Available online at www.sciencedirect.com

ScienceDirect

Advances in Space Research 57 (2016) 1928–1938

ADVANCES IN SPACE RESEARCH
(a COSPAR publication)

www.elsevier.com/locate/asr

Review and critical analysis of mass and moments of inertia of the LAGEOS and LAGEOS II satellites for the LARASE program

Massimo Visco [a,b], David M. Lucchesi [a,b,c,*]

[a] Istituto Nazionale di Astrofisica (INAF) – Istituto di Astrofisica e Planetologia Spaziali (IAPS), Via del Fosso del Cavaliere, 100, 00133 Roma, Italy
[b] Istituto Nazionale di Fisica Nucleare (INFN), Sez. Tor Vergata, Via della Ricerca Scientifica 1, 00133 Roma, Italy
[c] Istituto di Scienza e Tecnologie della Informazione (ISTI) – Consiglio Nazionale delle Ricerche (CNR), Via G. Moruzzi 1, 56124 Pisa, Italy

Received 18 November 2015; received in revised form 8 February 2016; accepted 9 February 2016
Available online 15 February 2016

Abstract

The two LAGEOS satellites, currently the best tracked satellites by the stations of the International Laser Ranging Service (ILRS), play a significant role in the fields of space geodesy and geophysics as well as in very precise measurements and constraints in fundamental physics. Specifically, for the measurements of tiny relativistic effects it is mandatory to build accurate models for the dynamics of the satellites, in particular concerning their spin evolution and the determination of their temperature distribution and thermal behavior under different physical conditions. Consequently, an accurate knowledge of both the external and internal structure of the laser-ranged satellites, and of their main dynamic parameters to be used within the orbit models, is of crucial importance. In this work we reconstruct information about the structure, the materials used, and the moments of inertia of the two LAGEOS satellites. The moments of inertia of LAGEOS resulted to be (11.42 ± 0.03) kg m^2 for the cylindrical symmetry axis and (10.96 ± 0.03) kg m^2 for the other two main axes. The analogous quantities for LAGEOS II are (11.45 ± 0.03) kg m^2 and (11.00 ± 0.03) kg m^2. We also built a 3D-CAD model of the satellites structure which is useful for finite element-based analysis. We tried to solve contradictions and overcome several misunderstanding present in the historical literature of the older LAGEOS, carefully reanalyzing the earlier technical papers. To test the results we obtained, we used our moments of inertia to compute the spin evolution of the two satellites obtaining a good agreement between measured and estimated values for the spin direction and the rotational period. We believe we now have accurate knowledge of the mass, moments of inertia, and composition of both LAGEOS satellites.

Keywords: Experimental tests of gravitational theories; Orbit determination and improvement; Satellite orbits; Rotational dynamics; Mass and density; Computer modeling and simulation

---

1. Introduction

The two LAGEOS (LAser GEOdynamics Satellite) are very simple and passive Earth orbiting satellites having a golf ball aspect: almost spherical in shape, their diameter is about 60 cm while their mass is about 400 kg and their aluminum surface is covered with 426 cube-corner retro-reflectors (CCRs) for laser tracking, see Fig. 1. In space technology, the adjective *passive* means that these geodetic satellites have no solar panels to derive power from the Sun, no instruments to perform a direct measurement of a physical quantity, no engines or thrusters for orbital maneuvers or attitude variation and no antennae for radio communications with a ground station.

The CCRs allow a very precise tracking of the satellites' orbit around the Earth by means of the powerful Satellite

---

* Corresponding author at: Istituto Nazionale di Astrofisica (INAF), Istituto di Astrofisica e Planetologia Spaziali (IAPS), Via del Fosso del Cavaliere, 100, 00133 Roma, Italy.
E-mail addresses: massimo.visco@iaps.inaf.it (M. Visco), david.lucchesi@iaps.inaf.it (D.M. Lucchesi).

http://dx.doi.org/10.1016/j.asr.2016.02.006
0273-1177/

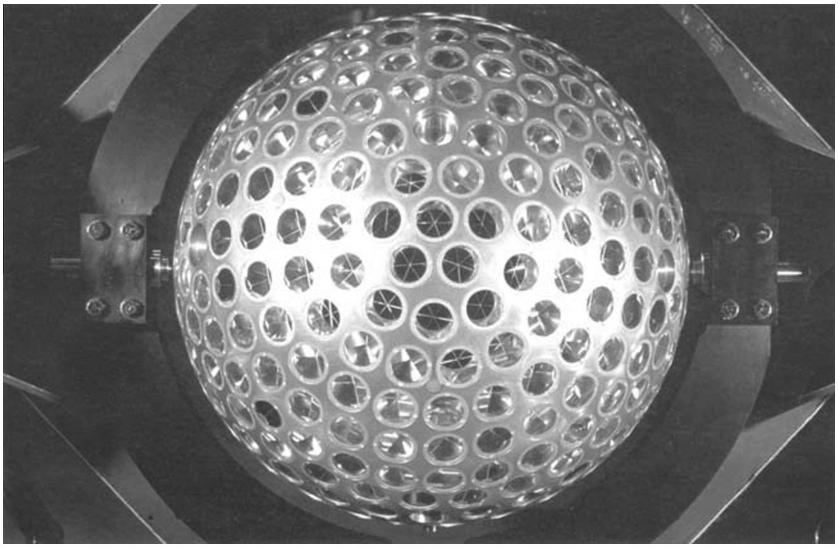


Fig. 1. Picture of the LAGEOS II satellite (courtesy of ASI). Launched by ASI/NASA space agencies at the end of October 1992, LAGEOS II is one of the best tracked satellites all over the word by the SLR technique. LAGEOS II is almost a twin of the older LAGEOS (NASA, 1976). In the case of LAGEOS II, the orbit has an inclination of about 53° over the Earth's equator, a semi-major axis of about 12,163 km and an eccentricity of about 0.014. LAGEOS has an orbit inclination of about 110°, a semi-major axis of about 12,270 km and an eccentricity of about 0.004. The smaller inclination of LAGEOS II has been chosen to obtain a better visibility from the network of the Earth laser ranging stations. The area-to-mass ratio (A/m) of these satellites is quite small in order to minimize the impact of the non-gravitational perturbations. Concerning the CCRs, 422 are made of fused silica while 4 are made of germanium and are disposed in a tetrahedral configuration. Note that the spatial distribution of the germanium CCRs is different for the two LAGEOS.

Laser Ranging (SLR) technique. The ranges provide high quality data gathered by the International Laser Ranging Service (ILRS), see Pearlman et al. (2002). The sub-cm precision of the SLR data allow a careful evaluation of a wide number of geophysical parameters after a data reduction of the satellites orbit through a least-squares fit; for details we refer to Yoder et al. (1983), Rubincam (1984), Cohen and Smith (1985), Smith et al. (1990), Lemoine et al. (1998), Bianco et al. (1998) and Cox and Chao (2002), just to cite a few references in the field of geophysical applications of LAGEOS and LAGEOS II data analysis. The great tracking precision also make the two LAGEOS satellites a powerful test bench for Einstein's theory of general relativity (Einstein, 1916), see for instance (Ciufolini et al., 1996; Ciufolini and Pavlis, 2004; Lucchesi and Peron, 2010; Lucchesi and Peron, 2014).

It is important to stress that presently, with the current level of development of the SLR technique, a further increase in the precision of the measurements of relativistic effects from the gravitational field of the Earth and, more specifically, in the final accuracy of such measurements, is only possible through the development of more accurate dynamical models for the orbit of such geodetic satellites and their subsequent inclusion in the software used for their precise orbit determination (POD). These models have to account for gravitational and non-gravitational perturbations, and their accuracy will impact on the reliability of the estimate of the final error budget in terms of the degree of knowledge of the various systematic error sources at work in a given measurement. An unrefined modeling may prevent a reliable measurement; for instance, a disturbing effect may mask (or even mimic) a relativistic effect. In this regard, another important issue is in the case of possible correlations among the relativistic parameters to be extracted from the data, and those related to classical effects poorly modeled or not modeled at all.

In this context, a crucial role is played by the non-gravitational perturbations (NGPs) which are characterized by subtle effects, very complex to model. These perturbations are due to surface forces which depend on the satellite structure and composition and are responsible for very long term effects in the orbital elements of the two LAGEOS satellites (see for instance Milani et al. (1987) and Andrés de la Fuente (2007) for a detailed, though not complete, discussion). Indeed, the small area-to-mass ratio of the two LAGEOS satellites, about 6.95×10^{-4} m^2/kg, has been chosen to minimize these disturbing accelerations.

Among the plethora of NGPs acting on the two LAGEOS satellites, one main source is due to thermal-drag perturbations, strictly connected to the evolution of the spin vector (orientation and rate) of the satellites, we refer to Bertotti and Iess (1991), Habib et al. (1994), Farinella et al. (1996), Vokrouhlický (1996) and Andrés de la Fuente (2007) for details. These are the Earth-Yarkovsky and Yarkovsky-Schach thermal effects, see Rubincam et al. (1987), Rubincam (1988), Afonso et al. (1989), Farinella et al. (1990), Scharroo et al. (1991), Slabinski (1996), Farinella and Vokrouhlický (1996), Rubincam et al. (1997), Métris et al. (1997), Métris et al. (1999) and Lucchesi (2002).

Indeed, in order to correctly model such effects, besides a detailed knowledge of the physical properties of the elements of the surface of the satellite — such as the absorptance α, the emissivity ϵ, the specific heat C_P and the heat capacity $\mathcal{H}$ — the knowledge of the overall mass distribution of the satellite is important. Through this knowledge, we are able to compute the values of the moments of inertia of the satellite which, in turn, contribute to the spin evolution.

The spin evolution is mainly provoked by the gravitational torque, which is due to the oblateness of the satellite, and by the magnetic torque, which arises because the satellite represents a conductor moving in a magnetic field: the field induces eddy currents (Foucault currents), thence a magnetic moment that in turn interacts again with the external geomagnetic field and produces the magnetic torque.

Therefore, the equation of motion for the rotational dynamics cannot be written without knowing the moments of inertia of the two LAGEOS satellites. These parameters were indirectly estimated but, unfortunately, not measured before the launch of the satellites. Of course, this is not surprising, as it was hard to imagine — at the epoch of launch of the two satellites — the great increase in the tracking precision obtained afterward with its consequent potential applications, as well as its remaining in full efficiency after so many years.

It is worth mentioning that when the satellites were injected in their orbit, a spin was induced in their motion: in the case of the older LAGEOS the initial rotational period was about 0.6 s, while for LAGEOS II it was about 1 s. It was expected, at the epoch of LAGEOS launch, that the spin rate would be fully decayed within a very short time (one year as written at page 3–2 of NASA (1975) or two years according to Johnson et al. (1976)).

In this paper we are interested in re-visiting the state of the art knowledge of the main dynamic parameters of the two LAGEOS satellites. On the one hand, let us say from the historical point of view, we were motivated to search for the cause (or the causes) of the contradictory information provided in the literature on some of these parameters. On the other hand, let us say for more meaningful practical reasons, we were motivated by the attempt to determine these parameters with the purpose of improving the dynamic models of the two LAGEOS satellites with respect to their current knowledge, as specified above. In particular, the construction of a refined three-dimensional (3D-CAD) model for the structure of the two satellites is of primary importance for a refined and reliable modeling of their thermal behavior based on finite element method.

This activity falls in the LARASE research program. The LAser RAnged Satellites Experiment (LARASE) aims to provide an original contribution in testing and verifying relativistic physics by means of the very precise measurements provided by the SLR technique together with a POD of a dedicated set of passive laser-ranged satellites. A baseline prerequisite for the successful outcome of this goal is represented by the availability of reliable dynamical models for the orbit of the considered satellites. Therefore, LARASE aims to improve the dynamical models of the currently best laser-ranged satellites, with special attention to the subtle non-gravitational forces, see Lucchesi et al. (2015a,b) for details.

Beside the two LAGEOS, also the new LARES (LAser RElativity Satellite) satellite will be subject of our investigations. LARES was launched by ASI on February 13, 2012, and its main objective is to provide a new and refined measurement of the Lense-Thirring effect, see Ciufolini et al. (2009) and Paolozzi and Ciufolini (2013) for details.

The enhancements in the reliability of the dynamical models of the orbit for these laser-ranged satellites will help also to reduce the usage of the so-called empirical accelerations. These accelerations are generally exploited to absorb for effects not currently accounted for by the overall dynamical model, as in the case of some NGPs and, more specifically, for the thermal-drag effects, at least in the case of the two LAGEOS.

Therefore, these studies, by contributing in improving the POD precision and accuracy, thanks to a reduction of possible systematic errors in the modeling and, consequently, in the differential correction procedure, will also contribute significantly in the improvements of all geophysical products of the ILRS.

For instance, the SLR solutions for the Earth Orientation Parameters (EOP) and the tracking stations position and velocity (which are derived from the orbit of a few selected satellites, among which the two LAGEOS), play a strong role in the practical realization of a reference system fixed to the Earth, the International Terrestrial Reference Frame (ITRF) which has its origin in the Earth's center of mass (the so-called geocenter), i.e., the point around which an Earth satellite orbit.

The rest of the paper is organized as follows. In Section 2, the apparent confusion in the knowledge about the mass and moments of inertia of the two LAGEOS satellites is described as given in the literature. In Section 3, starting from the original drawings of the satellites, we reconstructed the dimensions of the two LAGEOS and the materials used to build their internal structure. In Section 4, still by the use of the original drawings of the satellites, we describe the construction of a 3D-CAD model for both LAGEOS and LAGEOS II. This allowed us to independently estimate mass and moments of inertia of the two LAGEOS satellites and get what we believe are accurate values. In Section 5 the computed moments of inertia are used in a program to simulate the spin direction evolution as further test of their compatibility with measurements. Finally, in Section 6 our conclusions and recommendations are provided.

2. Mass and moments of inertia in the literature

Among the main parameters needed to model the dynamic behavior of a satellite we have to consider its mass, dimensions, center of mass position, and moments of inertia. Ideally, these physical quantities should be accurately measured before the launch of a satellite but, unfortunately, for the satellites we are considering, some of these measurements are absent or are characterized by a great uncertainty.

As underlined in the previous Section, the NGPs are proportional to the satellite area-to-mass ratio; consequently its value must be small to reduce orbit perturbations and very well known in order to model these perturbations properly.

The most likely values for the masses of the two LAGEOS satellites are those reported in the IRLS website[1] 406.965 kg for LAGEOS and 405.38 kg for LAGEOS II. The same value for the mass of LAGEOS II is reported by Cogo (1988) and by Fontana (1990). In these reports, the mass of LAGEOS II was computed as sum of the masses of the different parts of the satellite.

Unfortunately, in the case of the older LAGEOS satellite a clear reference to the value of its mass is not easy to find. In 1974, the baseline value for the mass of LAGEOS was 385 kg, successively increased to 411 kg as a result of launch vehicle modifications — which included the addition of a 4th-stage apogee-kick motor — as written by Siry (1975). The effective mass predicted for the flight model was 409.8 kg, as written in Section 5.2.1 of NASA (1975).

In the official documentation that we have been able to collect, it was not possible to find a direct report of the weighing of LAGEOS after the final assembly of the satellite. This weighing was surely performed, as it is documented by a photo at page 201 of Wong (1978). Unfortunately, we think that the author of this paper did not have direct access to information about the mass; she reported a value of 410.9 kg, higher than the most probable one and close to that of LAGEOS Press Kit (1976), i.e. 411 kg. The value of 411 kg is also reported by Kolenkiewicz et al. (1977), Fitzmaurice et al. (1977), Smith and Dunn (1980) and Rubincam (1982).

In Johnson et al. (1976) a value of 407.821 kg is provided for the mass of LAGEOS (one of the authors, Charles W. Johnson was the LAGEOS project manager at NASA-MSFC), very close to the more likely value, but Cohen and Smith (1985) are the first to provide the value of 407 kg for the mass of the satellite. We recall that David E. Smith was the project scientist of LAGEOS, and the quoted value for the mass of LAGEOS is also the most cited in the literature during the subsequent years. Indeed, afterwards this value was cited by Rubincam and Weiss (1986), Rubincam (1987), Rubincam et al. (1987) and Rubincam (1988).

From a chronological point of view the value of 406.965 kg for the mass of LAGEOS — the same as the one presently given by the ILRS website[1] — appears firstly in Smith et al. (1990) but referring to the previous work by Cohen and Smith (1985). Therefore, we believe that the value of 407 kg represents the rounding of 406.965 kg, where the significant digits are not connected to the accuracy of measurements but they came as sum of the weighing of the different components or from a conversion (not rounded) between pounds and kilograms.

Moreover, the Table IV of Slabinski (1996) — which cites in the final Reference list (Cohen and Smith, 1985) but not (Smith et al., 1990) — reports the value as 406.965 kg, but with a note (Note *f*) that reads [Mass from project engineer supplied by David E. Smith (GSFC)].

The situation is slightly more difficult when we are interested to the knowledge of the center of mass position and to the moments of inertia of the satellites.

In the NASA (1975) report the authors wrote that it had been decided not to measure the position of the center of mass and of the moments of inertia of the flight model of LAGEOS because they trusted the estimated values on the basis of the experience made on the balance model of LAGEOS.

In the same report the authors concluded that the 1 mm center of geometry to center of mass limit could not be exceeded if manufacturing tolerances were within specification and the predicted moments of inertia could be applied.

For the flight model of LAGEOS II the position of its center of mass was measured by Fontana (1990), we report his values in Table 4. We refer to Section 4.0 of this technical note for further details.

With regard to the moments of inertia of the satellites, none of them were measured for their flight model. Measurements were made only on the LAGEOS test model and on the structure of LAGEOS II without retroreflectors; see, respectively, Section 5.2.2 of NASA (1975) and Section 4.0 of Fontana (1989).

3. Dimensions and materials of the two satellites

We verify the estimates made in the past by computing the moments of inertia of the two satellites with numerical methods by the use of three-dimensional 3D-CAD models to overcome the lack of measurements.

The quality of such a work depends on the level of knowledge of the geometry of the satellites and on the density of their materials. A first obstacle was the lack of working drawings for LAGEOS and a great confusion present in the literature about shapes and materials used. On the other hand, the working drawings of LAGEOS II were available in Minott et al. (1993), in particular we refer to Section C of this technical note.

Anyway, the available information were equally sufficient because we reached the certainty that the two satellites were built using almost identical working drawings and that they differ only by the manufacture tolerances and the materials alloy.

---

[1] http://ilrs.gsfc.nasa.gov/missions/satellite_missions/current_missions/lag1_general.html.

The main evidence of the use of the same project for the two satellites comes from Cogo (1988). In this technical note the author on one hand declares to have access to LAGEOS working drawings[2] and on the other hand explains the measured mass differences on the basis of mechanical tolerances and the different alloys used in the two projects. The CCRs were also considered, but the conclusion about them was that they do not bring any significant contribution to the mass differences. If there were any small change in the working drawings of the two satellites, this would have been definitely pointed out by the author.

Furthermore, the title blocks of the LAGEOS II working drawings contain a reference to an "original" drawing ("originale" in Italian). The numbers there reported, whenever it is possible to check, are the same of those reported in the working drawings of LAGEOS. For example, LAGEOS drawings "30M20456–1" and "30M20457-1" relative to the brass core mass and to the tension stud, cited in flux diagrams at pages 7–4 and 6–1 of NASA (1975), appear also in the text block of the LAGEOS II working drawings at pages 1–7 and 1–8 of Minott et al. (1993).

Above these evidences, we tried to understand the origin of the uncertainties regarding size, shape and mass of the different parts of the satellites present in the literature.

We guess that the possible origin of many errors is related with the drawing apparently introduced for the first time in LAGEOS Press Kit (1976), see page 10 of that technical note. In that drawing of LAGEOS, sizes and mass are inconsistent with the density of the material indicated. In addition, Wong (1978) presents a photo of the internal structure of the test model of LAGEOS, that should be identical to the flight model except for the CCRs installations (see NASA, 1975). Very interestingly, the dimensions that can be reconstructed from the photo simply using a ruler, are absolutely incompatible with those reported in the cited drawing of LAGEOS Press Kit (1976).

We guess that this drawing refers to a preliminary model of LAGEOS, and that many authors have tried to find a consistency between sizes and masses of the different parts assuming different materials with respect to those actually used.

A long debate arose around the material used for the core mass and the threaded stud. In the drawing at page 10 of LAGEOS Press Kit (1976) is written that both are made of brass. Johnson et al. (1976) corrected this drawing, we believe wrongly, affirming that the core is made of copper beryllium and the stud of brass.

We believe that the description of materials and working procedures contained at page 6–2 in NASA (1975) can help us to overcome any doubt about the materials used. The description clearly refers more than once to a brass core weight and to a copper beryllium shaft. The descriptions of the machining and mounting procedures are so precise and detailed that it appears unlikely that the authors of this technical note were wrong about the materials.

The apparent error made on the core material by Johnson et al. (1976) was not echoed by others, being a general agreement about brass (see for instance Cohen and Smith, 1985) until Rubincam et al. (1987) refer again to Johnson et al. (1976) to state that the core of LAGEOS was made of copper beryllium. This information was also iterated by Slabinski (1996) who made a careful reconstruction of the materials, dimensions, masses and density of the satellite.

As pointed out by Andrés de la Fuente (2007), the analysis made by Slabinski (1996) brings to a contradiction between measured and computed moments of inertia. In his PhD thesis, Andrés concludes that either the size values reported in LAGEOS Press Kit (1976) and Cohen and Smith, 1985 were wrong or the materials given were not correct. He reports also that through a private communication from Slabinski, he had known that Métris in 1996 had reached the same conclusion even if he had not published anything about it.

The accuracy of the conclusion that we reached about materials and shapes needs to be validated by computing the masses and the moments of inertia of the test configuration of the satellites that have been measured and reported in the technical papers previously described. As it will be shown in the next Section, our results have proven to be very robust under this point of view.

4. 3D-CAD models of the two satellites

Using the working drawings of LAGEOS II available in Minott et al. (1993) and the information about materials by Cogo (1988), we built a complete 3D-CAD model of the two satellites.[3] A section view of the two satellites from our CAD model is shown in Fig. 2, where the main parts of the structure are visible: (i) two hemispheres of aluminum containing the CCRs, (ii) the brass core that contribute to increase the mass of the satellite, (iii) the copper beryllium shaft that allows to fasten the different parts of the satellites.

It is interesting to compare our Fig. 2 with Fig. 1 of Cohen and Smith (1985). The two figures differ for the internal dimensions and for the material of the shaft: copper beryllium vs brass.

Conversely, in Fig. 3 a CCR is shown with its three rings: a retainer ring (RR) and two mounting rings. The two mounting rings are those immediately in contact with the CCR, they are called upper ring (UR) and lower ring (LR).

With regard to the density of the materials, as a first approach we adopted for the different materials the density

---

[2] Indeed, on page 4 of Cogo (1988) it is explicitly stated [LAGEOS 1 drawings set (issued by NASA-MSFC)].

[3] We take advantage of SOLIDWORKS® 3D-CAD software and its capability to evaluate 3D model solid mass properties.

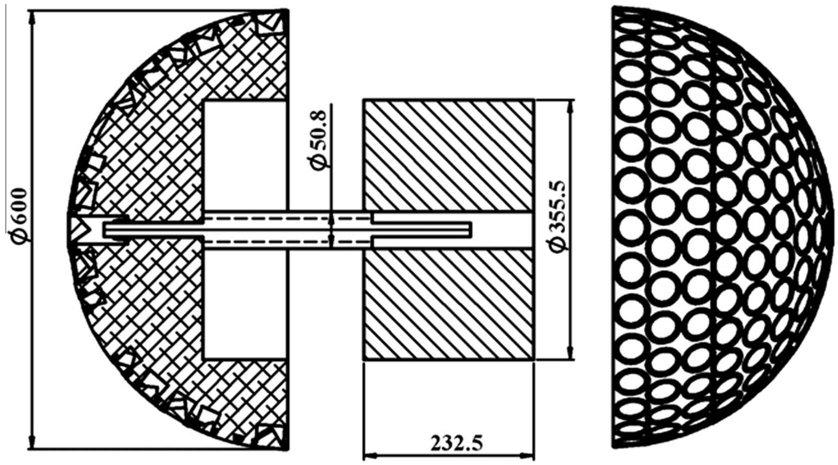
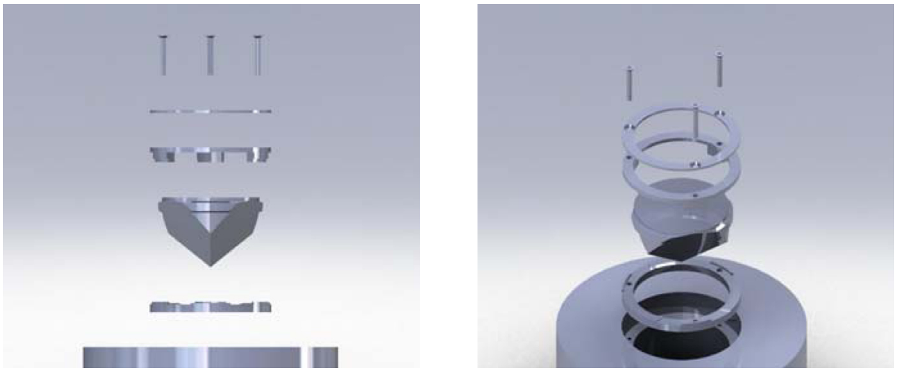


Fig. 2. The LAGEOS satellites assembly. The dimensions are in mm. The two aluminum hemispheres are shown with the section of the cavities containing the CCRs together with the internal brass cylinder and Cu-Be shaft.



(a) Orthogonal view (b) Perspective view

Fig. 3. A cube corner retroreflector (CCR) of the LAGEOS satellites: exploded view of our engineering model from two different points of view. The two mounting rings are machined from clear transparent PolyChloroTrifluoroEthylene (PCTFE), also known as KEL-F. Conversely, the retainer ring and the three screws are made of aluminum, the same of the hemispherical shells. The screws are used to fasten the entire assembly to the aluminum surface of the satellite. In our computations of the satellites' moments of inertia, the geometry and masses of the entire CCRs assembly elements were considered using the values provided in Minott et al. (1993).

values shown in Table 1 that, a priori, are the most likely ones. Hereinafter, these densities will be identified as nominal densities.

With the density values shown in Table 1 and the dimensions valid for the LAGEOS II satellite (see Fig. 2 or Minott et al., 1993) we obtained masses values close enough to those measured for both the satellites: 405.93 kg for LAGEOS (vs. 406.965 kg) and 404.97 kg for LAGEOS II (vs. 405.38 kg). As we can see, the fractional discrepancy is very small for both satellites, about $3 \cdot 10^{-3}$ in the case of LAGEOS and about $1 \cdot 10^{-3}$ for LAGEOS II. These results convince us that our hypothesis

Table 1
Materials used for the construction of the two LAGEOS satellites (Cogo, 1988) and their nominal densities.

Satellite	Material density ρ_n (kg/m^3)		
	Hemispheres	Core	Stud
LAGEOS	AA6061 2700[a]	QQ-B-626 COMP.11 8440[a]	Cu-Be 8230[b]
LAGEOS II	AlMgSiCu UNI 6170 2740[c]	PCuZn39Pb2 UNI 5706 8280[c]	Cu-Be QQ-C-172 8250[c]

[a] ASM International Handbook Committee (1990).
[b] Bauccio (1993).
[c] It is the value calculated in Cogo (1988) starting from the measured averaged composition.

about the internal sizes of the LAGEOS satellite are correct.

As a further check we calculated masses and moments of inertia for the two configurations of the satellites different from the flight model, whose parameters were measured and reported in the literature. The results are shown in Table 2.

We considered four cases on the basis of the previously discussed (and available) documentation: two cases focus on the flight arrangement of the two satellites, the other two cases concern the balance model of LAGEOS and the LAGEOS II model with no CCRs. In particular, we computed the moments of inertia in two different ways: (i) using the nominal density ρ_n for the various materials, see Table 1, (ii) by the use of a normalized density $\bar{\rho}$ which corrects the nominal density ρ_n by a factor equal to the ratio of the measured mass M_m for the two satellites with the mass M_c we computed using the nominal densities, that is:

$$\bar{\rho} = \rho_n \cdot \frac{M_m}{M_c}. \tag{1}$$

In this way, we aim to increase the precision of our model by modifying with the same percentage the density of the materials of each satellite in order to better fit to its measured mass M_m.

In the case of the LAGEOS flight arrangement our computed moments of inertia are in very good agreement with those computed in the technical note NASA (1975), differing by about 1% only. In the case of the LAGEOS balance model, the differences among our computed moments of inertia with those measured are even smaller, with a maximum discrepancy of about 0.3% and a minimum discrepancy of about 0.1%.

These results are well within the expected errors for the moments of inertia. Obviously, since the less significant digit is the second one, the order-of-magnitude of the error should be ≈ 0.01 kg m^2. To estimate an upper bound to the error to be attributed to the moments of inertia we had computed, we have modified dimensions and densities of the satellites within reasonable ranges. Using this method we arrived to give an error to our estimation of the moments of inertia of about $3 \cdot 10^{-2}$ kg m^2. This error takes also in account the change that can appear in the moments of inertia for thermal expansion of the different parts of the satellite.

For the flight arrangement of LAGEOS II our computed moments of inertia are in very good agreement with those computed by Fontana (1990); in fact they differ by less than 0.2%. In the case of the LAGEOS II test model, i.e. the one with no CCRs, the disagreement between our computations and the measurements is about 10%; however, our computed values are very close to those computed by Fontana (1989), with a maximum discrepancy $\lesssim 0.3\%$. We point out that the measurements of the moments of inertia of the test model of LAGEOS II appear only in the first version of Fontana (1989). Consequently, we can imagine either that the author was not so confident about the measurements or he believed to have an error so large as to consider the measurements obtained not so significant.

Table 2

Comparison of masses and moments of inertia for the two LAGEOS satellites. In the notation we follow NASA (1975). The x axis coincides (nominally) with the principal axis of inertia (the angle between the symmetry axis and the principal axis orientation was bound to be below 0.02 radians). Practically, this axis coincides with the initial rotation axis of the satellites.

Satellite origin of value	Mass (kg)	Moments of inertia (kg m^2)		
	M	I_{xx}	I_{yy}	I_{zz}
LAGEOS flight arrangement				
Computed value in NASA (1975)	409.8	11.516	11.084	11.084
Measured value in NASA (1975)	406.965	–	–	–
Values computed in the present work using nominal density of Table 1	405.93	11.40	10.93	10.93
LAGEOS balance model				
Computed value in NASA (1975)	440.3	13.14	12.71	12.71
Measured value in NASA (1975)	440.0	13.11	12.69	12.71
Value computed in the present work using nominal density of Table 1	437.68	13.09	12.62	12.62
Values computed in the present work using normalized density	440.00	13.16	12.68	12.68
LAGEOS II flight arrangement				
Computed values in Fontana (1990)	–	11.45	11.00	11.00
Measured value in Fontana (1990), Fontana (1989) and Cogo (1988)	405.38	–	–	–
Values computed in the present work using nominal density of Table 1	404.97	11.44	10.99	10.99
LAGEOS II without CCRs				
Computed value in Fontana (1989)	386.59	10.39	9.95	9.95
Measured value in Fontana (1989)	387.20	9.67	9.37	9.15
Values computed in the present work using nominal density of Table 1	386.71	10.41	9.95	9.95
Values computed in the present work using normalized density	387.20	10.42	9.96	9.96

Moreover, in the case of the flight arrangement of the two satellites, we corrected the computed moments of inertia for the normalized density $\bar{\rho}$. Using this method we find the most likely values for the moments of inertia which should be used in the future (from our point of view) for new analyses, see Table 3.

As we can see, in the case of LAGEOS II our estimated values for the moments of inertia are coincident with those computed by Fontana (1990), while for LAGEOS we have a discrepancy of just 0.8% for I_{xx} and 1% for I_{yy} and I_{zz} with respect to those computed in NASA (1975).

Finally, it is not useful to use our complete 3D-CAD models to evaluate the distance between the center of mass and the geometric center for the two satellites. In fact, these quantities do not depend on the design of the satellite, but derive mainly from the asymmetries introduced by the machining. We report in Table 4 the values for these quantities in the available literature.

5. Moments of inertia and spin evolution

In this Section we show how the previously calculated values for the dynamical parameters can be inserted in a model for the evolution of the spin direction, obtaining values well in agreement with the available observations.

By developing dedicated MATLAB routines, we built an environment able to simulate the spin evolution for LAGEOS-like satellites. This software simulates the spin evolution with different input models. Its applicability is not only restricted for values of the spin period much lower than the orbital period of the satellite (the so-called rapid-spin approximation) but it is also valid in the general case.

In order to verify the reliability of the calculated values for the moments of inertia, we have calculated the spin evolution for LAGEOS II in the rapid-spin approximation. Our model includes the gravitational torque due to the oblateness of the satellite, the magnetic torque due to the eddy currents induced in the rotating satellite (the equations used are those in Bertotti and Iess (1991) and Farinella et al., 1996 with some small corrections) and the torques due to the reflection asymmetry and the non coincidence between the center of mass and the geometrical center of the satellite (Vokrouhlický, 1996; Andrés de la Fuente, 2007; Lucchesi, 2003; Lucchesi, 2004). In Fig. 4, the evolution for the orientation of LAGEOS II spin vector is plotted.

The moments of inertia used in the calculations are those estimated by means of our work and reported in Table 3. As we can see, the agreement between model and observations, i.e. the measured values of the spin orientation in the inertial space, is remarkable and in line with the results provided by the LOSSAM model, see Fig. 3.18 of Andrés de la Fuente (2007).

To assess the sensitivity of our spin model to changes in the values of the moments of inertia, we modified them in the simulation within the errors we evaluated and reported in Table 3. In particular we calculated the spin orientation evolution using the maximum and minimum oblateness within the estimated errors. The result of this analysis is reported in Fig. 5.

Although other experimental parameters can equally effect the spin behavior, it is evident that an a priori estimation of the moments of inertia brings a big advantage in modeling the dynamics of a satellite, reducing the number of free parameters used in the model. This topic will be widely discussed in a paper in preparation.

Table 3
Mass and moments of inertia of LAGEOS and LAGEOS II to be used in the future. The masses are the one measured. The moments of inertia are those computed in the present work with normalized densities.

Satellite	Mass (kg)	Moments of inertia (kg m^2)		
	M	I_{xx}	I_{yy}	I_{zz}
LAGEOS flight arrangement	406.97	11.42 ± 0.03	10.96 ± 0.03	10.96 ± 0.03
LAGEOS II flight arrangement	405.38	11.45 ± 0.03	11.00 ± 0.03	11.00 ± 0.03

Table 4
Distance (offset) between the center of mass and the geometric center.

Satellite	Center of mass (mm)		
	X	Y	Z
LAGEOS (estimated)[a]	<1	<1	<1
LAGEOS II (measured)[b,c]	−0.053 ± 0.025	0.018 ± 0.040	0.008 ± 0.065

[a] NASA (1975).
[b] Fontana (1990).
[c] Fontana (1989).

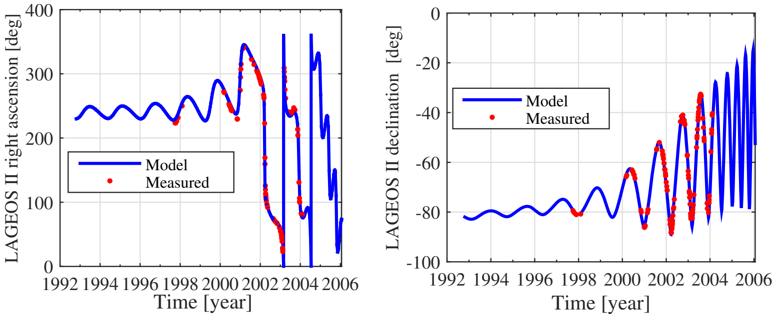


Fig. 4. LAGEOS II spin vector orientation in the J2000 inertial reference frame. Our values for the moments of inertia of LAGEOS II have been used and included in our spin model of the satellite in the so-called rapid-spin approximation. The blue line represents the model behavior while the red dots represent the measured values (i.e. the observations) of the spin orientation. The spin was directly measured up to a few years ago by means of a spectral analysis of the SLR data, see Bianco et al. (2001) and Kucharski et al. (2013), and for the three years between 2000–2002 using a photometric 'flash' technique by Otsubo et al. (2004). We refer to Chapter 3 and Appendix D of Andrés de la Fuente (2007) for further details. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

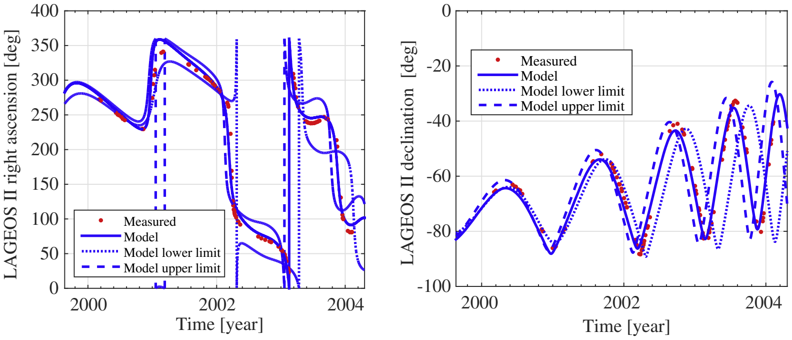


Fig. 5. LAGEOS II spin vector orientation in the J2000 inertial reference frame. The spin behavior shown in Fig. 4 has been redrawn over a smaller temporal interval (continuos line) and compared with the behavior that we obtained (still in the rapid-spin approximation) by varying the moments of inertia within their errors reported in Table 3 in such a way to vary the oblateness of the satellite between its corresponding lower and upper limit.

6. Conclusions and recommendations

As briefly highlighted in Section 1, the construction of a refined model for the structure of a satellite like LAGEOS is of primary importance for a refined and reliable modeling of its thermal behavior.

This is only one very important issue, among the several, that is necessary to consider in order to model more accurately the disturbing effects of thermal origin, such as the Yarkovsky-Schach and Earth-Yarkovsky thermal thrust perturbations (Rubincam et al., 1987; Rubincam, 1988; Afonso et al., 1989; Farinella et al., 1990; Scharroo et al., 1991; Slabinski, 1996; Farinella and Vokrouhlický, 1996; Rubincam et al., 1997; Métris et al., 1997; Métris et al., 1999; Lucchesi, 2002).

For instance, a characterization of the thermal properties of the various elements of the satellite — both internal and external — as well as the availability of a general model for its spin evolution — both in orientation and rate — are two additional issues of extreme importance (see Slabinski, 1996 and Andrés de la Fuente, 2007).

For these reasons, in the context of the LARASE activities (see Lucchesi et al., 2015a,b), we started an in-depth review of the dynamic parameters of the two LAGEOS satellites. More specifically, we began our work in this field from the analysis of the original drawings of the LAGEOS II satellite, see Minott et al. (1993).

This is the first paper which deals with the results we have obtained within these activities. Our results on the most significant dynamic parameters of LAGEOS and LAGEOS II can be summarized as below:

1. initially, as a first step, we reviewed the technical documentation and the main papers regarding the dimensions and masses of the satellites;
2. after studying the original drawings of the LAGEOS II satellite, we had the confirmation that these were exactly the same of those of the older LAGEOS satellite;
3. the two satellites are practically twins: they only differ — besides the well known different (tetrahedral) distribution of the four germanium CCRs — by manufacture tolerances and densities of the alloys used, and this is sufficient to explain a mass difference of about 1.6 kg between the two;
4. in particular, the internal cylinder is made of brass while the shaft is made of copper beryllium for both satellites;

5. we built a 3D-CAD model of the two satellites in the different configurations and using numerical methods we computed their masses and moments of inertia, see Table 2;
6. we propose as the best fitting values for the masses and moments of inertia of the satellites those reported in Table 3.

Indeed, a major product of this study has been the determination of the moments of inertia of the two LAGEOS satellites from their 3D model. Unfortunately, such physical parameters were not measured on the flight models and their values, similar to our approach, were simply computed by the engineers of NASA and AERITALIA, respectively for LAGEOS and LAGEOS II.

A great deal in this direction was performed also by Andrés de la Fuente (2007) in his PhD thesis. Thanks to the LOSSAM spin model, he has been able to estimate the moments of inertia of LAGEOS and he correctly concluded that the core of LAGEOS is made of brass as that of LAGEOS II. However, Andrés attributed the differences between the two satellites to their different internal sizes (see for instance Section 3.5.3 of Andrés de la Fuente (2007)).

We have also shown that the dimensions are not different for the two LAGEOS; the satellites' differences are due, as stated, to the tolerances in their manufacture and in the slightly different densities of the alloys.

As stated above, this activity falls in those of the LARASE program. These activities are focused on improving the dynamic models of the current best laser-ranged satellites in order to provide new and more refined measurements of relativistic physics in the field of the Earth, see Lucchesi et al. (2015a). In particular, we are interested in improving the knowledge of the dynamics of the two LAGEOS satellites as well as that of LARES. Such improvements are also very important for applications of these satellites in the fields of geophysics and of space geodesy.

We have developed a new model for the spin evolution of LAGEOS-like satellites, not restricted to the rapid-spin case — that is the one we briefly discussed in Section 5 — but valid for any value of the rotational period of the satellite. In this context, the 3D-CAD model of a satellite is particularly useful in comparing the analytic predictions with the numerical ones about the induced eddy currents, and the corresponding magnetic moment, as a function of the rotation frequency in the presence of the magnetic field of the Earth.

These and other aspects will be discussed in detail in a forthcoming paper, currently under preparation.

Acknowledgment

This work has been in part supported by the Commissione Scientifica Nazionale II (CSNII) on astroparticle physics experiments of the italian Istituto Nazionale di Fisica Nucleare (INFN) under the LARASE program. Special thanks for help in providing some of the documentation to L. Anselmo, C. Portelli, to P. Messidoro and in particularly to V.J. Slabinski. We are grateful to D.P. Rubincam for an accurate reading of the manuscript and for many useful suggestions. We express our gratitude to G. Bianco and V. Luceri for supporting our research. We thank G. Chionchio and his staff of Area della Ricerca di Tor Vergata - CNR for providing computation power and instruments. Finally, we thank P. Willis and three anonymous reviewers for their helpful comments and suggestions.

References

Afonso, G., Barlier, F., Mignard, F., Carpino, M., Farinella, P., 1989. Orbital effects of LAGEOS seasons and eclipses. Ann. Geophys. 7, 501–514.

Andrés de la Fuente, J.I., 2007. Enhanced Modelling of LAGEOS Non-Gravitational Perturbations (Ph.D. thesis). Delft University Press. Sieca Repro, Turbineweg 20, 2627 BP Delft, The Netherlands.

ASM International Handbook Committee, 1990. Metals Handbook, vol. 2 – Properties and Selection: Nonferrous Alloys and Special-Purpose Materials. ASM International 10th Ed.

Bauccio, M. (Ed.), 1993. ASM Metals Reference Book. ASM International, Material Perk OH.

Bertotti, B., Iess, L., 1991. The rotation of LAGEOS. J. Geophys. Res. 96, 2431–2440. http://dx.doi.org/10.1029/90JB01949.

Bianco, G., Chersich, M., Devoti, R., Luceri, V., Selden, M., 2001. Measurement of lageos-2 rotation by satellite laser ranging observations. Geophys. Res. Lett. 28, 2113–2116. http://dx.doi.org/10.1029/2000GL012435.

Bianco, G., Devoti, R., Fermi, M., Luceri, V., Rutigliano, P., Sciarretta, C., 1998. Estimation of low degree geopotential coefficients using SLR data. Planet. Space Sci. 46, 1633–1638. http://dx.doi.org/10.1016/S0032-0633(97)00215-8.

Ciufolini, I., Lucchesi, D., Vespe, F., Mandiello, A., 1996. Measurement of dragging of inertial frames and gravitomagnetic field using laser-ranged satellites. Nuovo Cim. A 109, 575–590. http://dx.doi.org/10.1007/BF02731140.

Ciufolini, I., Paolozzi, A., Pavlis, E.C., Ries, J.C., Koenig, R., Matzner, R. A., Sindoni, G., Neumayer, H., 2009. Towards a one percent measurement of frame dragging by spin with satellite laser ranging to LAGEOS, LAGEOS 2 and LARES and GRACE gravity models. Space Sci. Rev. 148, 71–104. http://dx.doi.org/10.1007/s11214-009-9585-7.

Ciufolini, I., Pavlis, E.C., 2004. A confirmation of the general relativistic prediction of the Lense-Thirring effect. Nature 431, 958–960. http://dx.doi.org/10.1038/nature03007.

Cogo, F., 1988. Weight discrepancy analysis between LAGEOS 1 and LAGEOS 2 satellites. Technical Report LG-TN-AI-035. Aeritalia.

Cohen, S.C., Smith, D.E., 1985. LAGEOS scientific results – Introduction. J. Geophys. Res. 90, 9217–9220. http://dx.doi.org/10.1029/JB090iB11p09217.

Cox, C.M., Chao, B.F., 2002. Detection of a large-scale mass redistribution in the terrestrial system since 1998. Science 297, 831–833. http://dx.doi.org/10.1126/science.1072188.

Einstein, A., 1916. Die grundlage der allgemeinen relativitätstheorie. Ann. Phys. 354, 769–822. http://dx.doi.org/10.1002/andp.19163540702.

Farinella, P., Nobili, A.M., Barlier, F., Mignard, F., 1990. Effects of thermal thrust on the node and inclination of LAGEOS. Astron. Astrophys. 234, 546–554.

Farinella, P., Vokrouhlický, D., 1996. Thermal force effects on slowly rotating, spherical artificial satellites-I. Solar Heating Plan. Space Sci. 44, 1551–1561. http://dx.doi.org/10.1016/S0032-0633(96)00073-6.

Farinella, P., Vokrouhlicky, D., Barlier, F., 1996. The rotation of LAGEOS and its long-term semimajor axis decay: a self-consistent solution. J. Geophys. Res. 101, 17861–17872.

Fitzmaurice, M.W., Minott, P.O., Abshire, J.B., Rowe, H.E., 1977. Prelaunch Testing of the Laser Geodynamic Satellite. Technical Report TP-1062. NASA.

Fontana, F., 1989. Physical properties of LAGEOS II satellite. Technical Report LG-TN-AI-037. Aeritalia.

Fontana, F., 1990. Physical properties of LAGEOS II satellite. Technical Report LG-TN-AI-037. Aeritalia.

Habib, S., Holz, D.E., Kheyfets, A., Matzner, R.A., Miller, W.A., Tolman, B.W., 1994. Spin dynamics of the LAGEOS satellite in support of a measurement of the Earth's gravitomagnetism. Phys. Rev. D 50, 6068–6079. http://dx.doi.org/10.1103/PhysRevD.50.6068.

Johnson, C.W., Lundquist, C.A., Zurasky, J.L., 1976. The LAGEOS satellite. In: Anaheim International Astronautical Federation Congress.

Kolenkiewicz, R., Smith, D.E., Rubincam, D.P., Dunn, P.J., Torrence, M. H., 1977. Polar motion and Earth tides from laser tracking. R. Soc. London Philos. Trans. Ser. A 284, 485–493. http://dx.doi.org/10.1098/rsta.1977.0024.

Kucharski, D., Lim, H.C., Kirchner, G., Hwang, J.Y., 2013. Spin parameters of LAGEOS-1 and LAGEOS-2 spectrally determined from Satellite Laser Ranging data. Adv. Space Res. 52, 1332–1338. http://dx.doi.org/10.1016/j.asr.2013.07.007.

LAGEOS Press Kit, 1976. NASA (1976) Project LAGEOS Press Kit release 76/67. Technical Report 76/67. NASA. National Aeronautics and Space Administration, Washington, DC.

Lemoine, F.G., Kenyon, S., Factor, J.K., Trimmer, R.G., Pavlis, N.K., Chinn, D.S., Cox, C.M., Klosko, S.M., Luthcke, S.B., Torrence, M. H., Wang, Y.M., Williamson, R.G., Pavlis, E.C., Rapp, R.H., Olson, T.R., 1998. The Development of the Joint NASA GSFC and the National Imagery and Mapping Agency (NIMA) Geopotential Model EGM96. Technical Paper 206861. NASA.

Lucchesi, D., Anselmo, L., Bassan, M., Pardini, C., Peron, R., Pucacco, G., Visco, M., 2015a. Testing the gravitational interaction in the field of the Earth via satellite laser ranging and the Laser Ranged Satellites Experiment (LARASE). Class. Quantum Grav. 32, 155012. http://dx.doi.org/10.1088/0264-9381/32/15/155012.

Lucchesi, D., Peron, R., Visco, M., Anselmo, L., Pardini, C., Bassan, M., Pucacco, G., 2015b. Fundamental physics in the field of the Earth with the laser ranged satellites experiment (LARASE). Metrology for Aerospace (MetroAeroSpace). IEEE, pp. 71–76. http://dx.doi.org/10.1109/MetroAeroSpace.2015.7180629.

Lucchesi, D.M., 2002. Reassessment of the error modelling of non-gravitational perturbations on LAGEOS II and their impact in the Lense-Thirring derivation-part II. Planet. Space Sci. 50, 1067–1100. http://dx.doi.org/10.1016/S0032-0633(02)00052-1.

Lucchesi, D.M., 2003. The asymmetric reflectivity effect on the LAGEOS satellites and the germanium retroreflectors. Geophys. Res. Lett. 30, 1957. http://dx.doi.org/10.1029/2003GL017741.

Lucchesi, D.M., 2004. LAGEOS satellites germanium cube-corner-retroreflectors and the asymmetric reflectivity effect. Celest. Mech. Dyn. Astron. 88, 269–291. http://dx.doi.org/10.1023/B:CELE.0000017171.78328.f1.

Lucchesi, D.M., Peron, R., 2014. LAGEOS II pericenter general relativistic precession (1993–2005): Error budget and constraints in gravitational physics. Phys. Rev. D 89, 082002. http://dx.doi.org/10.1103/PhysRevD.89.082002.

Lucchesi, D.M., Peron, R., 2010. Accurate measurement in the field of the earth of the general-relativistic precession of the LAGEOS II pericenter and new constraints on non-newtonian gravity. Phys. Rev. Lett. 105, 231103. http://dx.doi.org/10.1103/PhysRevLett. 105.231103.

Métris, G., Vokrouhlický, D., Ries, J.C., Eanes, R.J., 1999. LAGEOS spin axis and non-gravitational excitations of its orbit. Adv. Space Res. 23, 721–725. http://dx.doi.org/10.1016/S0273-1177(99)00142-8.

Métris, G., Vokrouhlický, D., Ries, J.C., Eanes, R.J., 1997. Nongravitational effects and the LAGEOS eccentricity excitations. J. Geophys. Res. 102, 2711–2729. http://dx.doi.org/10.1029/96JB03186.

Milani, A., Nobili, A.M., Farinella, P., 1987. Non-Gravitational Perturbations and Satellite Geodesy. Adam Hilger, Bristol.

Minott, P.O., Zagwodzki, T.W., Varghese, T., Seldon, M., 1993. Prelaunch Optical Characterization of the Laser Geodynamic Satellite (LAGEOS 2). Technical Report 3400. NASA Technical Paper 3400. National Aeronautics and Space Administration, Washington, DC.

NASA, 1975. LAGEOS Phase B Technical Report, NASA Technical Memorandum X-64915. Technical Report TMX-64915. Marshall Space Flight Center. Marshall Space Flight Center, Alabama 35812. February 1975.

Otsubo, T., Sherwood, R., Gibbs, P., Wood, R., 2004. Spin motion and orientation of LAGEOS-2 from photometric observation. IEEE Trans. Geosci. Remote Sens. 42, 202–208. http://dx.doi.org/10.1109/TGRS.2003.817191.

Paolozzi, A., Ciufolini, I., 2013. LARES successfully launched in orbit: satellite and mission description. Acta Astronaut. 91, 313–321. http://dx.doi.org/10.1016/j.actaastro.2013.05.011, arXiv:1305.6823.

Pearlman, M.R., Degnan, J.J., Bosworth, J.M., 2002. The international laser ranging service. Adv. Space Res. 30, 135–143.

Rubincam, D., 1982. On the secular decrease in the semimajor axis of lageos's orbit. Celestial Mech. 26, 361–382. http://dx.doi.org/10.1007/BF01230417.

Rubincam, D.P., 1984. Postglacial rebound observed by LAGEOS and the effective viscosity of the lower mantle. J. Geophys. Res. 89, 1077–1087. http://dx.doi.org/10.1029/JB089iB02p01077.

Rubincam, D.P., 1987. LAGEOS orbit decay due to infrared radiation from Earth. J. Geophys. Res. 92, 1287–1294. http://dx.doi.org/10.1029/JB092iB02p01287.

Rubincam, D.P., 1988. Yarkovsky thermal drag on LAGEOS. J. Geophys. Res. 93, 13805–13810. http://dx.doi.org/10.1029/JB093iB11p13805.

Rubincam, D.P., Currie, D.G., Robbins, J.W., 1997. LAGEOS I once-per-revolution force due to solar heating. J. Geophys. Res. 102, 585–590. http://dx.doi.org/10.1029/96JB02851.

Rubincam, D.P., Knocke, P., Taylor, V.R., Blackwell, S., 1987. Earth anisotropic reflection and the orbit of LAGEOS. J. Geophys. Res. 92, 11662–11668. http://dx.doi.org/10.1029/JB092iB11p11662.

Rubincam, D.P., Weiss, N.S., 1986. Earth albedo and the orbit of LAGEOS. Celestial Mech. 38, 233–296. http://dx.doi.org/10.1007/BF01231110.

Scharroo, R., Wakker, K.F., Ambrosius, B.A.C., Noomen, R., 1991. On the along-track acceleration of the LAGEOS satellite. J. Geophys. Res. 96, 729–740. http://dx.doi.org/10.1029/90JB02080.

Siry, J.W., 1975. The LAGEOS system. Technical Report TM-X-73072. NASA.

Slabinski, V.J., 1996. A numerical solution for LAGEOS thermal thrust: the rapid-spin case. Celestial Mech. Dyn. Astron. 66, 131–179. http://dx.doi.org/10.1007/BF00054962.

Smith, D.E., Dunn, P.J., 1980. Long term evolution of the LAGEOS orbit. Geophys. Res. Lett. 7, 437–440. http://dx.doi.org/10.1029/GL007i006p00437.

Smith, D.E., Kolenkiewicz, R., Dunn, P.J., Robbins, J.W., Torrence, M. H., Klosko, S.M., Williamson, R.G., Pavlis, E.C., Douglas, N.B., 1990. Tectonic motion and deformation from satellite laser ranging to LAGEOS. J. Geophys. Res. 95, 22013–22041. http://dx.doi.org/10.1029/JB095iB13p22013.

Vokrouhlický, D., 1996. Non-gravitational effects and LAGEOS' rotation. Geophys. Res. Lett. 23, 3079–3082. http://dx.doi.org/10.1029/96GL03025.

Wong, C., 1978. Watching the Earth move from space. Sky Telesc., 198–202

Yoder, C.F., Williams, J.G., Dickey, J.O., Schutz, B.E., Eanes, R.J., Tapley, B.D., 1983. Secular variation of Earth's gravitational harmonic J2 coefficient from LAGEOS and nontidal acceleration of Earth rotation. Nature 303, 757–762. http://dx.doi.org/10.1038/303757a0.