}(fw_{hm}(\sigma), \sigma = \tau(\sigma))$:
- $fw_{hm_ps1}(\sigma)$

$$\frac{2 \cdot n \cdot \sigma \cdot \sqrt{\ln(2)}}{c}$$

and the definition of $\alpha = \delta/\sigma$ allows to write:

- $fw_{hm_ps} := \delta \rightarrow \text{subs}(fw_{hm_ps1}(\sigma), \sigma = \delta/\alpha)$:
- $fw_{hm_ps}(\delta)$

$$\frac{2 \cdot n \cdot \delta \cdot \sqrt{\ln(2)}}{c \cdot \alpha}$$

Using numerical values for the speed of light in vacuum (in **mm/ps**) and the refraction index of the air (at STP conditions):

- $c = 0.299792458$:
- $n = 1.000293$:

gives a numerical expression for the FWHM in ps:

- $\text{float}(fw_{hm_ps}(\delta))$
- $$\frac{5.555833894 \cdot \delta}{\alpha}$$

in the case of a 50% contrast, we have:

- $fw_{hm_ps_50} := \delta \rightarrow \text{float}(\text{subs}(fw_{hm_ps}(\delta), \alpha = \alpha_c_{50}))$:
- $fw_{hm_ps_50}(\delta)$

$$18.24155482 \cdot \delta$$

and in the case of the "optimal" contrast, we obtain:

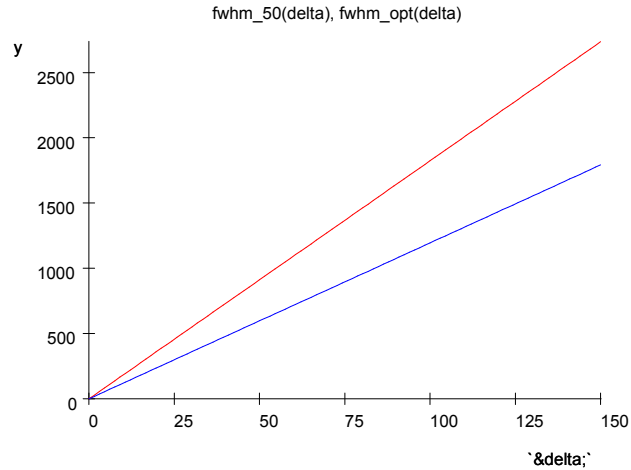
- $fw_{hm_ps_opt} := \delta \rightarrow \text{float}(\text{subs}(fw_{hm_ps}(\delta), \alpha = \alpha_c_{opt}))$:
- $fw_{hm_ps_opt}(\delta)$

$$11.94913167 \cdot \delta$$

The Graph 5 shows that values of α larger than 1.0 leads to very low contrast values, therefore the resolution of the instrument for short pulses will be limited to 1 or 2 ps.

The following graph shows the relation between δ and FWHM in both cases:

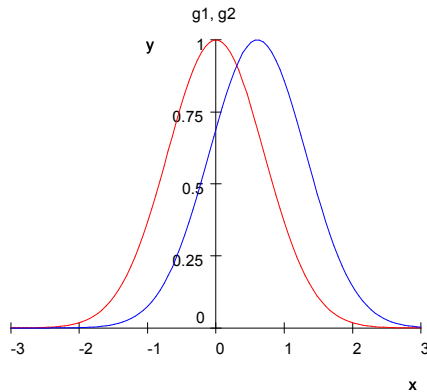
- $\text{Options} := \delta = 0..150, \text{Title} = "fw_{hm_50}(\delta), fw_{hm_opt}(\delta)":$
- $\text{plotfunc2d}(fw_{hm_ps_50}(\delta), fw_{hm_ps_opt}(\delta), \text{Options})$



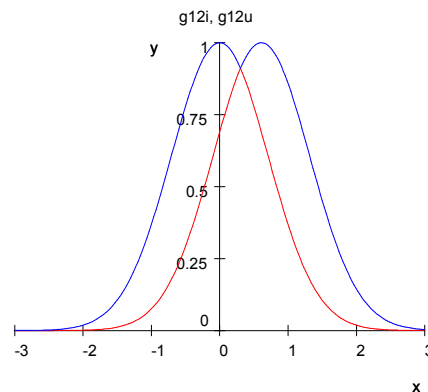
Graph 1: FWHM(δ) for 50% (red) & optimal (blue) contrasts

(for all the following graphs, a contrast of 50% and a $\sigma=1$ have been chosen)

- ``σ`=1.0:`
- `g2p:=subs(g2(`δ`,`δ`=delta_50(`σ`))):`
- `Options:=x=-3..3, Ticks=[Steps=1, Steps=0.25], Title="g1, g2":`
- `plotfunc2d(g1,g2p(x), Options)`
- `g12ip:=eval(subs(g12i(`δ`,`δ`=delta_50(`σ`))):`
- `g12up:=eval(subs(g12u(`δ`,`δ`=delta_50(`σ`))):`
- `Options:=x=-3..3, Ticks=[Steps=1, Steps=0.25], Title="g12i, g12u":`
- `plotfunc2d(g12ip,g12up, Options)`



Graph 2: Original (red) & delayed (blue) pulses.



Graph 3: Pulse Intersection (red) & Union (blue)

- `delete `δ`,`σ`:`
- `assume(`δ`,`Type::Interval(0,infinity)):`
- `assume(`σ`,`Type::Interval(0,infinity)):`
- `g12i_a_p:=float(g12i_a(`δ`)):`
- `g12u_a_p:=float(g12u_a(`δ`)):`

Area of the Intersection:

- $g12i_a_p$;

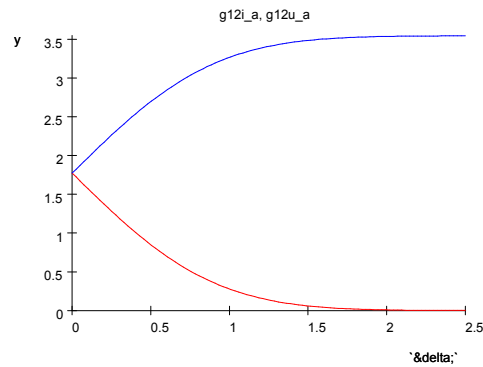
$$1.772453851 \cdot \sigma \cdot \operatorname{erfc}\left(\frac{\delta}{\sigma}\right)$$

Area of the Union:

- $g12u_a_p$;

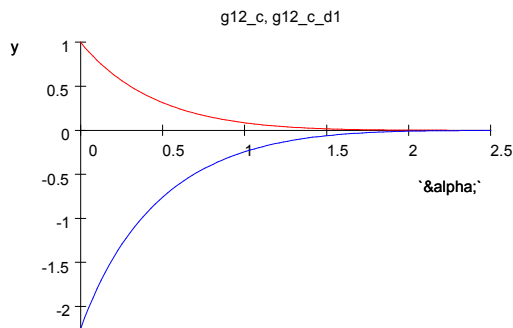
$$1.772453851 \cdot \sigma \cdot \left(-\operatorname{erfc}\left(\frac{\delta}{\sigma}\right) + 2.0\right)$$

- $\sigma := 1.0$;
- Options: $\delta = 0..2.5$, Ticks=[Steps=0.5, Steps=0.5], Title="g12i_a, g12u_a":
- $\text{plotfunc2d}(g12i_a_p, g12u_a_p, \text{Options})$

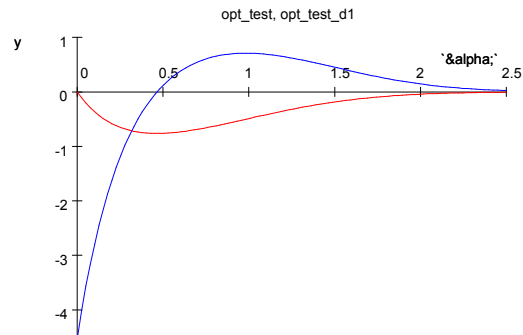


Graph 4: Intersection (red) & Union (blue) areas of the pulses.

- delete σ ;
- assume(σ , Type::Interval(0,infinity)):
- Options: $\alpha = 0..2.5$, Ticks=[6, 7], Title="g12_c, g12_c_d1":
- $\text{plotfunc2d}(g12_ca(\alpha), g12_ca_d1(\alpha), \text{Options})$
- Options: $\alpha = 0..2.5$, y=-4.5..1, Ticks=[6,6], Title="opt_test, opt_test_d1":
- $\text{plotfunc2d}(\text{opt_test}(\alpha), \text{opt_test_d1}(\alpha), \text{Options})$



Graph 5: Fringe contrast (red) & derivative (blue).



Graph 6: Test function (red) & derivative (blue).

II - Contrast of fringes in a balanced interferometer, with a density in one arm:

In order to make a contrast reference, a null path-difference interferometer with a density in one arm is used;

the transmission of a density d is given by:

- $t_d := d \rightarrow 10^{-d}$:
- $t_d(d)$

$$10^{-d}$$

since the light goes twice through the density, the resulting transmission is:

- $t_{d_2} := d \rightarrow \text{rewrite}(t_d(2*d), \text{exp})$:
- $t_{d_2}(d)$

$$10^{-2 \cdot d}$$

- *delete d*:
- *assume(d, Type::Positive)*:
- $g3 := d \rightarrow t_{d_2}(d) * \exp(-x^2 / \sigma^2)$:
- $g3(d)$

$$\frac{e^{-\frac{x^2}{\sigma^2}}}{10^{2 \cdot d}}$$

The contrast of the fringes obtained with $g1$ and $g3$ is:

- $g1_a := \text{int}(g1, x=-\text{infinity}..\text{infinity}, \text{Continuous})$:
- $g3_a := d \rightarrow \text{int}(g3(d), x=-\text{infinity}..\text{infinity}, \text{Continuous})$:
- $g13_c := d \rightarrow (g3_a(d) / g1_a)$:
- $g13_c(d)$

$$10^{-2 \cdot d}$$

In order to obtain a contrast of 50%, we need a density:

- $d_{50} := \text{solve}(g13_c(d) = 1/2, d)[1]$

$$\frac{\ln(2)}{2 \cdot \ln(10)}$$

- $\text{float}(d_{50})$:

$$0.1505149978$$

which is well closer to the standard density value of 0.15 than standard manufacturing tolerances. In fact, this density value leads to a contrast of:

- $g13_c(0.15)$

$$0.5011872336$$

In order to get the "optimal contrast", one needs a density:

- $d_{\text{opt}} := \text{solve}(g13_c(d) = g12_c_{\text{opt}}, d)[1]$

$$0.2323349772$$

We see that the contrast of 50% corresponds to a density very close to a somewhat standard

value (0.15), while the density required for the "optimal" contrast would need to be custom manufacturing.

- `delete `σ`, `δ`, d:`
- `assume(`σ`, Type::Interval(0, infinity)):`
- `assume(`δ`, Type::Positive):`
- `assume(d, Type::Positive):`
- `assume(c_d, Type::Interval(0, [1])):`

The density d_c corresponding to the contrast c_d is:

- `d_c := c_d -> -(1/2)*ln(c_d)/ln(10):`
- `d_c(c_d)`

$$-\frac{\ln(c_d)}{2 \cdot \ln(10)}$$

Contrast corresponding to a density of 0.2:

- `g13_d_02:=float(g13_c(0.2))`
0.3981071706

this corresponds to α :

- `alpha_d_02:=float(solve(g12_ca(`α`=g13_d_02, `α`))[1]`
0.4021996042

which gives:

- `fwhm_ps_02:=`δ` -> float(subs(fwhm_ps(`δ`), `α`=alpha_d_02)):`
- `fwhm_ps_02(`δ`)`
13.81362348· δ

Contrast corresponding to a density of 0.3:

- `g13_d_03:=float(g13_c(0.3))`
0.2511886432

this corresponds to α :

- `alpha_d_03:=float(solve(g12_ca(`α`=g13_d_03, `α`))[1]`
0.5931986952

and this gives:

- `fwhm_ps_03:=`δ` -> float(subs(fwhm_ps(`δ`), `α`=alpha_d_03)):`
- `fwhm_ps_03(`δ`)`
9.365890281· δ