

*Optimum Planning of LAGEOS Laser Ranging for Geodynamic
Programs.*

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For the determination of Earth Orientation Parameters with the use of satellite laser ranging a great amount of measurements should be processed and analysed, but in certain cases the additional data becomes useless due to significant effects of even weak correlation between measurements which are not shown when a moderate set of data is used. In this connection a problem of optimum distribution of the moments of ranging along the given time interval may be solved and only these measurements should be included in the analysis. A statement of a problem is particularly expedient because of a large number of satellites, which have to be observed from the global SLR network. A priority of the ranging may be chosen in accordance with the optimum plan of tracking for the given task. An optimum plan should be also taken into account when the temporary locations of the mobile SLR stations are considered.

A method of an optimum planning of laser ranging measurements from the global network, based on the "theory of planning of experiments" (Bachghian B. et al. 1980), is proposed. Two algorithms have been tested using LAGEOS laser ranging data from 21 sites. The problem was solved assuming that a correlation between measurements is not significant and values of the corrections to the predicted parameters do not exceed a few percents of the parameter, that is proved by the present accuracy of the IERS data.

For the every orbital arc at the moment of ranging an observation equation may be linearised in the form of

$$\left| \rho_o(t) - \rho_c(t, \Theta^o) = \nabla(\rho_c(t, \Theta - \Theta^o)), \right| \quad (1)$$

where $\rho_o(t)$ and $\rho_c(t, \Theta^o)$ are observed and calculated values of the satellite range, Θ - is the vector of estimated parameters with a dimension - m (in matrix operations all vectors are columns), Θ^o - a nominal (or predicted) value of this vector, $\nabla(\rho(t, \Theta))$ - a gradient of the range by Θ vector. We consider the case when Θ consists of the coordinates and velocities of a satellite at the initial time moment and of pole coordinates x_p and y_p at the given

epoch, i.e we assume $m = 8$, and values of other parameters (such as gravity constant, terms of gravity field, etc) we consider accurate enough not to be included in the linearised model (1).

To find the criteria of optimization an approach based on the variation representation of dispersions has been used. It makes possible to solve this problem with the use of linear programming. For calculations of elements of the matrix (1) the software ASTRA (Bayuk O., Tatevian S. 1995) are used. As a result of LAGEOS data (seven 5-days arcs) analyses it was found that the optimum solution can be obtained at the 1 day time interval and that modifying the number of control parameters s it is possible to improve their estimation. Different combinations of control parameters have been investigated: x_p or y_p ($s = 1$); x_p and y_p ($s = 2$); 3 components of the satellite velocity ($s = 3$); Pole coordinates and velocity components ($s = 5$); coordinates and velocity components of a satellite ($s = 6$); all enumerated parameters ($s = 8$).

Table 1. A distribution of the SLR data analysed for the optimum planning.

| <i>Stations</i> | <i>NP</i> | σ |
|-------------------|-----------|----------|
| 7840 HERSTMONCEUX | 5398 | 2.2 |
| 7843 CANBERRA | 2435 | 2.2 |
| 7210 MAUI | 3558 | 0.8 |
| 7838 SIMOSATO | 501 | 6.7 |
| 7105 WASHINGTON | 2925 | 1.0 |
| 7835 GRASSE | 1160 | 4.3 |
| 7939 MATERA | 783 | 13.8 |
| 7810 ZIMMERWALD | 2467 | 6.8 |
| 7811 BOROWIEC | 826 | 5.6 |
| 7836 POTSDAM | 1146 | 2.4 |
| 8834 WETTZELL | 4884 | 2.2 |
| 1884 RIGA | 1204 | 5.4 |
| 7837 SHANGHAI | 281 | 6.1 |
| 7831 HELWAN | 290 | 2.9 |
| 7236 WUHAN | 59 | 5.5 |
| 7237 CHANGCHUN | 547 | 5.7 |

The results obtained for one of the orbital arc (8.02.1994- 15.03.1994) will be discussed below. In Table 1 a distribution of the observations and SLR stations are shown. The values of the estimated mean-square errors for different sets of control parameters are presented in Table.1.

Table 2. Estimated dispersions errors for different sets of control parameters.
 (σ - is the diagonal element of the matrix of normal equations)

| S | x | y | z | V_x | V_x | V_x | x_p | y_p |
|-----|------|------|------|-------|-------|-------|-------|-------|
| 1 | 22.0 | 28.9 | 24.9 | 7.0 | 27.0 | 19.5 | 0.4 | 7.0 |
| 1 | 17.7 | 13.5 | 13.6 | 13.1 | 14.7 | 16.1 | 12.1 | 0.6 |
| 2 | 1.4 | 1.2 | 1.2 | 0.8 | 1.3 | 1.2 | 0.5 | 0.7 |
| 3 | 1.9 | 0.5 | 0.7 | 0.5 | 0.5 | 0.5 | 1.1 | 1.8 |
| 5 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 |
| 6 | 0.6 | 0.5 | 0.6 | 0.6 | 0.6 | 0.6 | 1.1 | 0.8 |

As it is seen from Table.2 the least values of dispersions for the Pole coordinates were obtained for the variants 2 and 5, while the satellite positions and velocities are estimated with the least dispersions for the case 5 and 6. The set $s=8$ requires a lot of time for computation and we consider this variant as unacceptable.

Table 3. Optimum basis for the period 49405.5 - 49406.5 MJD for the variant $s = 5$.

| n | t | dt | Stations | p |
|-----|----------------|------------|----------|---------|
| 1 | 49405.51674311 | 0.00000000 | 7835 | 0.16526 |
| 11 | 49405.52986577 | 0.01312266 | 7835 | 0.07056 |
| 12 | 49405.53382421 | 0.01708110 | 7105 | 0.06875 |
| 31 | 49405.61756961 | 0.10082650 | 7090 | 0.14255 |
| 33 | 49405.66257872 | 0.14583561 | 7835 | 0.17556 |
| 43 | 49405.68807653 | 0.17133342 | 7110 | 0.00263 |
| 47 | 49405.69302788 | 0.17628477 | 7105 | 0.05739 |
| 64 | 49405.71788675 | 0.20114364 | 7110 | 0.00735 |
| 110 | 49405.82591678 | 0.30917366 | 1884 | 0.04678 |
| 113 | 49405.82727205 | 0.31052894 | 7545 | 0.09658 |
| 114 | 49406.09216235 | 0.57541924 | 7403 | 0.08866 |
| 120 | 49406.10088920 | 0.58414609 | 7403 | 0.07794 |

Thus using the variant with 5 control parameters (pole position and satellite velocity components) it is possible to obtain the optimum plan of tracking (table 3) and to save a computation time.

Thus using the variant $s = 5$ (pole position and satellite velocity components) it is possible to obtain an optimum plan of tracking and to save a computation time. The optimum basis for this variant is shown in table 3, where n - numbers of basic points where the satellite should be observed from the every station located along this optimum basis. It is

evident that not all of the tracking stations, which were analysed (table.1), are included in the optimum plan.

To summarize the results of analyses we may conclude that proposed methodic of optimization allows to decrease dispersion of the estimated Pole coordinates by 30-40 percents and gives an effective opportunity to coordinate tracking plan. This methodic may be applied for different satellites but it is most effective for high orbital satellites with a slow longitude movement of the orbital Node.

The same methodic has been applied for optimization of the tracking program in the problem of station positioning with the use of "short arc" method. In this case an optimum plan contains 3 basic points, corresponding to the bounds of the tracking interval and to the culmination.

Bibliography

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