The Center of Mass Corrections of Spherical Satellites

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Introduction

This paper is part of a contribution prepared for the ILRS signal processing working group. Its purpose is the compilation of a homogenous set of Center of Mass corrections (CoM) for all existing spherical SLR satellites. Whereas the errors of the model assumptions are not well known, it seemed to be useful to provide a set of CoM values derived by identical procedures.

The CoM values given here are applicable for single photoelectron detecting systems working at very low return rate. The model of continuously distributed cube corners [1] has been adopted as described in the next chapter. Main difference to earlier work [12] is a revision of the reflectivity function.

Description of the Method

The standard procedure of SLR data preprocessing is the formation of mean values of the residuals in some time window. Details are described in the normal point generation procedure adopted at the Herstmonceux workshop. To refer the normal point ranges to the center of mass of the satellite, we are needing a model of the expected distribution of range residuals. This can be obtained quite straightforward for the case of a single photoelectron system working at low return rate. The effect of nonzero return rate can be modelled separately.

Because the orientation of the passive spherical satellites are unknown, averages over all orientations are computed. This can be done by computing the incoherent superposition of the signals of individual cube corners for a large set of orientations of the satellite and taking then the numerical average [6],[10]. The justification of using incoherent superposition lies in the fact, that for a sufficiently large data set the effect of coherent interference is averaging out. We are simplifying the averaging process by regarding first an individual cube corner at a given angle of incidence and averaging its relative return signal over all azimuth angles (rotating the cube corner around its axis of symmetry). This way we obtain a 'reflectivity function $\eta(\phi)$ ' depending from the angle of incidence only.

In the next stage the cube corners are regarded as homogeneously distributed over the spherical surface. This leads to a simple analytical expression for the expected residual distribution in terms of the function $\eta(\phi)$. Note that this does not necessarily imply that the cube corners are really uniformly distributed over the sphere. Even a single reflector mounted on the surface would produce the same distribution as long as all orientations of the satellite have the same probability.

Before discussing how to get reasonable estimates of $\eta(\phi)$ for the different cube corner types, let us reproduce here the array transfer function:

$$F(x(\phi)) = \frac{1}{N} \cdot \frac{\eta(\phi)}{R - \frac{L \cdot \cos(\phi)}{\sqrt{n^2 - \sin^2(\phi)}}} \qquad \text{Eq.1a}$$

$$x(\phi) = R \cdot \cos(\phi) - L \cdot \sqrt{n^2 - \sin^2(\phi)} \qquad \text{Eq.1b}$$

Eq.1a and Eq.1b shall be considered together as a parameter representation where:

F(x): probability density for the range residual to be inside x, x+dx (for infinitesimal short laser pulses)
x : distance from the satellite's center
ϕ: angle of incidence
R : radius of the satellite (distance from the center to the cube corner's front face)
L : vertex length of the cube corners
N: normalising factor

To compute F(x) in dependence from x, the inversion of Eq.1b may be useful:

$$\phi(x) = \cos^{-1}\left[\frac{x \cdot R + L\sqrt{(n^2 - 1)(R^2 - L^2) + x^2}}{R^2 - L^2}\right] \qquad \text{Eq.1c}$$

The factor N has been introduced to ensure proper normalisation of F(x):

$$N = \int_0^{\phi_c} \sin(\phi) \cdot \eta(\phi) d\phi \qquad \int_{x1}^{x2} F(x)dx = \int_0^{\phi_c} F(x(\phi)) \cdot \frac{dx(\phi)}{d\phi} \cdot d\phi = 1 \qquad \text{Eq.2a,b}$$

The earliest reflection point x1 corresponds to $\phi=0$ and the most far reflection x2 corresponds to the cut off angle ϕ_c. At the earliest reflection point the value of F(x) is:

$$F(x1) = \frac{1}{N} \cdot \frac{\eta(0)}{R - \frac{L}{n}} \qquad \text{Eq.3}$$

F(x) is monotonously decreasing from this value with increasing distance as can be seen from a typical example in Fig.1.

If the SLR system response (obtained from the distribution of calibration residuals) is symmetrical, for instance nearly Gaussian, the center of mass correction is simply the first moment of F(x):

$$CoM = \int_{x2}^{x1} x \cdot F(x)dx \qquad \text{Eq.4}$$

In general we first have to convolute the array transfer function F(x) with the system response of the SLR system , apply iterative data editing, and use the resulting modified distribution to compute the first moment.

In this study we model the system response by a gaussian of different width and then apply iterative 2.5 data editing.

We have now to come back to the estimate of the reflectivity function $\eta(\phi)$. The total intensity reflected back from a cube corner is first proportional to the (orientation dependent) active area multiplied by the transmission factors of all optical surfaces involved. From this light a small fraction is received by the SLR telescope depending on the location of the station in the far field diffraction pattern on ground. The effect of diffraction we are taking into account in this study by a weighting factor proportional to the active area [1],[6],[10]. The result is the following reflectivity function:

$$\eta(\phi) = \frac{1}{2\pi A_0} \int_0^{2\pi} A^2(\phi,\alpha) \cdot \rho(\phi,\alpha) \cdot d\alpha \qquad \text{Eq.5}$$

$A(\phi,\alpha)$: active area of an individual reflector including the effect of masking and recession

$\rho(\phi,\alpha)$: total optical transmission coefficient

ϕ,α : angle of incidence and azimuth resp.

Equations to compute the functions $A(\phi,\alpha)$ for cube corners with triangular, hexagonal and circular front faces can be found in [2]. Useful relations to find the conditions of partial loss of total internal reflection, which are necessary to find the function $\rho(\phi,\alpha)$ are compiled in [2] as well.

For the biggest spherical satellite, AJISAI, the resulting transfer function is plotted in Fig.1. The asymmetric shape is very pronounced even after convolution with a gaussian of 11.5 mm rms width. The shape of the AJISAI cube corners is somewhat unusual (hexagons with unequal sides, [10]) and there are no simple equations for the computations of the active area available. Therefore we used a table of $A(\phi,\alpha)$ kindly submitted by Otsubo.

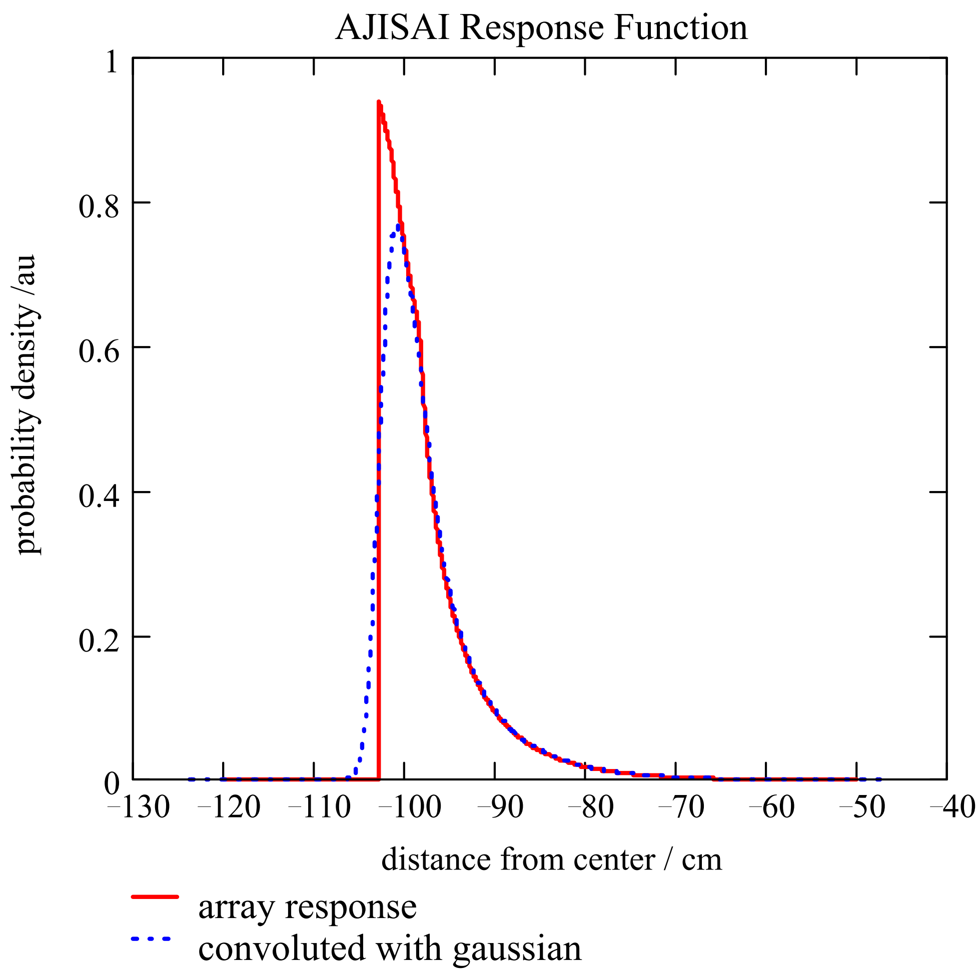

Fig.1: The Transfer Function of the AJISAI Satellite

Results

The described method has been used to tabulate the transfer functions of all spherical satellites with 0.25 mm spacing. This tables are then used for discrete convolution with gaussians of different width. In the last step iterative data editing has been applied using 3σ or 2.5σ as a criterion. In the following table the first column contains the CoM values without any editing (first moments of F(x)). In the following 3 columns the effect of 2.5σ– clipping is shown for gaussian systems with 5,10 and 15mm rms width respectively. In the last column some reference values are given , which seem to be generally adopted. Note that all data are for 532 nm wavelength.

Satellite	No clipping	2.5 σ – clipping, 10 Iterations			Standard [11]	Further Ref.
		5mm	10mm	15mm		
AJISAI	974.2	991.6	990.7	989.5	1010	971.1 [10]
ETALON	569.6	580.9	579.9	578.7	558	576 [6]
LAGEOS	245.8	248.6	247.6	247.0	251 [5]	243 [4],[12}
STARLETTE	77.7	78.6	78.1	77.9	75 [3]	
GFZ-1	60.1	60.1	60.1	60.1	58.5 [8],[9]	
WESTPAC	62.4	62.4	62.4	62.4	61.8 [7]	

As can be seen, our CoM values are not far from the standard values except for AJISAI and ETALON. But our results for AJISAI and ETALON are in good agreement with more recent

estimates [10],[6]. The small difference to Otsubo's value is due to the fact that he is using a slightly different weighting function ((active area)²cos(ϕ) instead of (active area)²cos²(ϕ), where the active area is measured on the prism front face).When using identical weighting functions we obtained perfect agreement with Otsubo's result.
The effect of data editing obviously is significant for LAGEOS and bigger satellites only. For the smaller satellites including STARLETTE/STELLA data editing has negligible effect on the mean because we assumed a gaussian system response. This might be different when using the realistic asymmetric system response which is typical for SPAD detectors.
The most critical point in this analysis is the choice of the weighting function $\eta(\phi)$ determining the shape of the distribution F(x). Comparison with the residual distributions of Herstmonceux data from LAGEOS and AJISAI seem to indicate that this model produces slightly too narrow distributions [13]. That means the CoM values given here may be slightly too high. In the case of LAGEOS earlier work estimating $\eta(\phi)$ from data tabulated in Ref.[4] resulted in a 3mm lower CoM value. Further comparisons with measurements using data from different stations are indicated.

References:

[1] J.J.Degnan: Millimeter Accuracy Satellites for Two Color Ranging, Proc. 8th Workshop on Laser Ranging Instr., Annapolis (1992), pp.7-36....7-51

[2] D.A.Arnold: Method of Calculating Retroreflector_Array Transfer Functions, SAO Special Report 382, (1979)

[3] D.A.Arnold: Optical Transfer Function of the STARLETTE Retroreflector Array, Grant NRG 09-015-002, Suppl. No.57, TechRep.RTOP 161-05-02, (1975)

[4] D.A.Arnold: Optical and Infrared Transfer Function Transfer Function of the LAGEOS Retroreflector Array, Final Report, Grant NRG 09-015-002, Suppl. No.57, (1987)

[5] P.O.Minott, T.W.Zagwodski, T.Vargehese, M.Seldon: Prelaunch Optical Characterization of the Laser Geodynamic Satellite (LAGEOS 2), NASA Tech.Paper TP-3400 (1993)

[6] N.T.Mironov, A.I.Emetz, A.N.Zaharov,V.E.Tschebotarev: Proc. 8th Workshop on Laser Ranging Instr., Annapolis (1992)

[7] V.B. Burmistov, V.P. Vasiliev, N.N. Parkhomenko, V.D.Shargorodsky, N.M.Soyuzova: The WESTPAC Satellite: Design Features and First Results of Return Signal Analysis, Proc. 11th Workshop on Laser Ranging Deggendorf , p. 297, (1998)

[8] R.König, A.Bode, Z. Chen, L.Grunwaldt, R.Neubert, Ch.Reigber, P.Schwintzer: The GFZ-1 Mission: Design, Operation and First Results, GFZ Scient.Tech.Reports STR96/09, (1996)

[9] V.D.Shargorodsky: The Device GFZ-1, Technical Description ORA-94-60 (1995)

[10] T.Otsubo, J.Amagai, H.Kunimori: Dependence of AJISAI's Center-of-Mass Correction on Laser Ranging System, Proc. 10th Workshop on Laser Ranging Instr., Shanghai, 1996

[11] D.E.Smith, R.Kolenkiewicz, P.J.Dunn, M.H.Torrence: Earth Scale Below a Part per Billion from Satellite Laser Ranging, Proc. IUGG Meeting Birmingham (1999)

[12] R.Neubert: Satellite Signature Model: Application to LAGEOS and TOPEX, Proc. Annular EUROLAS Meeting Munich, (1995)

[13] T.Otsubo: private communication