

Ideas on feedback in satellite ranging systems

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1.0 Introduction

Because laser ranging systems differ so significantly in funding and objectives, it is probably most useful here to discuss some general guidelines for specifying the operator feedback in such systems. To make the presentation more concrete, some examples from the system in Wetzell, Bundesrepublik Deutschland, are presented. They demonstrate one approach that has been reasonably effective in solving the problems of feedback.

2.0 Include the feedback in the initial design

The desired feedback should be at least as carefully thought out in the early design phases as is the hardware. Feedback needs and content should be determined with enough detail that (1) the proper computer and peripherals can be considered, (2) the hardware designers can plan to provide the feedback data in convenient form, (3) a reasonably accurate cost estimate can be made, and (4) the ultimate

success of the feedback can be assured.

3.0 Include a software designer

Nearly, if not as, important as adequate early feedback design is the need to involve a software professional in the project design team at the earliest possible date. The inclusion of an experienced programmer will work toward assuring (1) reasonable hardware-software tradeoffs, (2) balanced, achievable requirements for both the feedback and the system as a whole, and (3) a flexible software base for future expansion.

The Wettzell system is a case in point. The original request for proposal specified support for future expansion but made no stipulations about the language or operating system which would form the base of the tracking software itself. Because of existing in-house experience the first inclination of the system builder was to write the tracking software in PDP-11 assembly language and use a small, home-grown, real-time executive as the controlling software entity. The "future expansion" would be covered by providing as separate items a standard DEC operating system and a Fortran compiler.

The software engineer, however, argued for a more integrated system-wide approach using a DEC real-time operating system and the newly-available Fortran-IV-Plus compiler that generated in-line, fast machine code. That approach was viewed with skepticism by some of the project planners but was ultimately accepted. In the end, however, it was generally agreed that the fully integrated approach had been correct because the Wettzell system

proved quite adaptable to even unforeseen system requirements. At the same time a second, somewhat similar system being developed in parallel under the "in-house" approach was relatively rigid and unreceptive to change.

The important point of this history is the impact which the early use of the programmer's normal professional experience had on the ultimate success of the system. Similar rewards are to be expected on any ranging system that uses a significant amount of software.

A short, final note on picking a programmer:

- he must be a listener;
- he must be flexible enough to consider various software solutions for the very real hardware problems involved in a laser system;
- he must know that software is only one of several important means in such an undertaking;
- he must have the trust of the hardware people;
- he must be consulted regularly and often;
- he needs up-to-date knowledge so that his responses can be timely and effective.

Without this close cooperation, a haphazard software system will probably result, a system that satisfies no-one, which cannot attract and hold good programmers, and which never ceases to cost the project money.

4.0 Other factors

Beyond the overriding importance of including the software designer in the earliest design phases, several other factors will mold the feedback and its effectiveness in a ranging system.

4.1 What are the main system objectives?

Is the tracking data the only objective? Will someone want to detect dynamic events at the satellite (e.g., Wobble)? Will monitoring of laser performance be part of the computer's responsibility? Will the system be used to train inexperienced operators or students? Will the computer system perform complicated data analysis? Will graphs, maps, or charts be created? Is this a system which anticipates another system (that is, a feasibility study)?

In other words, potential aims as well as the possibly more restricted current ones should be considered. In feedback terms this may mean, for instance, designing a flexible system for handling CRT displays, plots or graphs. Such generalized packages need not be elaborate. They can be either purchased from vendors (though speed is often a problem with vendor software) or created by project personnel. In any case the rewards are fast in materializing as the ideas for feedback often change early in the project development.

As an example, in the Wettzell system standardized display and command entry formats were adopted from the earliest stages of the project. A software package to create and use these forms was written as the first major programming effort. The entire package was no more than fifty small subroutines. Yet use of that standard interface provided a valuable overall coherence and control to the entire system.

One example: when it was decided to add a standard control character on a system-wide basis, only one subroutine was changed.

4.2 Identify the users

Will experienced astronomers operate the system? If so, succinct phrases, jargon, or symbols might suffice on CRT or listing formats as opposed to more word-oriented approaches.

Technicians? An emphasis on numbers with accompanying graphs where appropriate might be more effective.

Undergraduates? Textual headers, graphs, generally more informative output is probably required.

The more technical, jargon-rich approaches limit system usage to more highly trained operators. On the other hand the wordy, self-explanatory solutions often become burdensome to anyone who gains familiarity with the system. These restrictions can become obviously confining only after the system is in operation.

The method which has proven effective in the Wettzell system for reducing "chatter" yet maintaining an atmosphere of operator assistance is "coded menus". The operator is shown on the CRT a list of the valid choices for the particular portion of the program in execution. Each choice is presented as a short descriptive title (about 35 characters) and a code (2 characters). A menu item is selected by typing the code. This approach provides enough prompting to allow the manual to remain on the shelf yet imposes a minimum amount of typing for those already knowledgeable in the system's operation.

4.3 What kind of information is familiar to the users?

Do they think easily in symbols? Graphs? Numbers? Charts? Sentences? Abbreviations? Will the long range

users of the system think this way? Frequent users can usually remember the form and meaning of many abbreviated entries and readouts. Infrequent users will be regularly driven to the manual (and the wall) by excessive shortenings.

4.4 Identify the computer resources

Does the computer have great speed? Large memory? Fast mathematical calculations? An operating system? Disk storage? Does the operating system support parallel tasking? Overlays? Inter-task communication? Powerful languages? Do the languages generate in-line code? Are multiple computers available? Are pre-coded packages for plotting and CRT display creation available?

All these factors influence speed, flexibility, and availability of programmer time to create various forms of feedback. For example, a system which has a relatively primitive operating system may require much programmer time (i.e., budget) to satisfy basic tasking and control needs leaving little opportunity for anything beyond bare-bones feedback features. Indeed, it is generally the case that expenditure of funds on flexible, expandable, vendor-supported computers, peripherals, and programming tools more than repays itself in improved productivity of the programmers.

Consider this example from Wettzell: the initial system was developed on a machine with only one small disk, a magnetic tape unit, and a primitive utility program for copying between tape and disk. This forced the software to reside on tape since only small subsets of the entire system could be stored on disk at one time. This, in turn, forced

programmers to spend a significant part of their expensive time (often as much as one hour per day) waiting for tape-disk transfers to occur. The "economy" of only one disk drive was in fact a major project expense.

4.5 Identify the operating environment

Is the facility dedicated to laser ranging? If so, extra computer time can be devoted to maintaining informative, dynamic displays or graphs. If not, the limited computer availability dictates stripped down, data-rich displays which give maximum information in the shortest possible computer execution time. Also in this case, on-line operator aids (e.g., a "help" command) might be desirable to allow the observer to make maximum, efficient use of the time available to him.

Is the facility noisy? Quiet? Bright? Dark? These factors influence the visual or auditory impact that the feedback must have to gain the operator's attention.

Are other computers available in a network? If so, non-time-critical processing can be assigned to them to provide extra feedback beyond the capability of the tracking computer.

4.6 Determine the required data

What must the operator see before him to detect success? At least as important, what does he require to be able to correct failure? Essential information certainly includes mount position, expected position, elapsed time of the mission, operator-entered system perturbations (such as AZ/EL offsets, a time delta), and an indicator of satellite

acquisition (perhaps a display showing the range delta).

4.7 Determine the presentation format

The way the data is presented to the operator usually represents a difficult tradeoff between (1) desirable or elegant presentations with visual impact, (2) the computer time and peripheral equipment requirements to provide such feedback, (3) cost - both equipment and programmer, and (4) the operator's ability to understand and retain the data presented to him.

For example, CRT displays of dynamic graphs showing azimuth and elevation positioning relative to predicted values are impressive visually and provide rapid operator understanding. However, they are also very costly in CPU utilization and programming time. Plots on paper provide excellent hardcopy records of the mission. But they are slow relative to CRT displays and, therefore, may not be adequate for providing real-time tracking feedback. A display of number registers allows a good deal of information to be shown on a single, relatively inexpensive, alphanumeric CRT screen. Such displays generally lack visual impact (a major aid to understanding and recognition) and require a great deal more operator concentration to detect trends. (For example, a gauge or dynamic graph will quickly show the azimuth reading and the direction of movement; a number register only shows the former. The operator must mentally integrate several successive register values to detect direction).

4.8 Make the computer do the work

Computer projects of all kinds are usually under budge-

tary pressure. Shortcutting the operator feedback system often appears to be a good way to save money. The programmers can work on the "important parts of the system".

What budgets really are, however, are disguised opportunities. They offer the chance to design and build the best feedback system in the most compact form at the best price without all the extras that no-one really needs anyway?

Shortcuts, on the other hand, are rigid. Unyielding beasts that lunge out at any programmer who dares enter their domain. They are entrenched shortsightedness. They deny the inevitable need for adaptability in something as complex, as evolving, as a laser ranging system.

But most importantly, shortcuts show a lack of concern for the people who must use the system. The computer is a tireless doer of drudgery. The human is not. The best systems incorporate these truisms. The results are pleasant to use, allow operators to solve their problems rather than the computer's, and open the system to a wide audience of users and onlookers (it helps to have a system that management can visually understand!).

In short, spend time for the humans. You have nothing to lose but complaints.

LAGEOS EPHEMERIS PREDICTIONS

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Abstract

The operational procedures used at the University of Texas to generate Lageos ephemeris predictions for the TLRS and MLRS are described. The procedures which have been adopted for Lageos use two months of quick-look range data to estimate the satellite orbit elements and a "drag" parameter at a specified epoch. These parameters are used to predict the ephemeris for four to six months beyond the two months used in the estimation process. The accuracy of this predicted ephemeris has been evaluated with specific cases. These cases show the ephemeris error to be less than 100 m after four months past the estimation interval. The operation and software design considerations used in the development of the TLRS and MLRS software for reconstructing the predicted ephemeris are described also. The ephemeris reconstruction software has been designed to use a simplified model of the satellite dynamics to enable its operation in a mini- or micro-computer environment, as well as to reconstruct the predicted ephemeris to meter-level accuracy. By adjusting the initial conditions, model mismatches between the prediction software and the reconstruction software can be accommodated. The performance and characteristics of this software is described.

Introduction

The successful operation of satellite laser ranging (SLR) systems is dependent, in part, on the ability to point the laser/telescope in the approximate direction of the satellite. The allowable pointing error is essentially determined by the laser beam divergence and the signal-to-noise ratio. This error stems from the following sources:

- errors in the predicted satellite ephemeris,
- errors in the predicted earth orientation,
- errors in the a priori knowledge of the SLR location,
- errors in pointing the instrument to a commanded position, and
- errors in the SLR clock.

If the site is permanent or has been previously occupied by an SLR system, the third source should contribute only a small error. However, sites which have not been occupied previously may require the approximate specification of coordinates from a topographic map, thus introducing errors of tens or hundreds of meters. Range acquisition can be seriously degraded if any of these error sources, singly or in combination, exceed the beam width at the satellite's altitude. Although all sources are important, the latter two are hardware-dependent and will not be treated in this paper.

The error in the predicted satellite ephemeris arises from two distinct sources. First, the ephemeris is obtained by estimating the satellite state from observations taken over some interval of

time. This estimation procedure is performed using an adopted model for the forces and kinematics used to describe the satellite motion. With the estimated state and the adopted model, the satellite equations of motion are solved and the predicted satellite states beyond the estimation interval are obtained. This predicted ephemeris will inevitably be in error because of 1) inaccurate state estimates, 2) inaccurate or incomplete models for the satellite dynamics/kinematics and 3) the approximation technique used to solve the satellite equations of motion. In general, however, the dominant characteristic of the ephemeris prediction error is in the satellite's "along-track direction," i.e., direction of motion. In the subsequent discussion, the predicted ephemeris will refer to the ephemeris generated in the manner described above, usually at a site with access to data from many sites and to significant computer facilities which may be required to analyze the available data.

The second error may be introduced at a laser site which uses the predicted ephemeris to generate on-site pointing predictions. This error is distinctly different from the ephemeris prediction error since it will result from the use of an approximation technique which is compatible with the limited computer resources available within the SLR system. Consequently, the SLR may not reproduce the predicted ephemeris exactly due to the inherent differences in the models or approximation methods used at the central site compared to those used in the SLR station. The ephemeris generated on-site within the SLR will be referred to as the reconstructed ephemeris. The selection of

a technique to generate the reconstructed ephemeris within the SLR will be based on the following factors:

- the amount of data which must be transmitted to the SLR,
- the frequency with which the data must be transmitted,
- the computer resources required by the SLR to provide a usable ephemeris from the transmitted data.

For example, a one-day ephemeris provided to the SLR at one-minute intervals would require 1440 time points with three components of satellite position at each time. To reduce the amount of data, only the data points from the predicted ephemeris which pertain to the satellite passes for a particular SLR should be provided; however, this procedure places severe sort/merge problems on the central site due to different observing schedules at each station. Furthermore, in this example, the SLR computer resources must still be used to interpolate between the available points for the instrument pointing. The disadvantage of providing a large number of points to the SLR becomes even greater when considering highly mobile laser ranging systems.

Highly mobile laser ranging systems which operate in remote areas may have limited access to high-speed communication resources, thus requiring the frequent transmittal of data via disk or cassette storage media. As a consequence, a highly mobile laser station must assume a significant degree of autonomy, especially for operation in undeveloped areas where access to even ordinary telephone communication may be difficult. In these circumstances, the station should be able to operate for long periods without external communica-

tion during which the on-site resources should be fully utilized for the following objectives:

- generate laser/telescope pointing predictions,
- update the predictions using previously acquired data,
- evaluate the quality of the laser pulse returns.

The first two objectives are necessary to acquire successful laser pulse returns, while the last objective is necessary to enable on-site evaluation and assessment of system performance. In addition, the computer implementation of the algorithms necessary to achieve the foregoing objectives within the computer capability of the station has further requirements, namely:

- The prediction software and procedures should not introduce abrupt changes in the predicted ephemeris except at infrequent and specified times when the ephemeris is updated from external sources.
- The prediction quality should gracefully degrade.
- The prediction software and hardware should be capable of computing considerably faster than real-time.
- The prediction software should not use overlaid and virtual memory but should occupy the directly addressable memory of typical minicomputers to enhance the transferral of software between distinctly different machines.
- Maximum ephemeris information should be compressed into a small number of parameters.
- To enhance the system performance, especially the signal detection, it is desirable that the maximum allowable prediction error be less than 100 m at all times for Lageos.

The design and analysis of algorithms and procedures to accomplish the previously stated objectives require further considera-

tion of the available computer resources within the station. Compact systems will have limited computer power, as represented by the central processor performance capabilities and the main memory storage capacity. These limitations are imposed by the following:

- available hardware technology and available space within the cabinetry,
- power requirements and available power supply,
- heat dissipation and available cooling system.

These three limitations are interrelated and, because of the speed at which technology is changing, are difficult to fully evaluate in the algorithm and procedure design process. Consequently, the design has been directed toward the hardware available in the Transportable Laser Ranging System-1 (TLRS-1) and the McDonald Laser Ranging Station (MLRS) developed for NASA by the University of Texas at Austin [Silverberg, 1981].

The following sections describe the procedures which have been adopted and which have evolved for the generation of the predicted ephemeris and the reconstructed ephemeris, particularly for Lageos. The accuracy of these predictions and the models used in their generation are described. The considerations for the TLRS prediction software development are discussed, and the software performance in generating the reconstructed ephemeris is summarized also.

The Predicted Ephemeris

The ability to predict the state (position and velocity) of a satellite is dependent on several factors, namely, 1) the accuracy of the satellite state at some epoch, 2) the accuracy of the models used to describe the forces acting on the satellite, 3) the accuracy of the models used to describe the kinematic contributions in the various coordinate transformations, 4) the accuracy of predictions which influence the representation of forces and kinematics, and 5) the accuracy of the method used to solve the differential equations of satellite motion. These factors, which are not completely independent of each other, are discussed in the following paragraphs.

Applying the principles of Newtonian mechanics, the motion of a satellite is described by a system of second-order ordinary differential equations which relate the inertial acceleration of the satellite's center of mass to the forces which act on it. This description is an initial value problem, hence, the position and velocity of the satellite, \bar{r}_0 and \bar{v}_0 , are required at some time t_0 . The solution of the initial value problem yields the position and velocity at other times, t . However, the initial values are typically not known to an adequate accuracy to achieve the required accuracy in prediction. As a consequence, observations must be used to determine the state at a selected epoch. The determination of the orbit, however, requires consideration of all of the other factors listed above. In general, the model characteristics and the techniques utilized in

the state estimation will be identical or nearly identical to those used to perform the state prediction.

In support of TLRS and MLRS, the determination of the orbit utilizes quick-look laser range data reported from the NASA systems, the Smithsonian Astrophysical Observatory (SAO) systems and participating European systems. These data are transmitted to the University of Texas at Austin through the NASA Communications Network (NASCOM), usually within a few hours of acquisition. The precision of the quick-look laser range data is system-dependent and varies from a few centimeters to several decimeters.

The models used for the estimation of the epoch state (\bar{r}_0 and \bar{v}_0) have evolved from analysis of over five years of Lageos data. These analyses have included several refinements in the force model based on "long-arc" solutions spanning the entire data set. However, because some small forces observed in these long arcs are not fully understood and cannot be accurately predicted, they have been partially ignored in the models adopted for the orbit determination aspect in support of the predicted ephemeris generation. The models adopted for the state estimation and prediction process are summarized in Table 1. To accommodate the unknown and variable forces, a constant along-track force is estimated simultaneously with the satellite state. In addition, the differential equations of motion are solved by a fourteenth-order multi-step numerical integration method described by Lundberg [1981]. The software used in the estimation

process is UTOPIA [Schutz and Tapley, 1980].

The interval of time used for the Lageos orbit determination portion of the prediction process will be referred to as the estimation interval, usually an interval of approximately two months. The state and parameters obtained from the estimation process, combined with the adopted models, are used to generate the predicted ephemeris for a period at least as long as the estimation interval. The interval of time beyond the estimation interval is referred to as the prediction interval. The estimation and prediction intervals of two recent predicted ephemerides are summarized in Table 2.

In order to evaluate the accuracy of the ephemerides summarized in Table 2, the available quick-look data for August and September, 1981, were used in a separate state estimation. The resulting estimated ephemeris has an accuracy of about one meter over the estimation interval and will be referred to as the "truth ephemeris." By comparing the truth ephemeris with the predicted ephemerides, the differences can be resolved into the radial, transverse (along-track) and normal (cross-track) differences. These differences are shown in Figures 1 to 3. In addition, for comparison, ephemerides generated at the NASA Goddard Space Flight Center (GSFC) are also shown. These ephemerides, referred to as GSFC Inter-Range Vectors (IRV), provide pointing predictions for the NASA network of laser stations.

It is of particular interest to note from Fig. 2 that the error in the predicted ephemeris LAG0013 has grown to about 80 m at

the end of a four-month prediction interval. In addition, the radial and normal components of the error are less than 10 m. The GSFC IRV's, however, exhibit significantly larger errors in this time period. While these large errors may not totally prohibit acquisition, they are likely to be a contributing factor to poor daytime tracking of Lageos and the signal-to-noise ratio of the return. The manner in which the errors illustrated in Figs. 1 to 3 are "seen" at a particular SLR site will be dependent on the viewing geometry of the satellite with respect to the SLR.

The Reconstructed Ephemeris

The ideal procedure for reconstruction of the predicted ephemeris at the SLR is to use the same dynamic and kinematic models used for the generation of the ephemerides. In such an ideal case, only the estimated position and velocity at a chosen epoch would be required for the SLR to completely reconstruct the predicted ephemeris. Consequently, only one set of six quantities would be required for a several-month prediction interval.

Because of the complexity of models used in the creation of the predicted ephemeris, it is not feasible with current technological and budgetary constraints to achieve the ideal case. Consequently, the SLR software, which reconstructs the predicted ephemeris, must operate within the available computer resources. Various procedures are feasible, for example:

- The predicted ephemeris fit with simple approximating functions, e.g., spline or Chebychev polynomials. In these cases, the parameters associated with the approximating function would be provided to the SLR;
- Analytical orbit theory, requiring mean orbit elements consistent with the theory;
- Simplified versions of the models and techniques used to generate the predicted ephemeris.

Although numerous factors were considered in the evaluation process, the overriding factors were maximal use of existing software and techniques, as well as the transmission of a small number of parameters for the reconstruction process. Consequently, the last procedure listed above was adopted for the TLRS operation.

In general, the use of a simplified model for the ephemeris reconstruction will cause a degradation at a rate which is dependent on the degree of simplification, assuming that initial conditions from the predicted ephemeris are directly used. An alternate procedure, however, is to provide the reconstruction software with initial conditions which have been adjusted with respect to the predicted ephemeris in order to reduce the reconstruction error. This "tuning" of the initial conditions is transparent in the use of the reconstruction software; the burden for generating the tuned initial conditions rests with the facility which generates the predicted ephemeris. The

procedure is illustrated in Fig. 4. It is important to note that the purpose of the tuning is to essentially eliminate prediction errors which can be introduced by the reconstruction software. With a chosen reconstruction interval, i.e., the time interval for which the initial conditions have been tuned, the software reconstructs the predicted ephemerides as illustrated in Fig. 5. Although discontinuities will exist between adjacent tuned intervals, the proper combination of simplified model and tuning interval will ensure small discontinuities.

The degree of simplification in the reconstruction software is dependent on the acceptable error magnitude and the reconstruction interval. After considering various intervals for providing the predicted ephemeris (in the form of adjusted initial conditions), the interval of one day was chosen as the nominal goal for TLRS. With this selection, only one set of satellite position and velocity per day need to be provided to the reconstruction software, thereby allowing even verbal transfer of several days of data to the SLR if it is required.

With the selection of a nominal one-day predicted ephemeris interval, consideration of various models for the reconstruction software was necessary. By faithfully reconstructing the predicted ephemeris, the satellite ephemeris available at the SLR will reflect only errors in the predicted ephemeris, instead of the errors from both the prediction and the reconstruction. With this consideration, the design criteria was adopted that the reconstruction error be less

than five meters RMS over the reconstruction interval.

Using UTOPIA, the software used for the generation of the predicted ephemeris, various experiments were performed to evaluate various simplified models for Lageos. These experiments utilized an ephemeris comparison mode in which points on an ephemeris can be used as "observations" for a least-squares adjustment of epoch state or other parameters. The nonlinear estimation process requires iteration; however, the initial iteration always began with the state directly available from the predicted ephemeris. To assist in the interpretation of results, the differences between the "observed (predicted) ephemeris" and the "computed ephemeris" are resolved into components in the radial (R), transverse (T) and normal (N) directions. These differences are expressed in terms of their Root Mean Square (RMS) for each iteration. Furthermore, only the epoch state was adjusted in the results which are described in the following paragraphs.

Because of the Lageos orbital characteristics, degree seven, order six spherical harmonic coefficients produce a 2.66-day long-period effect. Consequently, experiments have shown the necessity for inclusion of these terms in the reconstruction process, especially when the software will predict the state for one day from a single position and velocity. The influence of these coefficients is summarized in Tables 3 and 4.

Similarly, the moon produces an effect on the Lageos orbit which is characterized by 14- and 28-day periods, while the effect of the sun has annual and semi-annual periods. While the effect may be small within one day, the day-to-day effects can be expected to change, as exhibited in Tables 5 and 6. As shown in these tables, the use of points on the predicted ephemeris as the initial conditions for the reconstruction process without inclusion of the sun and moon yields errors of as much as 150 m. This violates the stated criteria that the ephemeris reconstruction be accurate to less than five meters. Even after tuning the initial conditions for this model deficiency, the error still exceeds the 10 m level. Inclusion of the sun and moon shows that the specified criteria can be met, even without tuning the initial conditions. This conclusion, however, assumes that the reconstruction process uses a model of the sun and moon coordinates, which is identical to that used in the generation of the predicted ephemeris, which requires a large supporting data base (DE-96).

Based on the preceding results, the reconstruction software was developed by Cuthbertson [1981] using the characteristics shown in Table 7. Particular characteristics include the use of Pines [1974] nonsingular formulation for the gravitational spherical harmonics, modified to directly use normalized coefficients. In addition, the software utilizes an analytical theory of the moon and sun to avoid the necessity of a large supporting data base. Evaluation of the accuracy of this approach compared to the use of DE-96 in the

reconstruction software is in progress. It is to be noted, however, that by appropriate tuning of the initial conditions most of the differences in the models can be eliminated.

The reconstruction software is coded in Fortran and has been operated on a PDP 11/60 minicomputer using the RSX11M operating system, as well as on a Data General Nova computer. The software uses less than 32000 16-bit words of memory for execution. With the system's hardware floating point processor, predictions for a full day require less than five minutes of computer time, typically about two minutes.

The reconstruction software has been evaluated through the use of points from the predicted ephemeris as well as with quick-look data. In the former case, a set of initial conditions from the predicted ephemeris were used for a one-day prediction and points generated by the reconstruction software were directly compared with points from the predicted ephemeris. The latter case used quick-look range data from Yarragadee, Australia, on October 10, 1981. Using the initial conditions from the predicted ephemeris, LAG0014, and without tuning, a range residual RMS of 5.1 m was computed, approximately equivalent to 34 ns in two-way propagation time. It is important to note that the last data used in the creation of this predicted ephemeris was 3 September 1981.

Conclusions

The results have demonstrated that the errors from a several-month prediction of the Lageos ephemeris are less than 100 m. With expected further improvements in the force models used for the state estimation and ephemeris prediction, simultaneous improvements in prediction accuracy will result also.

The ephemeris reconstruction accuracy is dependent on 1) the model used and 2) the interval of time for which the reconstruction is applied. The model requirements can be relaxed by adjusting initial conditions used in the reconstruction software to accommodate the model differences. This tuning of the information provided to the reconstruction software can produce meter-level accuracy in reconstructing the predicted ephemeris.

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Table 1

MODELS USED FOR LAGEOS ESTIMATION/PREDICTION (LAG0014)

Force Models:

Gravitational

GEM 10 (Lerch, et al., 1979); complete to degree and order 13 plus selected resonance coefficients

Sun and Moon; coordinates from JPL DE-96 [Standish, et al., 1976]

Solid Earth Tides*; Wahr (1981a) model using 1066A earth model

Ocean Tides*; Schwiderski (1980) nine-constituent model

Nongravitational

Solar Radiation Pressure; assumed constant

Drag-like Force; estimated parameter, assumed constant (LAG0014 value, $-2.60 \times 10^{-12} \text{ m s}^{-2}$)

Equations of Motion:

Mean equator and equinox of 1950.0 nonrotating reference system

Earth Orientation:

1976 IAU Precession [Lieske, et al., 1977]

Wahr Nutation (1981b)

Pole Position and UT1 from BIH Circular D; for prediction, values adopted from extrapolation of BIH Circular D

Laser Station Coordinates, LSC 80.11 [Tapley, et al., 1980]

Numerical Methods:

Fourteenth-order multi-step [Lundberg, 1981]

Software:

UTOPIA [Schutz and Tapley, 1980]

* Implementation and results are given by Eanes, et al. [1981].

Table 2

RECENT PREDICTED EPHEMERIDES FOR LAGEOS

<u>Ephemeris Identification</u>	<u>Estimation Interval*</u>	<u>Prediction Interval</u>
LAG0013	18 March 1981 to 10 May 1981 (54 days)	11 May 1981 to 19 September 1981 (131 days)
LAG0014	16 July 1981 3 September 1981 (49 days)	3 September 1981 to 1 February 1982 (151 days)

* Interval during which quick-look laser range data are used to estimate the epoch orbit elements and drag parameter.

Table 3

ONE-DAY EPHEMERIS RECONSTRUCTION: 4 x 4 GRAVITY
EPHEMERIS DIFFERENCES (m)

<u>Day</u>	<u>R</u>	<u>T</u>	<u>N</u>	<u>RMS Overall</u>	<u>RMS Tuned Overall</u>
1	3.4	44.3	4.5	25.8	3.0
2	3.6	30.9	3.9	18.1	2.9
3	3.3	32.5	5.4	19.1	2.9
4	4.5	13.1	4.5	8.4	3.2
5	4.0	19.5	2.6	11.5	3.1

Table 4

ONE-DAY EPHEMERIS RECONSTRUCTION: 7 x 7 GRAVITY

<u>Day</u>	<u>R</u>	<u>T</u>	<u>N</u>	<u>RMS Overall</u>	<u>RMS Tuned Overall</u>
1	0.8	5.2	0.3	3.0	0.6
2	0.3	4.8	0.5	2.8	0.6
3	0.8	3.7	0.4	2.2	0.5
4	1.1	2.6	0.3	1.7	0.6
5	0.8	2.7	0.3	1.7	0.6

Table 5

ONE-DAY EPHEMERIS RECONSTRUCTION: NO SUN/MOON
EPHEMERIS DIFFERENCES (m)

<u>Day</u>	<u>R</u>	<u>T</u>	<u>N</u>	<u>RMS Overall</u>	<u>RMS Tuned Overall</u>
1	8.9	523.3	73.8	152.4	21.9
2	5.2	52.2	127.7	79.7	19.6
3	4.9	27.7	109.2	65.1	16.3
4	4.8	16.2	90.0	52.9	13.5
5	5.7	14.2	79.5	46.8	12.9

Table 6

ONE-DAY EPHEMERIS RECONSTRUCTION: SUN/MOON
EPHEMERIS DIFFERENCES (m)

<u>Day</u>	<u>R</u>	<u>T</u>	<u>N</u>	<u>RMS Overall</u>	<u>RMS Tuned Overall</u>
1	0.7	6.7	0.6	3.9	0.5
2	0.3	4.1	1.2	2.5	0.7
3	1.0	4.2	1.7	2.7	0.6
4	1.0	2.4	1.1	1.6	0.6
5	1.0	3.1	0.8	1.9	0.7

Table 7

MODELS USED FOR LAGEOS EPHEMERIS
RECONSTRUCTION SOFTWARE

Force Models:

Gravitational

GEM 10, complete to degree and order 7

Sun and Moon; coordinates from analytical theory

Nongravitational

None

Equations of Motion:

True-of-date nonrotating reference system defined by
initial epoch

Numerical Methods:

Fourteenth-order multi-step integrator [Lundberg, 1981]

COMPARISON OF THE RADIAL ORBIT ERROR OF DIFFERENT LAGEOS EPHEMERIDES FOR AUGUST-SEPTEMBER 1981

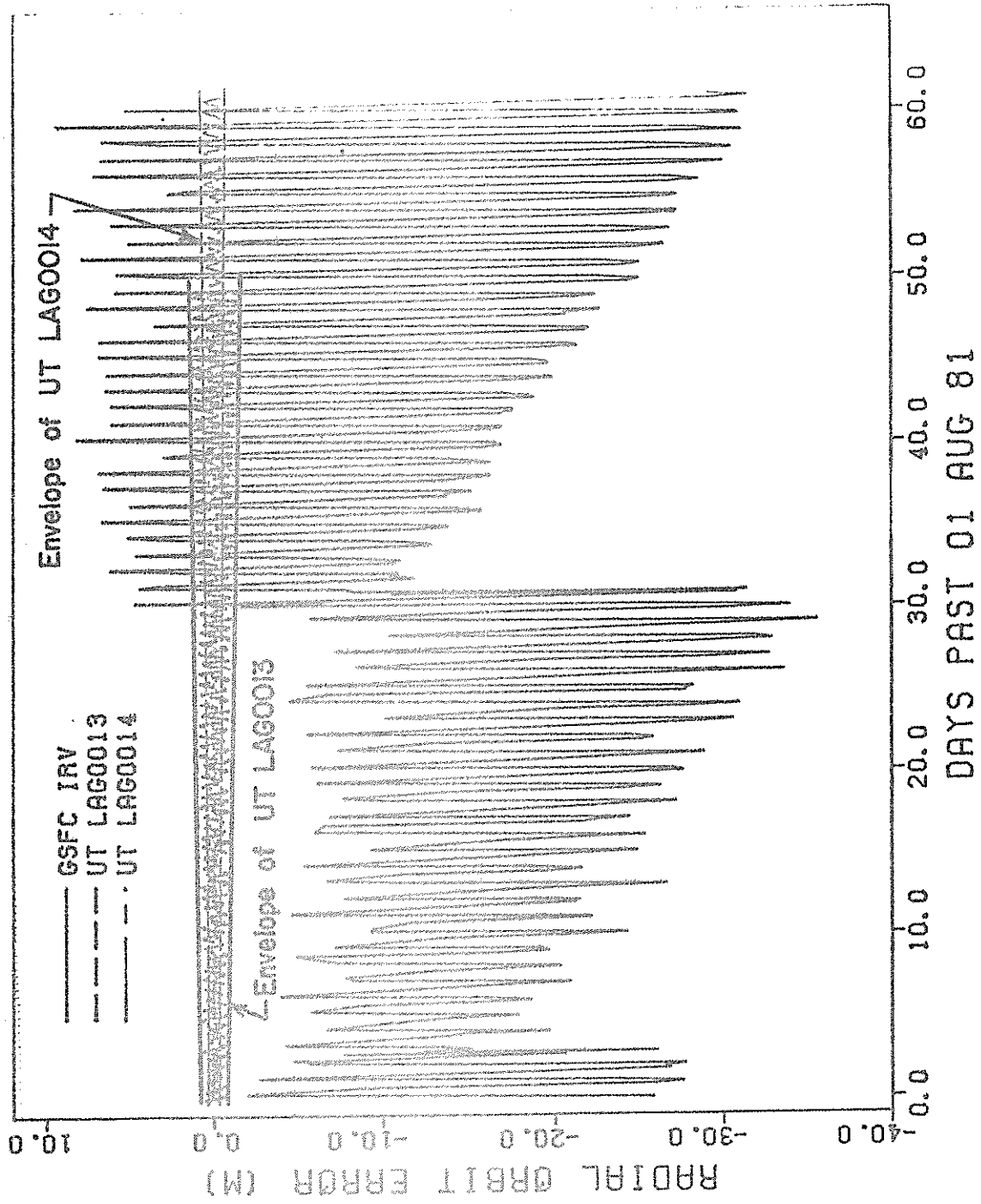


FIG. I RADIAL PREDICTION ERROR

COMPARISON OF THE TRANSVERSE ORBIT ERROR OF DIFFERENT
LAGEDS EPHEMERIDES IN AUGUST-SEPTEMBER 1981

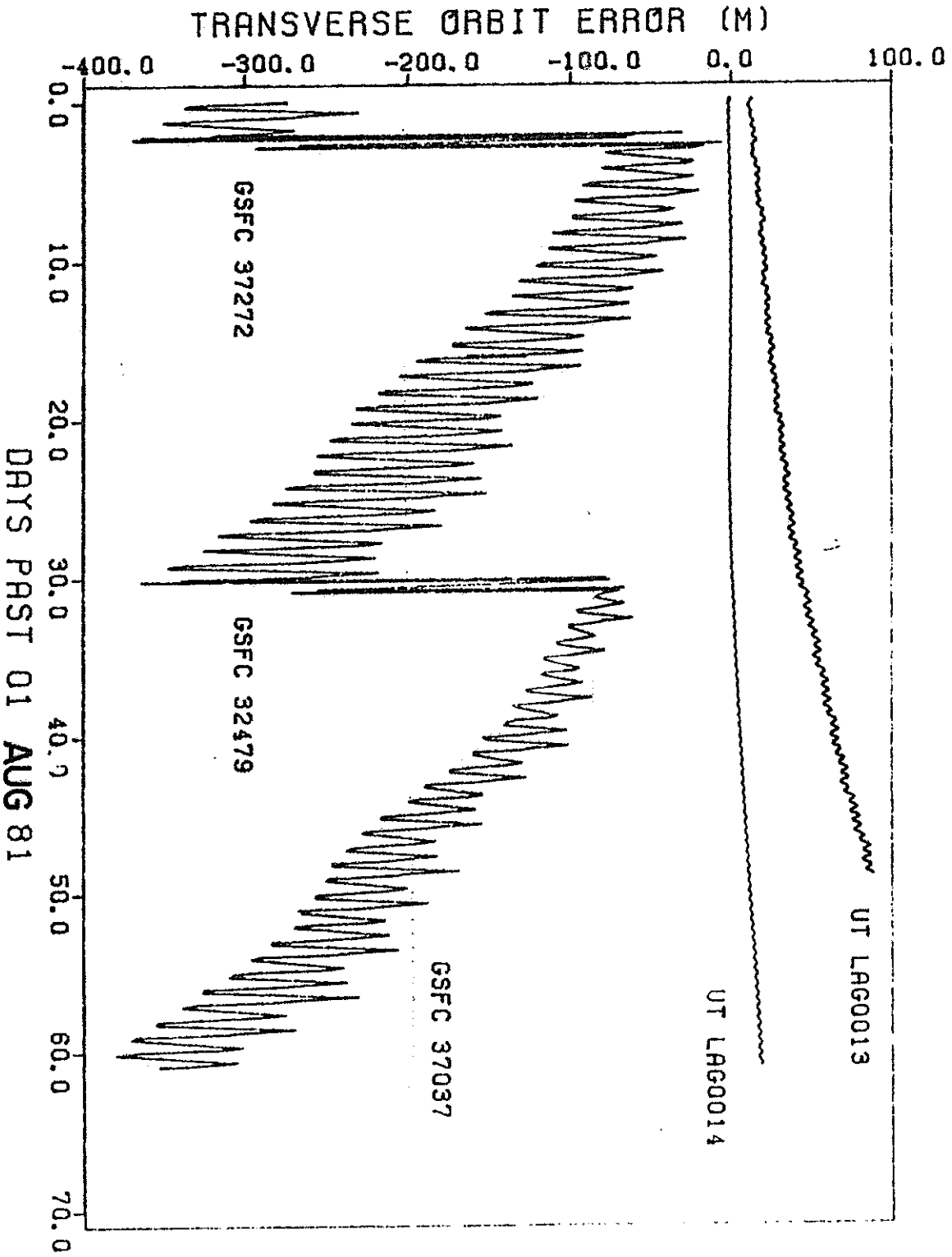


FIG. 2 TRANSVERSE PREDICTION ERROR

COMPARISON OF THE NORMAL ORBIT ERROR OF DIFFERENT
LAGEOS EPHEMERIDES FOR AUGUST-SEPTEMBER 1981

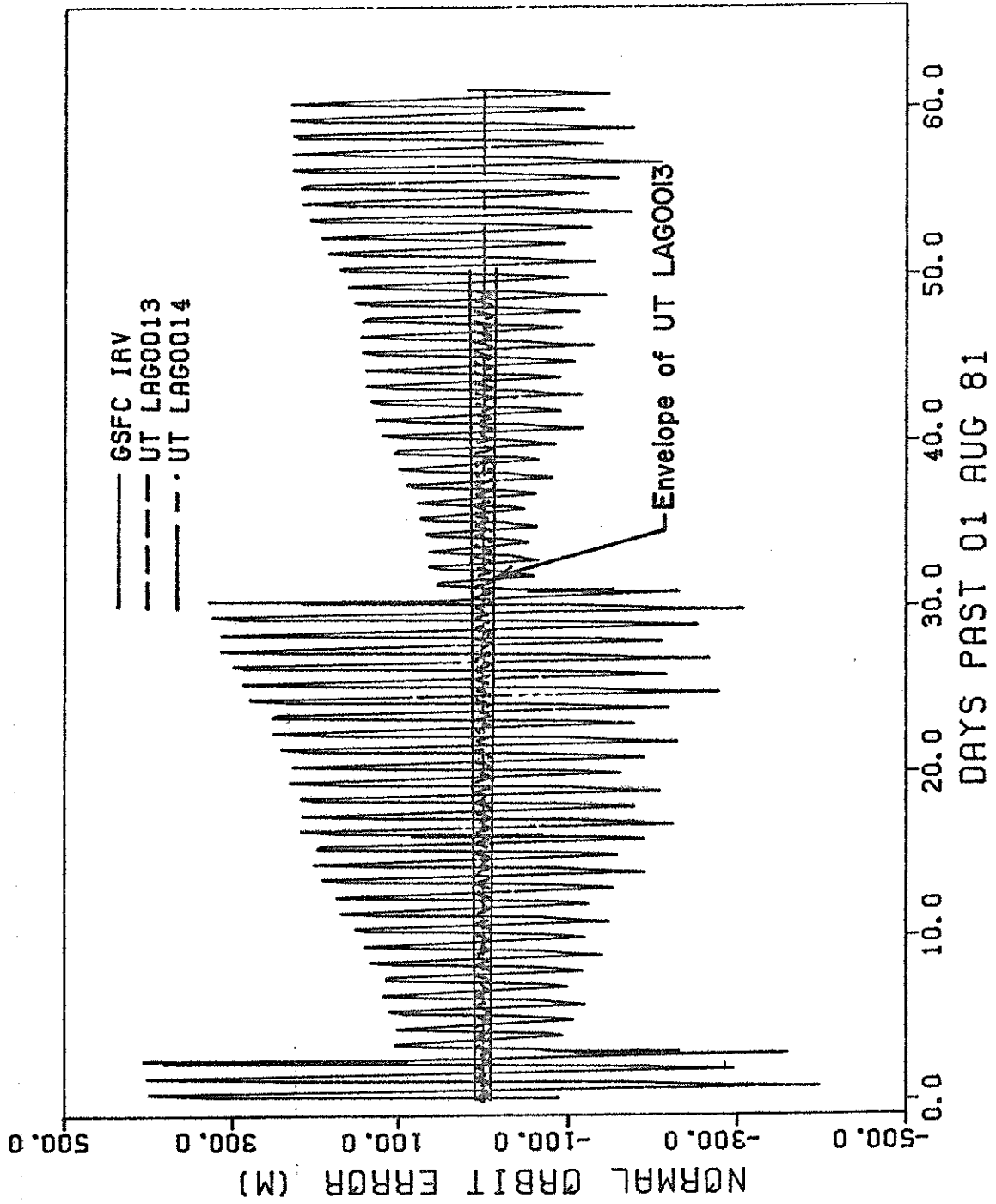
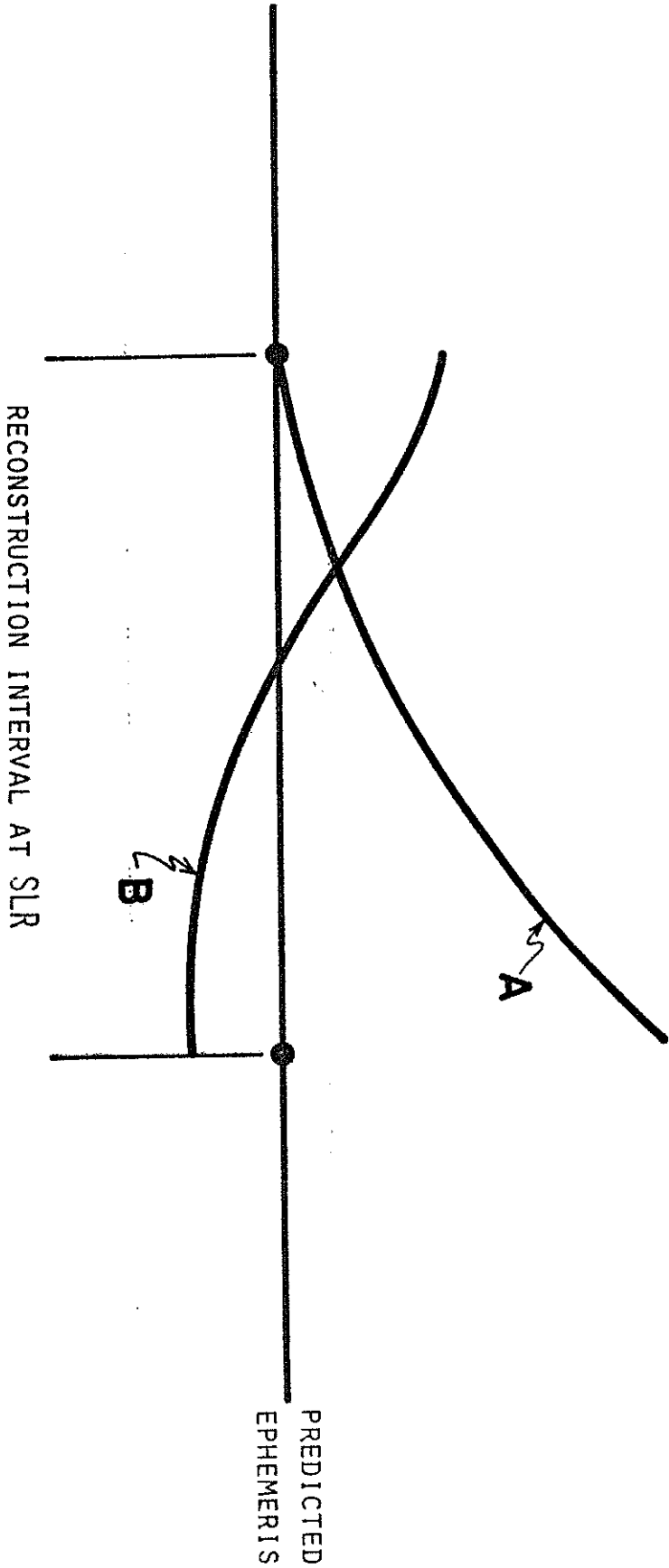


FIG. 3 NORMAL PREDICTION ERROR



A: RECONSTRUCTED EPHEMERIS USING PREDICTED EPHEMERIS FOR INITIAL STATE

B: RECONSTRUCTED EPHEMERIS USING INITIAL STATE TUNED TO BEST FIT SLR MODEL TO PREDICTED EPHEMERIS

FIG. 4 TUNED INITIAL CONDITIONS

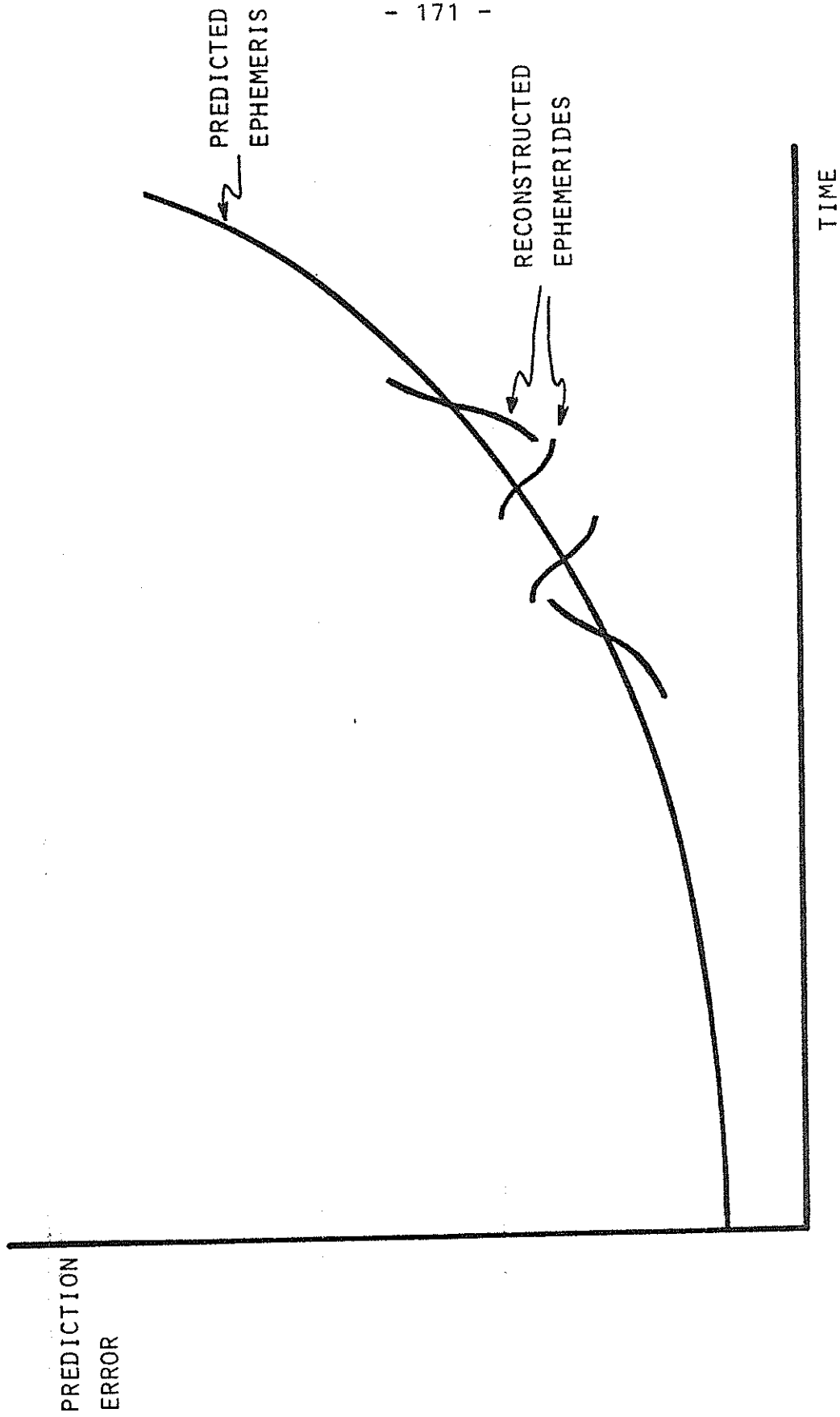


FIG. 5 PROCEDURE

PRACTICAL ASPECTS OF ON-SITE PREDICTION

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Abstract:

The amount of success of satellite laser ranging, apart from efforts in instrumental design, largely depends on the production of accurate satellite orbits by a prediction center.

This paper reviews some of the aspects of the interaction between a prediction center and mobile laser ranging systems.

Practical techniques for real-time prediction optimization are reviewed. The feasibility of on-site estimation of orbital parameters is discussed.

1. Introduction

Satellite laser ranging systems should be provided with orbital parameter messages, which are sufficiently compact

for ease of data-communication and which allow for efficient on-site computation of look-angles. Section 2 reviews techniques for this.

The frequency with which satellite orbits should be upgraded, from the reduction of recent quick-look data largely depends on orbital characteristics. Low flying satellites, especially those affected significantly by air drag, cannot be predicted accurately for more than 2 or 3 weeks ahead [Dunn, 1979].

The required prediction accuracy obviously depends on equipment characteristics. An important overall quality is the maximum beam divergence allowing for a reasonable probability of satellite return detection. In daylight, at high background noise levels, this value will be most critical.

If the a-priori quality of the look-angle predictions does not allow for "hands-off" tracking due to e.g. poor orbital information or, in case of mobile ranging, inaccuracies in site coordinates, mount orientation or timing, real-time facilities as described in section 3 could help optimising acquisition rates significantly.

In case of interrupted data communication with the prediction center, possibly happening to remotely operating mobile stations, a great deal of continuity of operation can be obtained from on-site orbital parameter estimation techniques. These are discussed in section 4.

2. Reproduction of the orbit on-site

Obviously it will be highly impractical to transmit sets of look-angles from a prediction center to the individual laser ranging stations. The prediction center

should define a set of parameters which describe the predicted orbit accurately and which allow for an efficient process of computing look-angles using on-site computing facilities. Thus one requires a parameterisation of the orbit to a sufficient degree of accuracy with a limited set of data involved on one hand, and reasonable simplicity in the reconstruction and in the subsequent computation of look-angles on the other hand.

A well known example of this approach is the SAO mean element message comprising Keplerian mean elements at an epoch and terms describing secular and long-periodic perturbations [Thorp, 1978]. The reconstruction of the orbit requires an analytical theory for updating the orbit to any epoch $t(i)$ and the addition of significant short-periodic perturbations. The famous SAO-Aimlaser programme does just that.

An alternative approach is the use of osculating elements (a state vector) which describe the actual satellite state at an epoch in a well-defined coordinate system, e.g. NASA IRV messages. The reconstruction of the orbit can be accomplished by numerical integration of the equations of motion derived from a dynamical model. Pertinent to this model a choice out of three cases could be made.

- a. The formally only correct way is to use the original dynamical model for the equations of motion as being used for the quick-look data analysis at the prediction center. This would precisely reconstruct the orbit as it has been predicted from the data analysis, at the unrealistic price however of heavily burdening the on-site computing facilities.
- b. Alternatively one could derive the equations of motion from a simplified model. The permissible degree of simplification depends basically on the satellite orbit

characteristics and the arc length required. The reconstructed orbit being tangential at the initial epoch will gradually diverge from the originally predicted orbit. In table 1 the committed error is summarized using a given state vector for STARLETTE and LAGEOS. Results are given at various arc lengths and using two very simple dynamical models, accounting for the influence of earth gravity up to the J2 term or up to degree and order 4 respectively, neglecting all other forces.

These results suggest the use of one state vector for each observable pass of STARLETTE (about 5 or 6 per day), where the J2-field is already sufficient and one state vector for every one-day arc of LAGEOS, utilising up to C,S(4,4). Thus the computational effort required on-site as well as the number of state vectors involved is quite limited.

- c. A third possibility is that the prediction center fits an arc of certain length to the predicted orbit in an adjustment process using a simplified dynamical model, yielding a new estimate for the initial state vector. Utilising this "mean" state vector, the fitted arc can be reproduced on-site by numerical integration applying the same simplified model. Again the permissible arc length and the degree of simplification are related and depend on the orbit. But the absence of the divergence occurring in the former case, as a result of the technique of approximation applied here, will allow for a considerably greater arc length to be derived sufficiently accurately from a single state vector. Fig 1 depicts the deviation from the original orbit for a 7-day STARLETTE arc, utilising a tailored dynamical model comprising 23 selected coefficients of the earth

STARLETTE	20 min	1 hour	1 day
J 2,0	77 m/ 16 "	205 m/ 43 "	-
J 4,4	15 m/ 3 "	63 m/ 13 "	-
LAGEOS	20 min	1 hour	1 day
J 2,0	-	71 m/ 2.4"	-
J 4,4	-	12 m/ 0.4"	229 m/ 7.9"

Table 1. Errors in satellite position due to truncation of the earth gravity model up to the J_2 -term and up to degree and order 4 respectively, at various arc lengths.

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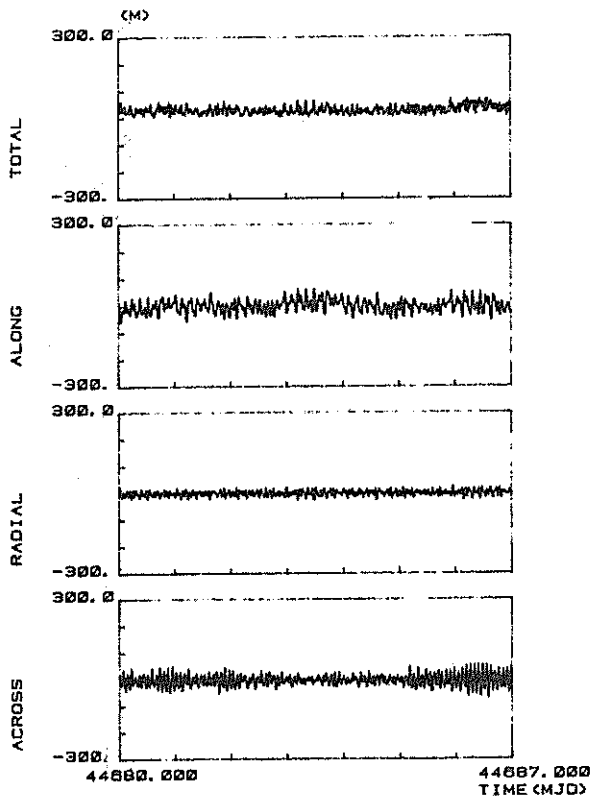


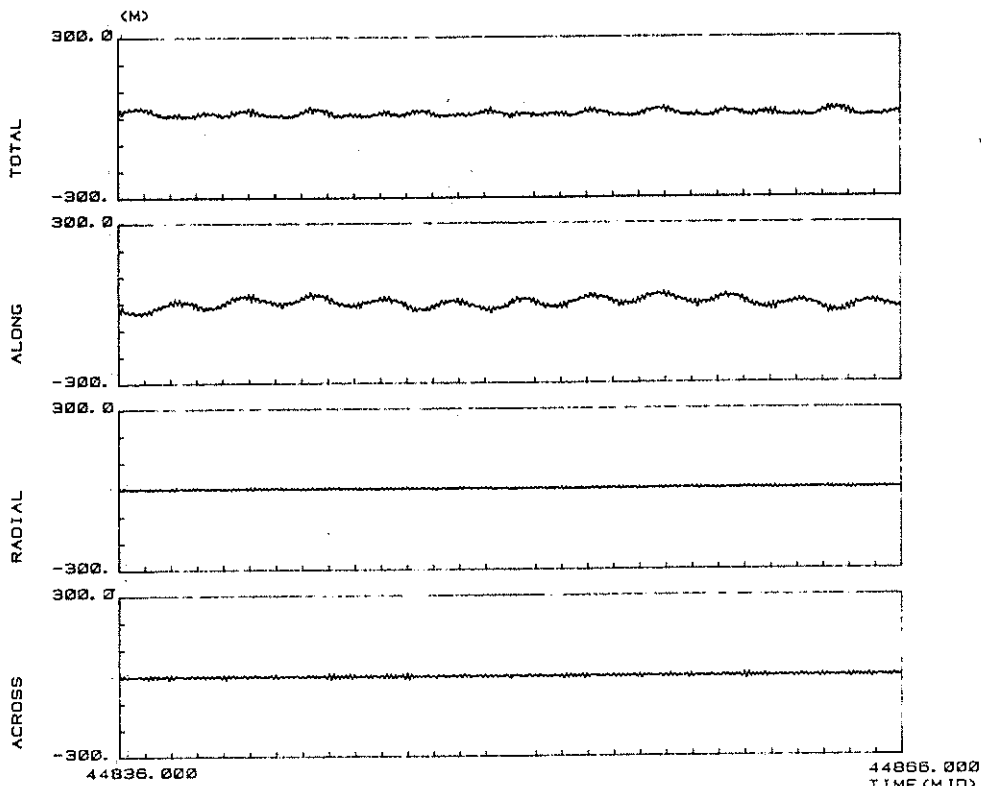
Figure 1.

Deviation of the approximated arc from a reference orbit. The tailored model comprises earth gravity terms up to degree and order 6 together with the sectorial terms of degree 13 and 14, as well as models for lunar gravity and air drag.

Figure 2.

A similar graph for LAGEOS. The tailored model comprises earth gravity terms up to degree and order 4, as well as models for solar and lunar gravity.

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gravity field basically to degree and order 6, as well as models for air drag and lunar gravity. Fig. 2 gives similar results for a 30-day arc of LAGEOS derived from an earth gravity field up to degree and order 4, and models for lunar and solar gravity.

In both cases the maximum error is less than 50 m in the along-track, radial and across-track components. More details on this approach are given in [Vermaat, 1981].

From these various approaches it can be concluded that ways exist to optimize both the amount of input data demanded from a prediction center and the simplicity of the on-site software producing the look-angle information required for tracking.

3. Real-time techniques for prediction optimisation

Basically three goals should be accomplished while tracking the satellite, in order to optimise the data acquisition:

- a. obtaining the satellite within the laser beam and obtaining the occurrence of its return signal within the time window,
- b. discriminating satellite returns from noise,
- c. deciding on corrections to the look-angles, delay and size of the time window to enhance the signal to noise ratio.

If a. is not accomplished automatically when tracking has commenced, eventually at the maximum permissible beam width and time window, there is little else to do then to engage some search-and-find process, primarily adjusting

the along-track component.

The second goal should obviously also be aimed at immediately, in order to be able to judge the amount of success in actually hitting the satellite. This discrimination could be facilitated by graphically displaying the occurred events e.g. in terms of observed - predicted values or by constructing histograms counting events in a number of classes defined in the travel time domain. This graphical data will require operator intervention to conclude about the actual offset in predicted range. Simple numerical techniques for processing range residual data could be imagined to arrive automatically at the range offset.

An increased acquisition rate resulting from the deployment of a multi-event timer, especially at high noise levels, could significantly speed up this process of locating the satellite.

Angular information could be derived from data sampling with a quadrant detection system, such as anticipated in the Delft mobile system [Visser, 1981]. This data, sampled with four independent PMT's, could likewise be processed either graphically or numerically, to decide on the angular displacement of the satellite with respect to the optical center of the detection system.

Once the satellite has been located with some degree of reliability, the corrections to be applied to the predicted delay or, in case of quadrant detection to the look-angles, can be decided upon easily by the operator or eventually from dedicated software. An example of a curve fitting technique rapidly concluding accurately on improved range prediction is given in fig. 3. As soon as a curve is fitted to 7 to 20 early range observations in the pass, utilising a second degree polynomial and a Keplerian model with three free parameters resp., the remaining range predictions in

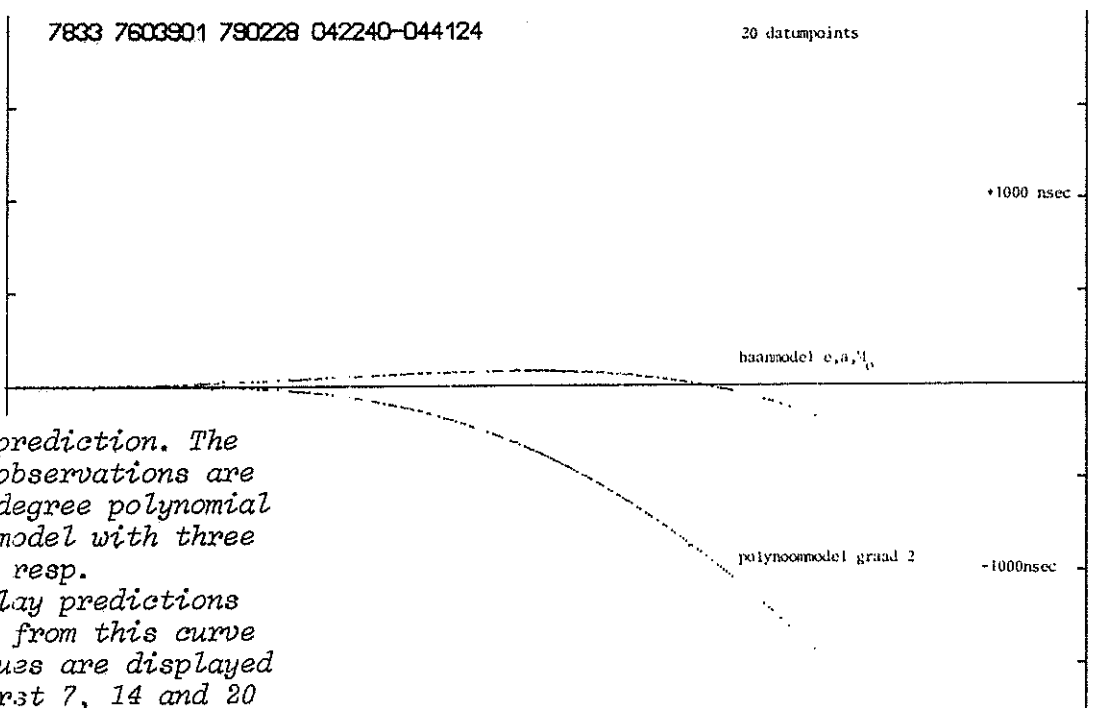
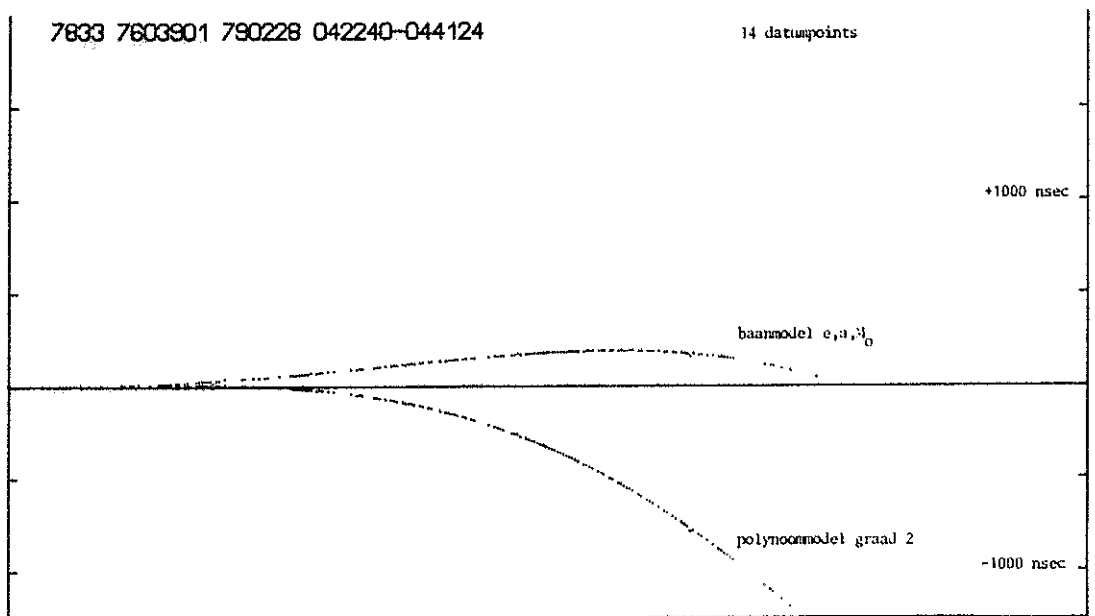
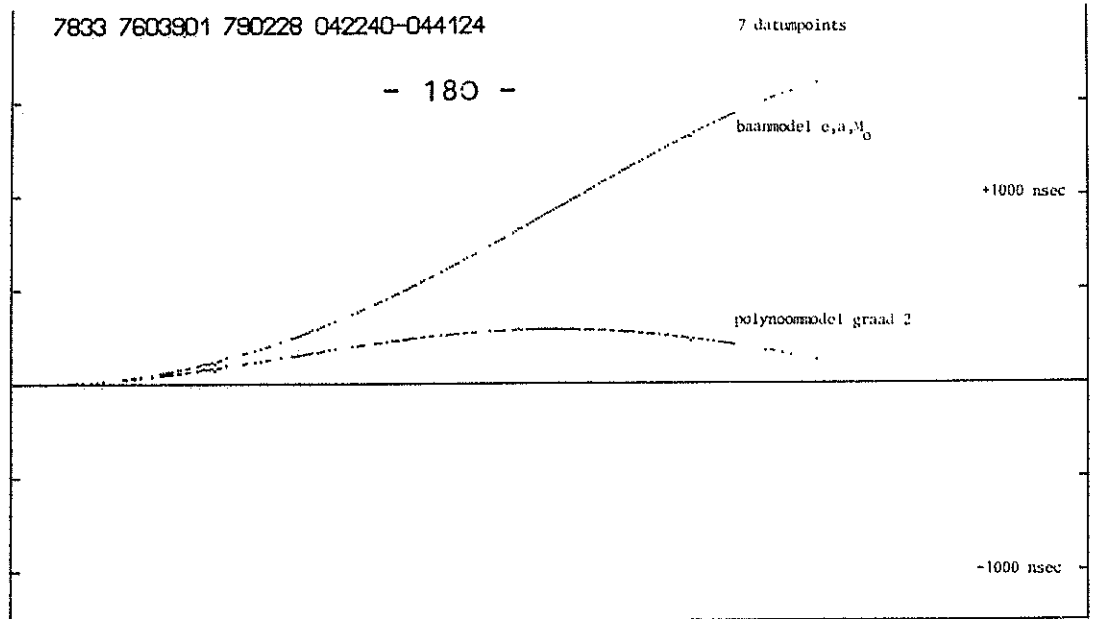


Figure 3.

Real-time delay prediction. The first few range observations are fitted to a 2nd degree polynomial and a Keplerian model with three free parameters, resp. The remaining delay predictions are extrapolated from this curve fit. The O-C values are displayed utilising the first 7, 14 and 20 datapoints resp.

the pass are extrapolated from the curve. The graphs display the residuals with respect to the actual observations. It is concluded that already after processing the first 14 range observations, the improved range predictions obtained from the Keplerian model result in range predictions accurate to at least 200 ns for the remainder of the pass.

Thus, with the techniques described, improved satellite predictions can be produced, allowing for the decrease of both the time window produced by the range gate generator and the width of the laser beam, which in turn yields improved signal to background noise levels. It is expected that these detection hardware facilities in concert with simple but cleverly designed software, will considerably improve the amount of data acquisition in case of initially marginal observability conditions like daylight ranging at low signal levels of poor a-priori information on either the satellite's orbit or on mount position and orientation, or in case of the occurrence of unknown clock offsets.

4. Orbital parameter estimation from on-site analysis

The quality of orbital predictions for low orbit satellites (say less than 1500 km), especially those with unfavourable area to mass ratio, tends to deteriorate quite rapidly in a few weeks time. Regular re-estimation based on recent tracking data is therefore important, requiring a prediction center collecting quick-look data from different tracking stations and transmitting newly derived orbital parameters regularly. If remotely operating mobile laser ranging systems encounter problems in data communication with the outside world, successful ranging on these satellites might become increasingly difficult. Some de-

gree of independence for these mobile systems in terms of on-site prediction capability therefore might add to the success of laser ranging campaigns.

Major drawbacks in this on-site data reduction and estimation process will be the poor geometry of the solution as a consequence of the use of one-station range data only and the severe simplifications in the dynamical model from which the equations of motion will be derived, dictated by the limited on-site computer capacity. The feasibility of this approach however has been indicated by several authors (e.g. [Schutz, 1978], [Wakker, 1981]).

The geometry problem will have to be solved by reducing data from a sufficient number of different passes, thus increasing the estimability of the orbital plane orientation parameters. But even in clearly non-singular solutions, systematic errors which might easily go undetected, could corrupt the significance of the estimated orbital parameters. Therefore this type of analysis always will have to be done with care.

The limited computer capacity will be most severe in real-time, because high priority programs will be in execution e.g. for monitoring the tracking and data formatting, etc. The necessity for orbital parameter estimation in real-time could be questioned however. Some of the more appropriate candidates for efficiently improving tracking parameters in real-time were outlined in the previous section. The corrections to the look-angles derived in this way, usually have limited validity for future passes however. Therefore, once serious offsets in the look-angles or delays due to poor orbital prediction quality have been encountered, programmes for data reduction and parameter estimation should be executed off-line in the periods between pass observations, thus exploiting the otherwise

idle computer capacity to the full.

The feasibility of the sequential KALMAN filter approach for this analysis has been clearly demonstrated in [Wakker,1981]. In this reference the application of the extended KALMAN filter technique to the problem of orbital estimation utilising one-station range data, is outlined. Of specific interest is the solution to the filter divergence problem due to non-linearity of the dynamical equations. The cautious application of a correlation correction factor slightly affecting the state covariance matrix prior to each observation-update step, effectively eliminates this instability. The notorious filter divergence problem occurs also in presence of undetected gross errors in the range data. [Vermeer,1981] describes a theoretically derived technique based on the "limited gain" philosophy, bounding the influence of gross errors. Without the use of this kind of techniques, the effect of gross errors, especially when occurring at the beginning of a new pass, is generally fatal to the filter stability. From these studies it can be concluded that with carefully designed software and cautious analysis, the KALMAN filter method proves to be a powerful tool to solve the problem of upgrading orbital parameters from on-site range data analysis.

For those who insist, this sequential technique could be applied in real-time, feasible for satellite ranging stations equipped with sufficient computer capacity.

Obviously batch type approaches could also be employed, exclusively in the off-line case. Reducing batches of data principally has the advantage of better control of the influence of gross errors in the data, than sequential techniques, although as indicated above, the quite poor geo-

metry due to the exclusive use of range data only from one station, might in many cases largely neutralise this quality, especially when a small number of passes has been observed. A candidate for a simplified dynamical model describing the satellite's orbital motion could be the model outlined in section 2, used for the approximation of an arc, estimating a "mean" state vector. This new initial state vector defines the arc with sufficient accuracy, deploying dynamical equations derived from the tailored force model. If this model would have been installed already for the computation of look-angles, it would require relatively little extra coding to allow for the reduction of range data, re-estimating the initial state in the same dynamical system. Promising results were obtained from a preliminary investigation upgrading a 7-day STARLETTE arc utilising the tailored model described in section 2. Fig. 4 illustrates the effect of the reduction of 9 passes of range data. In this simulation study, the initial state was corrupted causing a considerable divergence between the "real" and the corrupted arc, illustrated in the top figure. The graph at the bottom shows the deviation of the updated arc from the "real" arc after processing 9 passes of range data, taken within 48 hours from the beginning of the arc. It should be noted that the original initial state vector was derived from a 7-day arc. The latter 7 days depicted in this figure shows the results of sheer extrapolation, suggesting the feasibility of continuing this on-site analysis with recent data, thus obtaining a good deal of independence from external orbital information. Considerable interrupts in the data acquisition due to e.g. weather or site change would easily corrupt this process of "bootstrapping", compelling re-initialisation from external information.

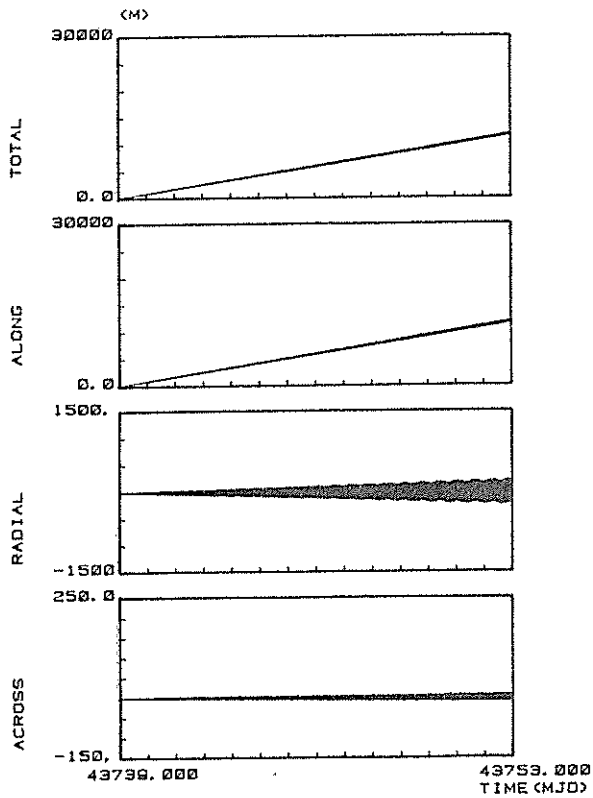
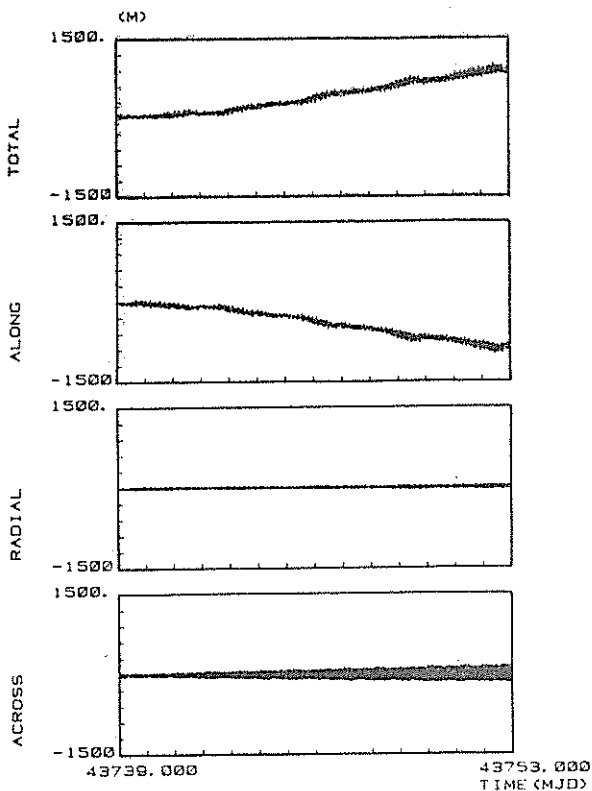


Figure 4.

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Satellite orbit improvement from on-site analysis. A poorly predicted arc of STARLETTE is upgraded from the reduction of one-station range data taken in 9 passes during the first 48 hours of the arc. The deviations from the reference orbit are displayed for the predicted arc (top) and the improved arc (bottom).

5. Concluding remarks

A dedicated prediction center communicating regularly updated orbital parameters for the low satellites to SLR stations, could eliminate all serious on-site prediction problems. Nevertheless a great deal of flexibility and reliability can be obtained from clever hardware/software features at the stations for real-time prediction optimization. Some degree of independence from data communication with the prediction center by means of on-site facilities for orbital parameter estimation, is advisable and feasible.

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