Edited by E. M. Gaposchkin

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SAO ISAGEX EXPERIENCE. I. DATA ACQUISITION

Edited by E. M. Gaposchkin

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\begin{abstract}
The International Satellite Geodesy Experiment (ISAGEX) has completed the data acquisition phase. This report describes the contributions and methods of the Smithsonian Astrophysical Observatory to the program. The report will provide users of the data with necessary supporting information. A sequel will be prepared when the analysis of the ISAGEX is completed.
\end{abstract}

\section*{INTRODUCTION}

\author{
E. M. Gaposchkin
}

The ISAGEX program is the third in a series of cooperative satellite-tracking campaigns. The first in 1967 and the second in 1968, organized by SAO, were primarily camera tracking programs. There were, respectively, four and five laser tracking instruments operating during those intervals. Where possible, the tracking schedules were established to accommodate these systems. In 1967, there were five satellites and in 1968, there were six satellites suitably equipped with corner reflectors. However, three of them were in almost identical orbital configurations, so for some purposes the number was, in reality, three and four.

ISAGEX was initiated in 1969 by the French CNES with its "Proposition for an International Laser and Photographic Observation Campaign on Satellites Equipped with Laser Reflectors." ISAGEX took on added importance with the increased number of laser systems (10) and of precision satellite-tracking cameras (30) and the launch of a seventh retroreflector satellite (Peole), by France.

There are many purposes of a tracking campaign, and the archives of data will be useful for applications not envisaged at the inception of the program. The cooperating groups proposed the following three broad objectives, which gave shape to the program:
A. To organize a well-coordinated tracking campaign of the seven satellites equipped with laser retroreflectors in such a way that its contribution to our knowledge of the gravity field of the earth and other geodetic parameters will be significant.
B. To collect the set of observations made by the different participating agencies and to make those data available to the scientific community with all information necessary for use in computations.
C. To further research and development of instrumentation and operations of high-precision tracking systems for future space experiments.

ISAGEX is primarily a program of coordinated observations and data exchange. The data are to be distributed to all participants as per the operations plan, and subsequent analysis is largely at the option of the individual agencies. The analysis objectives of the participants and others are given in the International Satellite Geodesy Experiment Plan, published by CNES on November 10, 1970.

The planning and execution of the program has been documented in several CNES reports. The program ran from January through August 1971 and, broadly speaking, all the objectives were met. The data reduction has been completed for the laser data, which have been forwarded to the CNES data bank. The reduction of photographic observations is now under way. Therefore, the first objective and part of the second have been achieved.

The purpose of this report is to describe SAO's experience and methods. Included are the information necessary to use the laser data and descriptions of the observing system, the calibration methods, the reduction methods, and the process of data validation. In addition, there is a discussion of the various aspects of data acquisition. We hope that with such a document, improvement of the data systems will be furthered. We have concentrated on the technological and operations aspects of ISAGEX and on laser tracking in general. Within the next year, a sequel will be prepared, on the scientific results to come from the ISAGEX data. Some results already in hand are reported here and elsewhere (Gaposchkin, Kozai, Veis, and Weiffenbach, 1971).

There were substantial objectives for camera observations during the campaign. Since the SAO Baker-Nunn data have already been discussed in considerable detail, this report is restricted to the SAO laser systems.

In addition to the above objectives, ISAGEX was a test bed. We have seen how successfully a multinational observing program can be carried out. This success, in the absence of any more than informal agreements, is due to the good faith and mutual interest of all parties concerned. This sort of cooperation is enormously
important for the future. With scientific objectives becoming ever more ambitious, the requirements for tracking data become more demanding. It is apparent that several groups pooling resources can achieve much more than can individuals alone. The future programs of all groups will be materially advanced by such cooperation, and it can even be argued that some programs are not feasible without it.

SAO agreed to participate with laser units that were in the process of construction. The fielding of these units and their subsequent operation taxed the resourcefulness of the whole organization. Indeed, it had to be considered an experiment to see whether such a program of fabrication, field installation, and immediate data acquisition in amounts and with a necessary precision could even be accomplished. The statistics attest to the increased volume of data as the program progressed.

ISAGEX was used as a period for improvement of the accuracy and reliability of the laser network under routine field operation. We faced the difficulties of repairing malfunctions and detecting operating problems while the observing program was continuing. This was a completely different situation from operating one or two systems, under essentially laboratory conditions, with all SAO technical personnel available at a domestic site. We had a mixed record as a result. Some problems slipped through the system until the validation process. Needless to say, the system has since been modified. In addition, studies were begun to improve the system's accuracy. Photographing the oscilloscope waveform for centroid detection was attempted on a routine basis. Analysis of these data is in progress, and preliminary results are reported here. This experiment will lead to improved signal detection and analysis in future operation.

ISAGEX was intended to provide a framework for individual agencies developing laser tracking to take data in an organized program. They could have routine predictions and an immediate evaluation of their data. This situation is very helpful for new systems. In the final accounting, only a few such new systems participated. However, they did have the benefits described, although on the whole, the data taken were too few to be geodetically significant. These systems have been in operation, and we hope they will be able to participate in a more substantial way during future programs such as EPSOC, currently being conducted by SAO.

The ISAGEX program has ushered in a new era of cooperative tracking programs. We have every indication that the laser data taken are of \(1-\mathrm{m}\) accuracy with \(60-\mathrm{cm}\) noise. We have good confirmation of the \(10-\mathrm{m}\) accuracy of our current geodetic tools, as well as the very real opportunity to obtain l-m geodesy using these and other data.

Everyone at SAO and many individuals at CNES, NASA, and other organizations contributed in a substantial way. It is impossible to acknowledge them all. The contributors of this report join me in expressing our gratitude to these people. The program has achieved what it has only through such cooperation.

\section*{REFERENCE}

GAPOSCHKIN, E. M., KOZAI, Y., VEIS, G., and WEIFFENBACH, G.
1971. Geodetic studies at the Smithsonian Astrophysical Observatory. Presented at the XVth IUGG General Assembly, Moscow, August.

\section*{SAO NETWORK DESCRIPTION}

\author{
J. M. Thorp and M. A. Bush
}

Fifteen years ago, SAO conceived the idea of a worldwide network of photographic observing stations to track the artificial satellites proposed for the International Geophysical Year (IGY). Since the United States planned to launch its IGY satellites from Cape Kennedy into orbits with low inclinations, the original locations of the astrophysical observing stations were selected to obtain the best practical coverage of such orbits. Later, the low-latitude network configuration was modified to recognize the existence and importance of high-inclination satellites, which allow analysis of atmospheric and geodetic conditions at high latitude.

The ISAGEX network configuration is depicted in Figure 1, showing the locations of 11 astrophysical observing stations and the station in Dakar, Senegal, which was operated in cooperation with CNES. Each site is equipped with a Baker-Nunn tracking camera and a highly precise timing system. In addition, five stations have been augmented with laser ranging systems. Table 1 lists the COSPAR number and the location of the sites used in the ISAGEX program.

Figure 2 shows the Baker-Nunn camera at San Fernando, Spain, and Figure 3, the new SAO laser ranging system at Natal, Brazil.

The Baker-Nunn camera is a modified Super-Schmidt f/1, of \(500-\mathrm{mm}\) focal length ( 20 inches) and \(500-\mathrm{mm}\) aperture. A pyrex spherical mirror 760 mm ( 30 inches) in diameter and three corrector elements, two positive and one negative, constitute the optics, designed by J. G. Baker. The focal surface is approximately spherical, and the film is stretched under tension on a specially designed pyrex spherical surface. The field is \(30^{\circ}\) along the tracking axis and \(5^{\circ}\) along the perpendicular one. When the satellite position is fairly well known, the field along the tracking axis can be reduced to \(15^{\circ}\), resulting in a considerable savings in film usage.

- SMITHSONIAN ASTROPHYSICAL OBSERVATORY SITES AND
\(\times\) COOPERATING AGENCIES
- french laser
Figure 1. Configuration of the Baker-Nunn ISAGEX network.

Figure 2. The Baker-Nunn camera at San Fernando, Spain.


Figure 3. The new SAO laser ranging system at Natal, Brazil.

Table 1. Sites used in the ISAGEX program.
\begin{tabular}{|c|c|c|}
\hline \multicolumn{2}{|l|}{Station} & \\
\hline Location & COSPAR number & Equipment \\
\hline San Fernando, Spain & 9004 & Baker-Nunn \\
\hline Naini Tal, India & 9006 & Baker-Nunn \\
\hline Maui, Hawaii & 9012 & Baker-Nunn \\
\hline Mt. Hopkins, Arizona & 9021 & Baker-Nunn \\
\hline Mt. Hopkins, Arizona & 7921 & Laser \\
\hline Olifantsfontein, South Africa & 9022 & Baker-Nunn \\
\hline Olifantsfontein, South Africa & 7902 & Laser \\
\hline Island Lagoon, Australia & 9023 & Baker-Nunn \\
\hline Dodaira, Japan & 9025 & Baker-Nunn \\
\hline Arequipa, Peru & 9027 & Baker-Nunn \\
\hline Arequipa, Peru & 7907 & Laser \\
\hline Debre Zeit, Ethiopia & 9028 & Baker-Nunn \\
\hline Dionysos, Greece & 9030 & Baker-Nunn \\
\hline Dionysos, Greece & 7930 & Laser \\
\hline Natal, Brazil & 9039 & Baker-Nunn \\
\hline Natal, Brazil & 7929 & Laser \\
\hline Dakar, Senegal & 9020 & Baker-Nunn \\
\hline Dakar, Senegal & 7820 & Laser \\
\hline
\end{tabular}

The standard film is Kodak Royal-X pan-recording 2475 (extended red) emulsion on a \(4-\mathrm{mm}\) estar base. The scale on the film is \(2.46 \mu \mathrm{arcsec}^{-1}\), and \(80 \%\) of the light is placed on a \(20-\mu\)-diameter disk. The camera can photograph stars of 14 th mag with a \(20-\) sec exposure. A barrell-type shutter, rotating in front of the focal surface
at a precise angular velocity, chops the trails of the stars (or of the satellites, if the camera is stationary) and provides the breaks that are used as references for the reduction of the film. The shutter (or chopper) rotates five times, making five breaks per exposure. When the shutter is in the middle of the central break, an electrical contact strobes a flashing tube that records the time from a slave clock in the camera. Time is thus recorded on the same film on which the satellite and stars appear. With the use of a phase shifter, the shutter can be synchronized to time signals so as to produce the middle of the central break at a predetermined time. This method can be used to perform simultaneous observations from several stations whose shutters are in full synchronization.

The Baker-Nunn can be operated in two basic modes, stationary and tracking. In the first, more simple mode, the camera is held stationary while the images of the satellite trail along the film. In the tracking mode, the body of the camera is driven at the same rate as the apparent angular velocity of the satellite, holding the image of the satellite to a point of light on the film.

The precision timing system is composed of an EECo clock that utilizes a \(5-\mathrm{MHz}\) Sulzer crystal oscillator as its frequency standard, a high-frequency receiver to monitor the WWV signal, and a VLF receiver. The VLF receiver monitors the very accurate frequency tones transmitted by various VLF stations around the world. The frequency of the crystal oscillator is continuously compared with these frequency tones, and the difference is displayed on both a chart recorder and an accumulated time-deviation counter. The oscillator can therefore be adjusted periodically by means of a tuning capacitor. As a further aid to more accurate timekeeping, a portable clock is carried from station to station to measure the relative settings of the clocks. Timing to within \(100 \mu \mathrm{sec}\) is routinely achieved at all camera stations. At the laser sites, timing is maintained to within \(50 \mu \mathrm{sec}\), an accuracy necessary for laser ranging.

Since 1966, SAO has been improving the accuracy of its tracking technique by installing laser tracking systems at several of its camera locations. The Baker-Nunn provides very accurate directional data, and the laser provides the added dimension of range or the accurate determination of a satellite's height above the earth. The
increased accuracy of the Smithsonian tracking program as a result of the addition of lasers has innumerable research benefits.

SAO currently operates five laser tracking systems collocated with Baker-Nunn cameras at tracking stations in Mt. Hopkins, Arizona; Natal, Brazil; Arequipa, Peru; Olifantsfontein, South Africa; and Athens, Greece. With the exception of the last, which was assembled and operated in cooperation with the NTU of Athens, all systems were designed and built to SAO specifications and represent near state-of-the-art ruby-laser technology.

The following characteristics apply to the four systems operated by SAO: The type is ruby Q-switched; the peak power is 400 Mw ; the pulse width is 18 nsec ; the energy output is 7 J ; the maximum pulse repetition rate is 4 pulses \(\mathrm{min}^{-1}\); the beam divergence can be varied from 0.5 to 6 mrad (or from 2 to 20 arcmin ); the pointing is automatic-static, which permits day and night ranging and does not require that the satellite be sunlit; and the range resolution of the counter is 1 nsec . The laser transmitter utilizes two ruby rods, one for the oscillator and one from the amplifier stage. The rods are stimulated by the discharge of xenon high-voltage lamps. At lasing, some of the output energy is sampled by a photodetector that triggers the range measuring counter. The pulse, after traveling in space from the transmitter to the satellite, is reflected back from the retroreflectors mounted on the spacecraft and is focused by a 20 -inch Cassegrain telescope onto a photomultiplier tube. The signal generated by the photomultiplier stops the counter, which then displays the elapsed time.

One of the problems encountered in laser ranging is the variation in signal strength of the pulse reflected from the satellite. This variation is due in part to the fact that the signal varies inversely as the fourth power of the satellite range and in part to an observed "scintillation, " or random effect. The variations in signal strength affect the range measurements. As we said, satellite range is obtained from a time-interval counter that is started by the transmitted pulse and stopped by the receiving pulse. The resolution of the counter is 1 nsec , but the duration of the pulse is 18 nsec. Hence, the counter reading changes significantly if it stops at different points on the pulse's leading edge. The counter stops when it reaches a threshold
that has been set near the half-amplitude point of a weak return pulse. Since the system is calibrated for such a pulse and setting, errors are introduced when the return pulse varies from its average value. These errors, however, can be corrected if a photograph of each return pulse displayed on an oscilloscope is obtained. An automatic recording system capable of doing this for every pulse has been devised and is currently being field-tested. This correction can reduce the error in range measurements to 1 ft or less.

\section*{DESCRIPTION OF THE SAO LASER SYSTEM CURRENTLY DEPLOYED}

IN BRAZIL, PERU, AND SOUTH AFRICA
P. W. Sozanski

The main components of SAO's laser system are the laser transmitter, the staticpointing pedestal, the telescope photoreceiver, the data system, and the epoch timing system (see Figure 1).


Figure 1. Laser ranging system.

To describe the operation of the laser system in simple terms, the laser transmitter head and the telescope photoreceiver are pointed by the static-pointing pedestal (see Figure 2) to the altitude and azimuth coordinates in accordance with predictions generated in Cambridge. The laser is then pulsed, under electronic or manual control, at the appropriate epoch, and a very short pulse of monochromatic light in a narrow beam is projected from the laser transmitter head toward the satellite. The transmitted pulse is detected at the transmitter by a photodiode whose output is an electrical pulse that starts the range interval counter and reads out the station clock to mark epoch. The light is reflected back from the satellite by its cube-corner reflectors and is detected photoelectrically by the telescope photoreceiver whose output is an electrical pulse that stops the range interval counter. The range from the laser system to the satellite is then calculated from the elapsed time, with due corrections for atmospheric and other effects.

Procurement started in early 1969, and the systems were fielded in late 1970.

A detailed description of the SAO laser system components follows:

\section*{1. LASER TRANSMITTER}

The laser transmitter was purchased in March 1969 from:
Spacerays, Inc.
Northwest Industrial Park
Burlington, Massachusetts 01803
(617)272-6220

The system is a flash-pumped, Q-switched ruby system with an oscillator and one amplifier stage. The output is a \(4-\) to \(5-J\) pulse 18 nsec wide. The beam is collimated with a Galilean telescope with an aperture of \(12.7-\mathrm{cm}\) diameter, the beam divergence is variable from 0.3 to 6.0 mrad (measured at full width, half-power points), the repetition rate is 4 ppm , and the wavelength of the output is 694 nm .

The laser transmitter system consists of three major units: the laser transmitter head, the power supply and control electronic units, and the cooling unit.


Figure 2. Laser transmitter head (right) and telescope photoreceiver (left) mounted on static-pointing pedestal.

\section*{2. PEDESTAL}

The pedestal was purchased in May 1969 from:
Tinsley Laboratories, Inc. 2448 Sixth Street Berkeley, California 94710 (415) \(843-6836\)

This pedestal is a static-pointing, open-loop unit of an altitude-over-azimuth biaxial configuration. Its overall accuracy is within \(0: 008\) (great circle error of 0.5 arcmin or less).

The pedestal is a static-pointing unit, i.e., it moves to a given pointing direction, waits for the satellite to pass through that direction, and then moves on to the next such static point.

The unit is of the open-loop type, i.e., it does not operate as a servomechanism and does not require a feedback error signal. It relies instead on starting at a known pointing direction of two orthogonal axes and on simple addition and subtraction of known increments of motion about those axes to arrive at a new predetermined pointing direction. The known increments of motion are provided by reliable, precision, incremental-stepping motors fed by precomputed number-of-steps input data. The initial starting position is established by optical goniometers, and the continuous addition and subtraction is maintained by solid-state arithmetic units, counting registers, comparison logic circuitry, and visual displays.

The pedestal is positioned by manual decade-switch selection or by using a prepunched paper-tape input.

\section*{3. TELESCOPE PHOTORECEIVER}

The telescope photoreceiver was purchased in May 1969 from several vendors (see below). It contains three subsystems (see Figure 3). The first, the main subsystem, contains two components, a \(53-\mathrm{cm}\)-diameter f/4 paraboloidal primary and a
14.6-cm-diameter flat secondary. The primary has at least a 50 -cm-diameter clear aperture and when combined with the secondary produces a field greater than 20 arcmin in diameter, an overall accuracy of better than \(1 / 4\) wave; after aluminizing and SiO overcoating, the primary has a minimum combined reflectance greater than \(60 \%\) between 400 and 700 nm .


Figure 3. Telescope photoreceiver.

The second subsystem, the photomultiplier tube (PMT) optical subsystem, transfers the beam reflected by the main optical system through its components and onto the face of the PMT. The beam passes through a holder for a stack of gelatin filters, then through a field lens that directs the beam through the field stop wheel containing six apertures. These apertures are sized to produce fields of \(2,4,8,12,16\), and 20 arcmin in diameter. The diverging beam passes next through a collimating lens and then through a tiltable interference filter, a final imaging lens, and onto the face of the PMT. This system is designed to produce a \(3.8-\mathrm{cm}\)-diameter spot on the face of the PMT independent of field stop settings.

The third subsystem, the auxiliary viewing subsystem, consists of \(1 / 4\)-wave flat flip mirror that, when inserted into the beam, directs the beam through the exit lenses onto a front surface diagonal mirror that reflects the beam through an illuminated reticle and out to the eyepiece for visual viewing.

A detailed breakdown of the photoreceiver is as follows:

Telescope. A \(50-\mathrm{cm}\) telescope is used to detect the laser return and was bought from:

Tinsley Laboratories, Inc. 2448 Sixth Street Berkeley, California 94710 (415)843-6836

Photomultiplier Tube. A RCA Model 7265, selected for a quantum efficiency of \(4.5 \%\) or greater at 694 nm and a gain of \(2 \times 10^{7}\) or greater at 2400 v , is used to trigger the counter. It was purchased from:

Radio Corporation of America
Industrial Tube Division
New Holland Pike
Lancaster, Pennsylvania 17604
(717)397-7661

Photomultiplier Tube Housing. A Products for Research Model PR 2100 (modified) housing, used to hold the PMT, was purchased from:

Products for Research
78 Holten Street
Danvers, Massachusetts 01923
(617)774-3250

Interference Filters. A \(20 \AA\) interference filter and a \(7 \AA\) interference filter were purchased from:

Thin Films Products Division
Infra-Red Industries
80 4th Avenue
Waltham, Massachusetts 02154
(617) 894-8410

\section*{4. DATA SYSTEM}

The data system was purchased during the interval from March 1969 through September 1970 from several vendors (see below). The system (Figure 4) consists of the measurement instrumentation as well as the digital-control and data-handling systems for the laser transmitter. The four functional subsystems are described below.

Counter. An Eldorado ElectroData Model 796 (modified) counter with a 1-nsec resolution is used to obtain the satellite range times. This unit was purchased from:

Eldorado ElectroData Corporation
601 Chalomar Road
Concord, California 94520
(415) 686-4200

Oscilloscope. A Tektronix Type R454 oscilloscope with modification 163D and a Tektronix Model C-40 oscilloscope camera are used to provide a means for visual monitoring and photographic recording of the laser transmitter output and the laser return pulses. The oscilloscope and camera were purchased from:

Tektronix, Inc.
P. O. Box 500

Beaverton, Oregon 97005
(503)644-0161

Figure 4. Laser data system.

Control System. In addition to providing the basic time-interval measurement for satellite ranging, the laser data system must also record the observation epoch time (system clock); program the operating sequence of the laser transmitter unit, the pedestal, and the data system itself (tape reader, tape control, and laser control); condition the stop channel and the return pulse (range-gate generator and amplifier detector/monitor); and print out the digital data (intercoupler, digital printer, and tape perforator). All the control-system components with the exception of the digital printer and tape perforator were built by SAO. The digital printer and tape perforator (Model ASR-32, modified by SAO) were purchased from:

> The Teletype Corporation
> 5555 Touhy Avenue
> Skokie, Illinois 60076
> (312) \(982-2000\)

Racks, Power, and Cabling. A Western Devices rack and blower unit, used to hold most of the data system, was purchased from:

Zero Manufacturing Company
1121 Chestnut Street
Burbank, California 91503
(213) 849-5521

A Bud Radio Company Model 2707 Series 60 rack is used to hold the tape-reader system and was purchased from:

Gerber Electronics
852 Providence Highway
Dedham, Massachusetts 02026
(617)329-2400

A General Radio Model 1581-ALR2 voltage regulator is used to supply regulated AC power to the data system. It was purchased from:

General Radio Company
300 Baker Avenue
West Concord, Massachusetts 01781
(617)369-4400

A Northeast Scientific regulated high-voltage power supply Model RQE-3001-21230 provides high voltage to the photomultiplier tube. It was purchased from:

Northeast Scientific Corporation 30 Wetherbee Street Acton, Massachusetts 01720 (617)263-7706

\section*{5. EPOCH TIMING SYSTEM}

The epoch timing systems, a Model ZA 34675 single-channel unit and a Model ZA 34685 dual-channel unit, were purchased in March 1965 from:

Electronic Engineering Company of California
1601 East Chestnut Avenue
Santa Ana, California 92700
(714) 547-5651

The EECo timing system (see Figure 5) is used to provide epoch. It has a display resolution of \(10 \mu \mathrm{sec}\) and an electrical resolution of \(1 \mu \mathrm{sec}\). The single-channel unit consists of a crystal oscillator, accumulator, oscilloscope, VLF receiver, chart recorder, WWV receiver, and a battery backup system. The dual-channel unit consists of the above plus an additional crystal oscillator, accumulator, and VLF receiver.


Figure 5. Epoch timing system (EECo). Solid lines show single-channel unit, while the dual-channel unit is represented by the solid lines and the dotted lines.

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CALIBRATION
C. R. H. Tsiang

The ranging accuracy of the laser system is calibrated through a procedure of ranging on a fixed land-based target at a surveyed distance, generally on the order of 0.25 to 1 mi . A calculation can be made to obtain the expected range time based on the surveyed distance and the atmospheric refractivity. Once an average range time is obtained from a series of target measurements, it is possible to compute a calibration number \(\tau_{c}\), which can be reported along with the satellite range times. This number covers delays in the range counter, cabling, telescope, output detector, photomultiplier tube, and signal amplifier. It does not provide a means for obtaining a calibration factor for atmospheric delays, but otherwise accounts for all components in the range measuring path to and from the satellite.

The following formula can be used in computing the atmospheric refractivity N :
\[
\mathrm{N}=80.29 \frac{\mathrm{P}}{\mathrm{~T}}-11.9 \frac{\mathrm{e}}{\mathrm{~T}},
\]
where \(P\) is the measured barometric pressure (in millibars), e the partial pressure of water vapor, and T the temperature \((\mathrm{K})\). The calculation for two-way range time has been based on
\[
\tau_{S}=\frac{R_{S}}{0.15}\left(1+\mathrm{N} \times 10^{-6}+6.917 \times 10^{-4}\right)
\]
where \(\tau_{S}\) is the calculated range time ( nsec ) for a surveyed distance of \(R_{S}(\mathrm{~m})\). After noting that local temperature and pressure variations at any one location never change N by more than \(\pm 10 \%\) and that the range equation gives subnanosecond variations in \(T_{S}\) for such changes, we decided that fixed values of \(N\) could be determined for each station. Rather than each station calculating its mean barometric pressure, we prepared a chart, which gives a direct conversion from the station altitude above the geoid in kilometers to values of N (see Figure 1).


Figure 1. Atmospheric refractivity as a function of station height for an ambient temperature of \(15^{\circ} \mathrm{C}\) at \(6943 \AA\).

The system-calibration number reported in word eight of the 33333 observation message is obtained by subtracting the observed range time \(\tau_{m}\) from \(\tau_{s}\), the range time calculated from the surveyed laser-to-target range:
\[
\tau_{S}-\tau_{m}=\tau_{c}
\]

The resulting system-calibration number \(\tau_{c}\) is reported as a signed quantity, which is added to all range measurements in the 33333 message. Generally, \(\tau_{c}\) is negative in the SAO laser systems.

In theory, the calibration of the instrument should change only if its components are changed, moved, or affected by environmental fluctuations or aging. By using the whole system to range on a fixed land-based target, we hoped that all such factors could be covered by the single system-calibration number. Attempts were made to simulate real operational conditions by regulating the pulse-repetition rate, photomultipliertube voltage, counter thresholds, amplifier gain, return signal level, output power level,
etc. Several unavoidable differences existed between the ranging measurements on the satellites and those on the target - viz., corner cubes vs. nonspecular reflecting surface of the target; small solid angle subtended by the satellite vs. full-beam reflection by the target ( \(8-\mathrm{ft} \times 8-\mathrm{ft}\) wooden surface painted flat white); point-source satellite image vs. off-axis, near-field reflection by the target; short air-path length to ground-based target vs. full atmosphere to satellite, etc. None of these differences is trivial, but for operation of the laser at the original design levels ( \(0.5-\mathrm{m}\) resolution and \(1-\mathrm{m}\) accuracy), the calibration procedure and results appear to be satisfactory.

For the three stations with the new lasers, the variations in the reported calibration number over the course of most of ISAGEX did not exceed 15 nsec . All the new stations did experience some problems with calibration ranging when first set up, but they were confined to the first week of operations and apparently were cleared up by the beginning of the second ISAGEX period. For the most part, the variations in the reported calibrations were due to the following:
A. Replacement or relocation of components in the laser head, photoreceiver, or data system.
B. Readjustment of signal operating levels in the target-ranging procedure.
C. Readjusted survey figures for the laser-to-target distance.

Attempts at standardizing and improving the reliability of the calibration tests were made throughout ISAGEX as more experience and knowledge of the instrumentation was gained. During investigations into the error-reducing capabilities of photographically determined range-time corrections, certain observations led to the establishment of procedures that would minimize the effects of variations in signal level during target observations. These effects - the most detrimental ones for satellite ranging - were reduced to a level so that the most significant systematic error component of the calibration lay in the accuracy of the ground-survey information. Therefore, where survey information was questionable, additional measurements were made in an attempt to allow no more than a 4 -cm error.

Unfortunately, this has led to the establishment of more than one survey target distance at two of the South American stations. Table 1 shows the results obtained by local surveyors using conventional techniques. Note that certain sites have been measured to greater resolution than others. Only where special comments are included should this be considered significant. Ultimately, corrections will be applied to all ISAGEX calibration data when the stations are resurveyed with a laser geodimeter.

Apart from the systematic error contributed by the differences in survey results, the net range-time uncertainty introduced by the calibration should generally be better than \(\pm 2\) nsec. Photographic reduction of pulse images offers the possibility of decreasing the error to less than \(\pm 1 \mathrm{nsec}\) for many of the periods after May 1971. Reduction of these photographic data and distribution of the results will be made in 1972.

Table 1. ISAGEX target range history: Effective dates of change to new values of survey distance and refractivity constant N.
\begin{tabular}{|c|c|c|c|c|}
\hline \multicolumn{2}{|l|}{Station} & \multirow[b]{2}{*}{\[
\begin{gathered}
\text { Date } \\
\text { (1971) }
\end{gathered}
\]} & \multirow[b]{2}{*}{Survey distance (m)} & \multirow[b]{2}{*}{Refractivity constant N} \\
\hline Location & COSPAR number & & & \\
\hline \multirow[t]{2}{*}{Arequipa, Peru} & 7907 & January 5 & 313.135 & 250 \\
\hline & & October 1 & 312.844 & 250 \\
\hline Mt. Hopkins, Arizona & 7921 & January 5 & 776.329 & 172 \\
\hline Olifantsfontein, South Africa & 7902 & February 12 & 404. 48 & 290 \\
\hline \multirow[t]{4}{*}{Natal, Brazil} & 7929 & January 5 & 316.08 & 248 \\
\hline & & April 28 & 316.08 & 340 \\
\hline & & April 29 & 314.67 & 340 \\
\hline & & September 23 & 315.62 & 340 \\
\hline \multirow[t]{3}{*}{Dionysos, Greece} & 7930 & January 5 & 327.25 & 0 \\
\hline & & April 9 & 327.973* & 0 \\
\hline & & April 28 & 327.973 & 340 \\
\hline
\end{tabular}

\footnotetext{
*Measurement made by laser geodimeter.
}

\author{
D. A. Arnold and J. M. Thorp
}

Epoch time is maintained at each station by use of the EECo precision time system (see Thorp and Bush, this volume) and by reference to UTC(USNO). A portable clock is used to set the station clock, which is then maintained by referencing the frequency of a \(5-\mathrm{MHz}\) Sulzer oscillator to a known frequency, broadcast by one of the various VLF stations. Each observing station maintains an estimate of its timing uncertainty in two ways: First, the accuracy of the original clock set from a portable clock is expressed as an uncertainty (usually \(\pm 5 \mu \mathrm{sec}\) ). Time is maintained at each station on one main channel, with one or more alternate channels keeping time independently for backup. If the main channel has to be reset to one of the backup channels, an additional uncertainty is added (usually \(\pm 5 \mu \mathrm{sec}\) ). The second uncertainty is the deviation of the oscillator caused by its drift in frequency. The oscillator drift is determined by comparing the phase of the VLF station with that of the oscillator at a particular time each day. Each station steers or guides its oscillator to keep its time-drift uncertainity as small as possible (usually \(\pm 50 \mu \mathrm{sec}\) ).

In addition to the above uncertainties, two sets of time corrections are added in order to have time equivalent to UTC(USNO). One set consists of corrections of hours, minutes, seconds, or parts of seconds when a failure has occurred in the main timekeeping channel. These corrections are confirmed by referring to the alternate timekeeping channel and the WWV time signals. If all channels fail, time reference is lost and a reset is necessary. The second set of corrections is added in Cambridge; this consists of the computed phase differences between the average VLF phase for a period (usually a month) and the phase of the VLF at the time the clock is set. These corrections, determined from data published in USNO time-service bulletins, are generally on the order of \(<20 \mu \mathrm{sec}\).

Two files of time corrections are maintained by the Data Services Division at SAO. The first gives the difference between A.S and UT1, and the second, the difference between A.S and the clocks at the observing stations. The time system A. \(S\) is related to UTC(USNO) by the expression
\[
\mathrm{A} . \mathrm{S}-\mathrm{UTC}(\mathrm{USNO})=6 . \mathrm{S} .140768+0.002592000(\mathrm{~T}-39856.0)
\]
for the period February 1, 1968, to January 1, 1972; T is the time in Modified Julian Days; 39856.0 is January \(1.0,1968\); and the difference is given in seconds. The A. S-A. 1 difference is about 0.8983 msec .

UT1 data are obtained from "Circular D, " published monthly by the BIH. Values of UT1-UTC(BIH) and AT - UTC (BIH) are listed at 5-day intervals. The difference A.S - AT is currently 35.3 msec . A. S - UT1 is calculated by the relation
\[
\text { A.S - UT1 = (A.S - AT })+[\mathrm{AT}-\mathrm{UTC}(\mathrm{BIH})]-[\mathrm{UT} 1-\mathrm{UTC}(\mathrm{BIH})]
\]

A second-order polynomial is fitted to the A.S - UT 1 values, and the coefficients are punched on cards. Usually, each polynomial covers a 50 -day period. If the values change too rapidly, the interval can be reduced to 25 days.

The difference between the station clocks and UTC(USNO) is recorded by STADAD as described. The corrections are added to the A.S - UTC(USNO) difference to obtain the correction from the station clock to A.S time. Cards are punched giving these corrections as a series of straight-line segments specifying the values of the corrections at the beginning and end of each interval. A new card must be used whenever there is a gap, discontinuity, or change of slope in the time correction.

\section*{ATMOSPHERIC REDUCTION OF LASER DATA}

\author{
C. G. Lehr
}

Laser ranges determined by using the value of the velocity of light in a vacuum must be corrected for the fact that the laser pulse travels at a lower velocity during its passage through the earth's atmosphere. The correction is currently made by means of the following formula (obtained in a personal communication from Gordon D. Thayer):
where \(r_{v}\) is the uncorrected range ( \(m\) ), \(r_{m}\) is the corrected range ( \(m\) ), \(P\) is the atmospheric pressure ( mb ) at the laser station, \(T\) is the temperature \((\mathrm{K})\) at the laser station, \(h_{s}\) is the laser's elevation above mean sea level (km), and \(a\) is the altitude angle of the satellite. The formula holds for a ruby laser, which operates at 694 nm . It should be used only when \(\theta_{0}>5^{\circ}\), where \(\theta_{0}\) is the apparent altitude angle (i.e., the altitude angle uncorrected for atmospheric bending).

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\author{
PULSE ANALYSIS
}

\author{
C. R. H. Tsiang and C. G. Lehr
}

The effects of variations in pulse amplitude and shape must be carefully considered in attempting to reduce the noise and bias in laser range measurements. The simplest mode of operation in making time-interval measurements employs only a fixed-voltage threshold discriminator. Range times obtained this way are susceptible to errors caused by phenomena such as leading-edge walk and leading-edge pulse distortion. Attempts were made during operations to keep these effects to a minimum, and further work was done to record some of the laser passes on film. Errors on the order of \(\pm 5 \mathrm{nsec}\) can be expected when point-to-point amplitude changes affect the fixed threshold counter triggering circuit. Reduction of the photographic images of the outgoing- and return-pulse oscilloscope traces may produce range-correction figures to decrease the net range-counter errors to about \(\pm 1.7 \mathrm{nsec}\). No work has yet been done to evaluate fully the accuracy of the photographic data collected during ISAGEX, and further study is necessary to substantiate empirically the estimated accuracy of \(\pm 1.7 \mathrm{nsec}\). The process of photoreduction of the data and discussion of the system-accuracy potential is given in Lehr, Pearlman, and Scott (1970a). Early attempts at testing the effectiveness of this technique are presented in Lehr, Pearlman, and Scott (1970b).

Table l lists the satellite passes covered by photography. Most returns were obtained on Polaroid film, rather than on \(35-\mathrm{mm}\) film, because of the former's quick developing process. This rapid feedback was advantageous to the laser operator for adjusting the amplification level in the return-signal circuitry. Saturation of the amplifier, the limited resolution range of the oscilloscope at a fixed gain, and the minimum voltage imposed by the counter threshold were the constraints that had to be satisfied. Returns were photographed successfully at all three new laser stations in spite of early problems, such as signal adjustment and operation of the laser with new procedures and with an already heavily burdened crew of observers.

The listing of satellite passes gives some information of the quality and quantity of the data, even though little has been done so far to reduce the images for range correction factors. The column marked "Total reported observation points" refers to the number of range measurements reported by the station for each pass, and the next column gives the number of attempted photographic images. The actual number of traces of return or noise pulses is given in the column labeled "Images." Of these frames, only a few are of sufficient quality that they could be measured reliably. Those of reduction quality should produce measurements of at least \(\pm 3-\mathrm{nsec}\) consistency, and probably no worse than 1.5 nsec rms . Those images that do not qualify to be counted in the "Reduction quality" column are usually very irregularly shaped, low-level returns or extremely strong returns that go off the oscilloscope screen or are distorted by the saturation of the amplifier. In some cases, there are images of noise pulses that have falsely stopped the range counter. Even in these rejected images, there is useful information about the behavior of the return-signal circuitry under extreme amplitude conditions. Aside from the measurement of the images for range correction figures, the most important additional data come from the cataloging of return-pulse amplitudes and shapes photographed under routine laser tracking procedures. These data can be applied to studies on the scintillation of returns and to the calculation or prediction of return amplitudes. The amount of photographic data amassed during the last part of ISAGEX is insufficient in itself to have much value. Some reduction work is planned, however, so that the results can be used to evaluate the effectiveness of the photographic technique and to improve the operational procedures of the data-recording system.

\section*{REFERENCES}

LEHR, C. G., PEARLMAN, M. R., and SCOTT, J. L. 1970a. A photographic technique for improved laser-ranging accuracy. In Laser and Radar Investigations, ed. by Computer Sciences Corp., NASA, Washington, vol. III, pp. 51-56.
1970b. Range corrections from oscilloscopic displays of laser returns. Smithsonian Astrophys. Obs. Laser Rep. No. 4, 27 pp.

Table 1. Satellite returns.
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multirow[b]{2}{*}{Date} & \multirow[b]{2}{*}{\[
\begin{aligned}
& \text { Time } \\
& \text { (UT) }
\end{aligned}
\]} & \multirow[b]{2}{*}{Satellite} & \multirow[t]{2}{*}{Total reported observation points} & \multicolumn{3}{|c|}{Number of frames} \\
\hline & & & & Total & Images & Reduction quality \\
\hline \multicolumn{7}{|c|}{Brazil} \\
\hline June 17 & \(22^{\mathrm{h}} 53^{\mathrm{m}}\) & 7010901 & 7 & 4 & 4 & 0 \\
\hline June 18 & 0823 & 6800201 & 9 & 2 & 2 & 0 \\
\hline June 19 & 0606 & 7010901 & 3 & 2 & 2 & 0 \\
\hline August 5 & 2234 & 6508901 & 39 & 45 & 17 & 0 \\
\hline August 11 & 2053 & 6508901 & 31 & 24 & 12 & 0 \\
\hline August 11 & 2122 & 6508901 & 11 & 15 & 7 & 0 \\
\hline August 13 & 2314 & 7010901 & 1 & 3 & 1 & 1 \\
\hline August 14 & 0803 & 6508901 & 20 & 15 & 10 & 4 \\
\hline August 14 & 2220 & 6508901 & 13 & 3 & 3 & 3 \\
\hline August 22 & 0214 & 6503201 & 5 & 3 & 1 & 0 \\
\hline August 22 & 0632 & 6508901 & 19 & 9 & 6 & 0 \\
\hline August 23 & 2133 & 6800201 & 10 & 15 & 3 & 0 \\
\hline August 24 & 0328 & 6503201 & 1 & 2 & 1 & 0 \\
\hline August 31 & 0353 & 6503201 & 7 & 6 & 4 & 1 \\
\hline September 1 & 0556 & 7010901 & 9 & 4 & 3 & 1 \\
\hline September 23 & 2227 & 6508901 & 25 & 22 & 11 & 3 \\
\hline September 27 & 2331 & 6800201 & 18 & 18 & 15 & 8 \\
\hline October 1 & 2258 & 6800201 & 20 & 21 & 15 & 3 \\
\hline October 1 & 2259 & 6508901 & 20 & 9 & 2 & 1 \\
\hline October 3 & 2224 & 6503201 & 15 & 9 & 7 & 0 \\
\hline October 4 & 2111 & 6508901 & 15 & 3 & 3 & 0 \\
\hline October 16 & 0658 & 6508901 & 14 & 3 & 3 & 3 \\
\hline October 24 & 0522 & 6508901 & 15 & 7 & 6 & 2 \\
\hline
\end{tabular}

Table 1 (Cont.)
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multirow[b]{2}{*}{Date} & \multirow[b]{2}{*}{\[
\begin{aligned}
& \text { Time } \\
& \text { (UT) }
\end{aligned}
\]} & \multirow[b]{2}{*}{Satellite} & \multirow[t]{2}{*}{Total reported observation points} & \multicolumn{3}{|c|}{Number of frames} \\
\hline & & & & Total & Images & Reduction quality \\
\hline \multicolumn{7}{|c|}{Peru} \\
\hline May 19 & \(17^{\mathrm{h}} 59^{\mathrm{m}}\) & 6508901 & 41 & 26 & 25 & 9 \\
\hline May 19 & 2209 & 6800201 & 33 & 26 & 26 & 8 \\
\hline May 20 & 1755 & 6508901 & 16 & 28 & 28 & 6 \\
\hline May 22 & 0253 & 6508901 & 8 & 2 & 2 & 0 \\
\hline May 28 & 0323 & 6508901 & 17 & 1 & 1 & 1 \\
\hline June 3 & 0136 & 6508901 & 31 & 20 & 20 & 0 \\
\hline June 8 & 1301 & 6508901 & 60 & 35 & 27 & 5 \\
\hline June 8 & 2301 & 6800201 & 17 & 16 & 14 & 0 \\
\hline June 8 & 2358 & 6508901 & 34 & 28 & 21 & 1 \\
\hline June 9 & 0013 & 6701101 & 16 & 5 & 5 & 0 \\
\hline June 9 & 1304 & 6508901 & 49 & 14 & 14 & 7 \\
\hline June 10 & 0001 & 6508901 & 43 & 29 & 20 & 7 \\
\hline June 10 & 1120 & 6800201 & 21 & 14 & 12 & 0 \\
\hline June 10 & 1310 & 6508901 & 45 & 13 & 9 & 2 \\
\hline June 11 & 0006 & 6508901 & 31 & 15 & 13 & 0 \\
\hline June 11 & 1316 & 6508901 & 55 & 48 & 37 & 22 \\
\hline June 11 & 2208 & 6800201 & 4 & 3 & 2 & 0 \\
\hline June 12 & 1320 & 6508901 & 51 & 39 & 33 & 20 \\
\hline June 13 & 1324 & 6508901 & 29 & 17 & 12 & 0 \\
\hline June 13 & 2216 & 6508901 & 22 & 15 & 15 & 0 \\
\hline June 13 & 2247 & 6800201 & 32 & 20 & 19 & 0 \\
\hline June 15 & 2222 & 6508901 & 26 & 12 & 9 & 0 \\
\hline July 6 & 1735 & 6508901 & 20 & 18 & 16 & 7 \\
\hline August 17 & 2121 & 6508901 & 14 & 6 & 9 & 0 \\
\hline
\end{tabular}

Table 1 (Cont.)
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multirow[b]{2}{*}{Date} & \multirow[b]{2}{*}{\[
\begin{aligned}
& \text { Time } \\
& \text { (UT) }
\end{aligned}
\]} & \multirow[b]{2}{*}{Satellite} & \multirow[t]{2}{*}{Total
reported
observation
points} & \multicolumn{3}{|c|}{Number of frames} \\
\hline & & & & Total & Images & Reduction quality \\
\hline \multicolumn{7}{|c|}{South Africa} \\
\hline August 19 & \(00^{\mathrm{h}} 08^{\mathrm{m}}\) & 6508901 & 45 & 60 & 33 & 5 \\
\hline August 20 & 0011 & 6508901 & 28 & 44 & 27 & 4 \\
\hline August 23 & 0024 & 6508901 & 42 & 54 & 22 & 10 \\
\hline August 24 & 0026 & 6508901 & 31 & 56 & 12 & 1 \\
\hline August 24 & 0716 & 6800201 & 5 & 31 & 8 & 1 \\
\hline August 24 & 1752 & 6800201 & 12 & 25 & 3 & 0 \\
\hline August 25 & 0607 & 6800201 & 9 & 36 & 16 & 0 \\
\hline August 26 & 0037 & 6508901 & 28 & 49 & 20 & 7 \\
\hline August 26 & 2236 & 6508901 & 12 & 41 & 8 & 0 \\
\hline August 26 & 2239 & 6508901 & 14 & 4 & 3 & 0 \\
\hline August 27 & \(17 \quad 19\) & 6508901 & 15 & 24 & 14 & 2 \\
\hline August 27 & 2240 & 6508901 & 29 & 49 & 16 & 0 \\
\hline August 28 & 0047 & 6508901 & 10 & 32 & 20 & 0 \\
\hline August 29 & 2248 & 6508901 & 40 & 57 & 15 & 0 \\
\hline August 30 & 1815 & 6800201 & 14 & 23 & 14 & 0 \\
\hline August 30 & 2253 & 6508901 & 37 & 55 & 36 & 0 \\
\hline August 31 & 0610 & 6800201 & 3 & 32 & 17 & 0 \\
\hline August 31 & 2256 & 6508901 & 41 & 56 & 35 & 0 \\
\hline September 1 & 1706 & 6800201 & 10 & 16 & 7 & 0 \\
\hline September 4 & 2190 & 6508901 & 35 & 47 & 24 & 0 \\
\hline
\end{tabular}

\section*{STATISTICS: RETURNS AND FAILURES}

\author{
B. R. Miller
}

The following statistics represent the SAO and Air Force Baker-Nunn optical and the SAO laser returns during the months of intensive tracking on the individual satellites.

The first 2 months' predictions were generated with a paucity of observations, which may account for the sparse number of returns. During the rest of ISAGEX, as more observations became available for use in predicting orbits, the number of returns increased also.

The numbers here reflect the data collected during ISAGEX after gross errors were removed in the orbit computations for predictions.

The data were processed for validation purposes before being used in analysis. Therefore, there is a discrepancy in these figures and the final data used in the analysis.

\section*{Preceding page blank}
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{2}{|l|}{Station} & \multirow[b]{2}{*}{January} & \multirow[b]{2}{*}{March} & \multirow[b]{2}{*}{April} & \multirow[b]{2}{*}{Total} \\
\hline Location & COSPAR number & & & & \\
\hline \multicolumn{6}{|c|}{Optical Returns} \\
\hline San Fernando, Spain & 9004 & & 10 & 5 & 15 \\
\hline Naini Tal, India & 9006 & & 6 & 6 & 12 \\
\hline Maui, Hawaii & 9012 & & 5 & 7 & 12 \\
\hline Mt. Hopkins, Arizona & 9021 & 6 & 3 & 13 & 22 \\
\hline Olifantsfontein, South Africa & 9022 & 5 & & & 5 \\
\hline Island Lagoon, Australia & 9023 & 13 & 8 & 5 & 26 \\
\hline Dodaira, Japan & 9025 & 8 & 1 & 9 & 18 \\
\hline Arequipa, Peru & 9027 & & & 1 & 1 \\
\hline Debre Zeit, Ethiopia & 9028 & & & 2 & 2 \\
\hline Dionysos, Greece & 9030 & & 7 & 13 & 20 \\
\hline Natal, Brazil & 9039 & 1 & & 5 & 6 \\
\hline Rosamund, California & 9113 & 5 & 1 & & 6 \\
\hline Cold Lake, Canada & 9114 & 2 & 16 & & 18 \\
\hline Johnston Island & 9117 & 1 & & & 1 \\
\hline Mt. John, New Zealand & 9119 & 9 & 21 & 1 & 31 \\
\hline San Vito, Italy & 9120 & - & \(\underline{2}\) & \(\underline{13}\) & 15 \\
\hline Total & & 50 & 80 & 80 & 210 \\
\hline
\end{tabular}

\section*{Laser Returns \({ }^{*}\)}
\begin{tabular}{llllll} 
Arequipa, Peru & 7907 & & & \(6(18)\) & \(6(18)\) \\
Mt. Hopkins, Arizona & 7921 & & \(1(3)\) & \(6(21)\) & \(7(24)\) \\
Natal, Brazil & 7929 & \(1(1)\) & & \(3(8)\) & \(4(9)\) \\
Dionysos, Greece & 7930 & - & \(\underline{3(5)}\) & - & \(3(5)\) \\
\multicolumn{1}{c}{ Total } & \(\vdots\) & \(1(1)\) & \(4(8)\) & \(15(47)\) & \(20(56)\) \\
\hline
\end{tabular}

\footnotetext{
*The first number is the number of passes of the satellite; the number in parentheses is the total number of points in those passes.
}
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{2}{|l|}{Station} & & & & \\
\hline Location & COSPAR number & February & March & August & Total \\
\hline \multicolumn{6}{|c|}{Optical Returns} \\
\hline San Fernando, Spain & 9004 & 6 & 1 & 30 & 37 \\
\hline Naini Tal, India & 9006 & 14 & 3 & & 17 \\
\hline Maui, Hawaii & 9012 & 12 & 2 & 42 & 56 \\
\hline Mt. Hopkins, Arizona & 9021 & 8 & 5 & & 13 \\
\hline Olifantsfontein, South Africa & 9022 & 20 & 3 & 21 & 44 \\
\hline Island Lagoon, Australia & 9023 & 47 & 6 & 11 & 64 \\
\hline Dodaira, Japan & 9025 & & 8 & 7 & 15 \\
\hline Arequipa, Peru & 9027 & 2 & & 8 & 10 \\
\hline Debre Zeit, Ethiopia & 9028 & 3 & & 1 & 4 \\
\hline Dionysos, Greece & 9030 & 2 & 2 & 44 & 48 \\
\hline Natal, Brazil & 9039 & 5 & & 5 & 10 \\
\hline Rosamund, California & 9113 & 2 & 3 & & 5 \\
\hline Cold Lake, Canada & 9114 & 5 & 3 & 20 & 28 \\
\hline Johnston Island & 9117 & 10 & & 3 & 13 \\
\hline Mt. John, New Zealand & 9119 & 34 & 4 & 26 & 64 \\
\hline San Vito, Italy & 9120 & & & 68 & 68 \\
\hline Total & & 170 & 40 & 286 & 496 \\
\hline \multicolumn{6}{|c|}{Laser Returns} \\
\hline Olifantsfontein, South Africa & 7902 & 4 (6) & 1 (2) & & 5 (8) \\
\hline Arequipa, Peru & 7907 & 2 (9) & 11 (30) & 25 (181) & 38 (220) \\
\hline Mt. Hopkins, Arizona & 7921 & 2 (4) & 2 (4) & 2 (3) & 6 (11) \\
\hline Natal, Brazil & 7929 & 7 (20) & 2 (6) & 13 (46) & 22 (72) \\
\hline Dionysos, Greece & 7930 & & 1 (3) & \(10 \quad(37)\) & \(11 \quad(40)\) \\
\hline Total & & 15 (39) & 17 (45) & 50 (267) & 82 (351) \\
\hline
\end{tabular}
Geos 1 (6508901)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{2}{|l|}{Station} & \multirow[t]{2}{*}{January} & \multirow[t]{2}{*}{February} & \multirow[t]{2}{*}{March} & \multirow[t]{2}{*}{April} & \multirow[t]{2}{*}{May} & \multirow[t]{2}{*}{June} & \multirow[t]{2}{*}{July} & \multirow[t]{2}{*}{August} & \multirow[t]{2}{*}{Total} \\
\hline Location & COSPAR number & & & & & & & & & \\
\hline \multicolumn{11}{|l|}{Optical Returns} \\
\hline San Fernando, Spain & 9004 & 6 & 45 & 48 & 5 & 12 & 33 & 49 & 3 & 201 \\
\hline Naini Tal, India & 9006 & 13 & 23 & 16 & 13 & 4 & & 5 & & 74 \\
\hline Maui, Hawaii & 9012 & & & 9 & 12 & 17 & 21 & 27 & 22 & 108 \\
\hline Mt. Hopkins, Arizona & 9021 & 19 & 26 & 28 & 11 & 6 & 32 & 15 & 1 & 138 \\
\hline Olifantsfontein, South Africa & 9022 & 1 & 6 & 17 & 40 & 3 & 9 & 3 & 8 & 87 \\
\hline Island Lagoon, Australia & 9023 & 8 & 10 & 15 & 22 & 6 & 11 & 14 & 45 & 131 \\
\hline Dodaira, Japan & 9025 & 27 & 21 & 18 & 3 & 1 & 39 & 1 & & 110 \\
\hline Arequipa, Peru & 9027 & 4 & & 25 & 36 & 1 & 9 & 13 & 75 & 163 \\
\hline Debre Zeit, Ethiopia & 9028 & 8 & 15 & 11 & 12 & & 15 & 9 & 2 & 72 \\
\hline Dionysos, Greece & 9030 & 10 & 19 & 24 & 7 & 24 & 14 & 36 & 3 & 137 \\
\hline Natal, Brazil & 9039 & 4 & 7 & 8 & 13 & & 18 & 3 & 24 & 77 \\
\hline Rosamund, California & 9113 & 12 & 17 & 15 & & 9 & & & & 53 \\
\hline Cold Lake, Canada & 9114 & 3 & 27 & 14 & & & 17 & 5. & & 66 \\
\hline Johnston Island & 9117 & 3 & 5 & & & & 41 & 10 & 4 & 63 \\
\hline Mt. John, New Zealand & 9119 & 39 & 23 & & 30 & 108 & 28 & 167 & 162 & 557. \\
\hline San Vito, Italy & 9120 & & 33 & 12 & 1 & 38 & - & 63 & - & \(\underline{147}\) \\
\hline Total & & 158 & 276 & 260 & 205 & 229 & 287 & 420 & 349 & 2184 \\
\hline \multicolumn{11}{|l|}{Laser Returns} \\
\hline Olifantsfontein, South Africa & 7902 & & 7 (33) & 13 (220) & 26 (349) & 32 (390) & 36 (658) & 49 (1362) & 46 (1152) & 209 (4164) \\
\hline Arequipa, Peru & 7907 & 5 (10) & 2 (12) & 13 (113) & 23 (187) & 41 (461) & 58 (1442) & 12 (404) & 38 (742) & 192 (3371) \\
\hline Mt. Hopkins, Arizona & 7921 & 2 (3) & 4 (9) & 7 (63) & 13 (74) & 16 (53) & 2 (11) & & & 44 (213) \\
\hline Natal, Brazil & 7929 & 5 (13) & 16 (68) & 1 (7) & 19 (79) & 22 (110) & 22 (108) & \(12 \quad\) (74) & 19 (231) & 116 (690) \\
\hline Dionysos, Greece & 7930 & & \(4 \quad\) (8) & 3 (8) & 4 (8) & 6 (26) & \(14 \quad(66)\) & \(4 \quad\) (17) & & 35 (133) \\
\hline Total & & 12 (26) & 33 (130) & 37 (411) & 85 (697) & 117 (1040) & 132 (2285) & 77 (1857) & 103 (2125) & 596 (8571) \\
\hline
\end{tabular}

D1C (6701101)
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{2}{|l|}{Station} & & & & \\
\hline Location & \begin{tabular}{l}
COSPAR \\
number
\end{tabular} & March & April & June & Total \\
\hline \multicolumn{6}{|c|}{Optical Returns} \\
\hline San Fernando, Spain & 9004 & 6 & 17 & 51 & 74 \\
\hline Naini Tal, India & 9006 & 6 & 14 & & 20 \\
\hline Maui, Hawaii & 9012 & 15 & 18 & 47 & 80 \\
\hline Mt. Hopkins, Arizona & 9021 & 8 & 27 & 35 & 70 \\
\hline Olifantsfontein, South Africa & 9022 & 11 & 7 & 5 & 23 \\
\hline Island Lagoon, Australia & 9023 & 20 & 10 & 5 & 35 \\
\hline Dodaira, Japan & 9025 & & 16 & 1 & 17 \\
\hline Arequipa, Peru & 9027 & & 3 & 6 & 9 \\
\hline Debre Zeit, Ethiopia & 9028 & 3 & 4 & 2 & 9 \\
\hline Dionysos, Greece & 9030 & 2 & 19 & 33 & 54 \\
\hline Natal, Brazil & 9039 & 1 & 2 & 4 & 7 \\
\hline Rosamund, California & 9113 & & 10 & 14 & 24 \\
\hline Cold Lake, Canada & 9114 & & & & \\
\hline Johnston Island & 9117 & & & 12 & 12 \\
\hline Mt. John, New Zealand & 9119 & 1 & 7 & & 8 \\
\hline San Vito, Italy & 9120 & - & 32 & 11 & 43 \\
\hline Total & & 73 & 189 & 226 & 488 \\
\hline \multicolumn{6}{|c|}{Laser Returns} \\
\hline Arequipa, Peru & 7907 & & & 11 (134) & 11 (134) \\
\hline Mt. Hopkins, Arizona & 7921 & 1 (6) & & 6 (19) & 7 (25) \\
\hline Natal, Brazil & 7929 & & & 2 (11) & 2 (11) \\
\hline Dionysos, Greece & 7930 & - & & 36 (187) & 36 (187) \\
\hline Total & & 1 (6) & & 55 (351) & 56 (357) \\
\hline
\end{tabular}
DID (6701401)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{2}{|l|}{Station} & & & & & & & \\
\hline Location & COSPAR number & January & February & March & April & May & July & Total \\
\hline \multicolumn{9}{|l|}{Optical Returns} \\
\hline San Fernando, Spain & 9004 & 7 & 5 & 9 & 3 & 28 & 76 & 128 \\
\hline Naini Tal, India & 9006 & 15 & 8 & 19 & 3 & 7 & & 52 \\
\hline Maui, Hawaii & 9012 & & 8 & 25 & 4 & 50 & 55 & 142 \\
\hline Mt. Hopkins, Arizona & 9021 & 12 & 8 & 13 & 5 & 12 & 18 & 68 \\
\hline Olifantsfontein, South Africa & 9022 & 8 & 11 & & 14 & 2 & 5 & 40 \\
\hline Island Lagoon, Australia & 9023 & 24 & 18 & 9 & 14 & 7 & 5 & 77 \\
\hline Dodaira, Japan & 9025 & 4 & 1 & 6 & 4 & 2 & 5 & 22 \\
\hline Arequipa, Peru & 9027 & 1 & & & 4 & 4 & 5 & 14 \\
\hline Debre Zeit, Ethiopia & 9028 & 2 & 5 & & 2 & 3 & 6 & 18 \\
\hline Dionysos, Greece & 9030 & 1 & 2 & 7 & 5 & 21 & 46 & 82 \\
\hline Natal, Brazil & 9039 & & 12 & 1 & & & 4 & 17 \\
\hline Rosamund, California & 9113 & 10 & 1 & 4 & 2 & 13 & & 30 \\
\hline Cold Lake, Canada & 9114 & 2 & & & & & 5 & 7 \\
\hline Johnston Island & 9117 & & & & & & 5 & 5 \\
\hline Mt. John, New Zealand & 9119 & 27 & 22 & & 29 & 1.2 & 10 & 100 \\
\hline San Vito, Italy & 9120 & - & 10 & 6 & 15 & 6 & 94 & 131 \\
\hline Total & & 113 & 111 & 99 & 104 & 167 & 339 & 933 \\
\hline \multicolumn{9}{|l|}{Laser Returns} \\
\hline Arequipa, Peru & 7907 & 4 (5) & & & 2 (15) & 23 (272) & & 29 (292) \\
\hline Mt. Hopkins, Arizona & 7921 & 1 (2) & 2 (3) & 4 (17) & & 56 (439) & & 63 (461) \\
\hline Natal, Brazil & 7929 & & & & 2 (8) & 5 (26) & 3 (19) & 10 (53) \\
\hline Dionysos, Greece & 7930 & & - & - & 1 (2) & 21 (57) & 14(75) & 36 (134) \\
\hline Total & & 5 (7) & 2 (3) & 4 (17) & 5 (25) & 105 (794) & 17 (94) & 138 (940) \\
\hline
\end{tabular}
Geos 2 (6800201)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{2}{|l|}{Station} & \multicolumn{2}{|l|}{\multirow[t]{2}{*}{March April}} & & \multirow[t]{2}{*}{June} & \multirow[t]{2}{*}{July} & \multirow[t]{2}{*}{August} & \multirow[t]{2}{*}{Total} \\
\hline Location & COSPAR number & & & May & & & & \\
\hline \multicolumn{9}{|l|}{Optical Returns} \\
\hline San Fernando, Spain & 9004 & 3 & 10 & 2 & & 11 & 32 & 58 \\
\hline Naini Tal, India & 9006 & 1 & 18 & 1 & & & 2 & 22 \\
\hline Maui, Hawaii & 9012 & & 19 & 18 & 12 & 19 & 25 & 93 \\
\hline Mt. Hopkins, Arizona & 9021 & 5 & 30 & 5 & 1 & 3 & 1 & 45 \\
\hline Olifantsfontein, South Africa & 9022 & 25 & 15 & 5 & 1 & 3 & 8 & 57 \\
\hline Island Lagoon, Australia & 9023 & 23 & 23 & 9 & 7 & 12 & 17 & 91 \\
\hline Dodaira, Japan & 9025 & 3 & 16 & 1 & & & 3 & 23 \\
\hline Arequipa, Peru & 9027 & 24 & 21 & 14 & 3 & 18 & 60 & 140 \\
\hline Debre Zeit, Ethiopia & 9028 & 3 & 16 & 5 & 1 & 10 & 23 & 58 \\
\hline Dionysos, Greece & 9030 & 4 & 22 & 6 & & 4 & 37 & 73 \\
\hline Natal, Brazil & 9039 & 4 & 8 & 1 & 3 & 6 & 35 & 57 \\
\hline Rosamund, California & 9113 & 3 & 10 & 4 & 4 & & & 21 \\
\hline Cold Lake, Canada & 9114 & 10 & 10 & & & 3 & 26 & 49 \\
\hline Johnston Island & 9117 & & 2 & & 1 & 8 & 8 & 19 \\
\hline Mt. John, New Zealand & 9119 & 19 & 40 & 43 & 40 & 58 & 74 & 274 \\
\hline San Vito, Italy & 9120 & 1 & 15 & - & - & 1 & 22 & 39 \\
\hline Total & & 128 & 275 & 114 & 73 & 156 & 373 & 1119 \\
\hline \multicolumn{9}{|l|}{Laser Returns} \\
\hline Olifantsfontein, South Africa & 7902 & 7 (51) & 18 (108) & 20 (181) & 29 (328) & 20 (257) & 23 (239) & 117 (1164) \\
\hline Arequipa, Peru & 7907 & 2 (8) & 16 (106) & 23 (243) & 31 (594) & 13 (233) & 15 (173) & 100 (1357) \\
\hline Mt. Hopkins, Arizona & 7921 & 3 (17) & 10 (44) & 15 (95) & 1 (6) & & 1 (1) & 30 (163) \\
\hline Natal, Brazil & 7929 & & 9 (49) & 10 (66) & 12 (51) & 9 (77) & 14 (102) & 54 (345) \\
\hline Dionysos, Greece & 7930 & 2 (3) & 2 (2) & 2 (2) & & \(3 \quad(7)\) & 9 (40) & \(18 \quad(54)\) \\
\hline Total & & 14 (79) & 55 (309) & 70 (587) & 73 (979) & 45 (574) & 62 (555) & 319 (3083) \\
\hline
\end{tabular}
Peole (7010901)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{2}{|l|}{Station} & \multirow[t]{2}{*}{January} & \multirow[t]{2}{*}{February} & \multirow[t]{2}{*}{March} & \multirow[t]{2}{*}{April} & \multirow[t]{2}{*}{May} & \multirow[t]{2}{*}{June} & \multirow[t]{2}{*}{July} & \multirow[t]{2}{*}{August} & \multirow[t]{2}{*}{Total} \\
\hline Location & COSPAR number & & & & & & & & & \\
\hline \multicolumn{11}{|l|}{Optical Returns} \\
\hline Maui, Hawaii & 9012 & & 8 & 8 & 3 & 7 & 2 & 9 & 9 & 46 \\
\hline Olifantsfontein, South Africa & 9022 & & & 1 & 2 & & 2 & & & 5 \\
\hline Arequipa, Peru & 9027 & & & 4 & 10 & 10 & 11 & 8 & 18 & 61 \\
\hline Debre Zeit, Ethiopia & 9028 & 10 & 8 & 6 & 10 & 14 & 2 & 6 & 7 & 63 \\
\hline Natal, Brazil & 9039 & 11 & & 4 & 7 & 12 & 9 & 6 & 16 & 65 \\
\hline Johnston Island & 9117 & 1 & 14 & 2 & 1 & 1 & \(\underline{2}\) & 7 & - & 28 \\
\hline Total & & 22 & 30 & 25 & 33 & 44 & 28 & 36 & 50 & 268 \\
\hline \multicolumn{11}{|l|}{Laser Returns} \\
\hline Arequipa, Peru & 7907 & & & 6 (30) & 5 (14) & 2 (22) & 27 (208) & & 2 (14) & 42 (288) \\
\hline Natal, Brazil & 7929 & 1 (1) & & & - & \(8(21)\) & 37 (192) & \(\underline{2(20)}\) & 15 (58) & 63 (292) \\
\hline Total & & 1 (1) & & 6 (30) & 5 (14) & 10 (43) & 64 (400) & 2 (20) & 17 (72) & 105 (580) \\
\hline
\end{tabular}

\section*{EVALUATION OF LASER OPERATIONS}

\section*{J. Thorp}

We selected a 3-month period during ISAGEX for collecting laser data in order to evaluate the potential of the SAO laser system. We chose June, July, and August for its good weather and the South Africa station since it was ranging to only the Geos 1 and Geos 2 satellites. The evaluation, therefore, did not have to consider the questionable orbits or the bad aspect angles of the magnetically stabilized satellites.

Statistics were generated by using laser passage information compiled at each station. We considered both individual points (Figure 1) and total arcs (Figure 2). Two to 65 points were predicted per arc during this period, with an average of 35 points per arc. Arcs started and ended at \(20^{\circ}\) above the horizon.

For simplicity in preparing the graphs, each attempted point was considered an attempted arc and each successful point was considered a successful arc. This, however, did create some apparent discrepancies between the two graphs. The category "not attempted (N.A.) other," which includes pass conflicts, observer errors, and attempts not made owing to safety, shows a decided decrease when points are compared to arcs. The main reason for this is that points at the beginning and end of many passes were not attempted because of the hazards of operating below \(30^{\circ}\).

Another discrepancy is apparent when successful points are compared to successful arcs. However, as the following statistics show, 44 to \(64 \%\) of the arcs have less than 16 successful points:
\begin{tabular}{lrcccccc}
\hline & \multicolumn{5}{c}{ Percentage of arcs } & & \\
\cline { 2 - 5 } & \multicolumn{5}{c}{ Number of points } & & \(\begin{array}{c}\text { Total } \\
\text { predictions } \\
\text { (points/arcs) }\end{array}\)
\end{tabular} \(\left.\begin{array}{c}\text { Total } \\
\text { successes } \\
\text { (points/arcs) }\end{array}\right]\)


Figure 1. Percentage of total predicted points from South Africa.


Figure 2. Percentage of total predicted arcs from South Africa.

The percentage of successful points per pass seems to be related to weather (July had the best weather). During August, a period was reported as not attempted owing to a malfunction, but, in fact, the maintenance was scheduled for and performed during cloudy weather.

From the statistics, we can expect that, in general, with the laser system operating properly, about \(30 \%\) of the predicted points will be successful. When dealing with arcs, we see a higher percentage of overall success, in that about half the predicted arcs are successful. If we discount weather completely, the success ratio is higher, around \(70 \%\). The cases studied are ideal, and we feel that this is the best the system can do under present circumstances. When bad weather occurs and when satellites with less stable orbits are added, the success ratio drops substantially.

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\section*{PREDICTION PROBLEMS}

\section*{J. Latimer}

Generating accurate predictions is crucial to the successful use of static-pointing laser systems. The prediction-observation-orbit-determination cycle is a selfsustaining process when it works properly. In general, the process functioned reasonably well during ISAGEX, except for the Peole satellite, which, with a perigee of 500 km , is subject to a great deal of atmospheric drag.

Section 1 discusses the prediction accuracies obtained, and Section 2 presents a technique for improving poor predictions. Section 3 deals with the drag problem, especially as it relates to Peole.

\section*{1. PREDICTION ACCURACIES}

In Table 1 we have estimated the accuracy of predictions for the ISAGEX satellites during the saturation observing periods. The best way to express accuracies seems to be in topocentric arcminutes, since this bears most directly on the static-pointing laser system. In general, because of the beamwidth, prediction accuracies to 10 arcmin are desirable; a smaller accuracy presents problems for acquisition (although the problem is not completely insurmountable, as discussed in Section 2).

Predictions degrade exponentially in time. We give accuracies for the first and last day of each week's predictions. Interpolation will provide estimates for the other days. The figure for the first day is actually the rms of the orbit determination. Since we never found any discontinuity between the rms orbit fit and the residuals of the first day's observations using the extrapolated orbit, this seems an appropriate measure of the starting value.
Table 1. Prediction accuracies (arcmin) from the first to the last day of each week.
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline \begin{tabular}{l}
ISAGEX \\
period
\end{tabular} & Week & Peole & Geos 1 & Geos 2 & D1D & BE-B & BE-C & D1C \\
\hline \multirow[t]{2}{*}{I} & Jan. 6-13 & No predictions & 1-27 & & & 1. 3-42 & & \\
\hline & Jan. 13-20 & Non-SAO orbit ( \(25-\) min time difference) & 1.8-54 & & & 2. 3-20 & & \\
\hline \multirow[t]{5}{*}{II} & Jan. 20-27 & 4-X & 1. 3-3. 5 & & & 11. 5-X & & \\
\hline & Feb. 10-18 & 4. 5-1155 & 1-2 & & & & 1-3.5 & \\
\hline & Feb. 18-24 & 4-215 & 1.8-2.5 & & & & 1-3.8 & \\
\hline & Feb. 24-Mar. 3 & 4-90 & 0.8-2 & & & & 1-7 & \\
\hline & Mar. 3-10 & 4. 8-350 & 0.8-4.8 & & & & 1-X & \\
\hline \multirow[t]{3}{*}{III} & Mar. 24-31 & 4-63 & 0.8-13 & 1-10 & & 10.8-37 & & \\
\hline & Mar. 31-Apr. 7 & 5-167 & 0.8-2.5 & 0. 8-6.8 & & 0.8-2.8 & & \\
\hline & Apr. 7-14 & 7. 3-253 & 0.5-2.8 & 0.5-7.5 & & 0.8-X & & \\
\hline \multirow[t]{3}{*}{IV} & Apr. 28-May 5 & 0.8-850, 115 & 0.7-1.3 & 0.8-1.5 & 0. 8-10 & & & \\
\hline & May 5-12 & 5-232, 29 & 0.7-9 & 0.8-1.5 & 0. 5-9 & & & \\
\hline & May 12-19 & 4-54, 87 & 0.7-6 & 0.5-0.8 & 0.4-21 & & & \\
\hline \multirow[t]{4}{*}{v} & June 5-9 & 1. 5-220, 54 & 0.4-12 & 0. 4-12 & & & & 1-31 \\
\hline & June 9-16 & 2. 4-53, 38 & 0.4-5 & 0. 5-2 & & & & 1-22 \\
\hline & June 16-23 & 3. 5-38, 22 & 0.4-7 & 0. 3-11 & & & & 0.7-78 \\
\hline & June 23-30 & 0. 3-69, 55 & 0.3-6 & 0. 3-2 & & & & 0.5-X \\
\hline \multirow[t]{2}{*}{VI} & July 14-21 & 2-490, 81 & 0.5-1.3 & 0.5-2.5 & 0. 8-2 & & & \\
\hline & July 21-28 & 4.8-X, 59 & 0.5-4 & 0. 5-13 & 0.7-14 & & & \\
\hline \multirow[t]{3}{*}{VII} & Aug. 11-18 & 4.8-X, 116 & 0.5-5 & 0. 5-4.5 & & & 1-2.5 & \\
\hline & Aug. 18-25 & 1.3-53, 87 & 0.5-3.4 & 0.5-1.4 & & & 1-9 & \\
\hline & Aug. 25-Sept. 1 & 1. 5-132, 86 & 0.5-2 & 0. 5-2 & & & 0.8-1.3 & \\
\hline
\end{tabular}

The final day's accuracy is determined by a direct comparison of pointing angles from the expiring and the fresh predictions. This overlap day was generated to ensure operation even in the case of communications delays. Occasionally, there was no overlap day, and a comparison could not be made. This is indicated by an "X" in Table 1. Clearly, in any week, the final day's figure is uncertain by the amount of error in the next first day's prediction, which is almost always relatively small. The error is predominantly in the along-track direction; the across-track component is relatively insignificant.

Notice in the table the large errors in the Peole predictions. In an attempt to improve the accuracy, we generated Peole predictions twice a week beginning with period IV. From period IV on, we give two overlap-day comparisons in addition to the first orbit fit. Although there was noticeable improvement, Peole predictions remained troublesome (owing to atmospheric drag, see Section 3).

Finally, since we intended for these values to represent worst case situations, we chose to compare pointing angles at the culmination of each pass. Culmination, or the point of closest approach, is more sensitive to orbital errors than are lower elevation points.

\section*{2. FIELD UPDATING OF PREDICTIONS}

One of the useful properties of laser ranging systems is their ability to operate at low elevation angles. This permits errors in the predicted satellite mean anomaly to be quickly detected and corrected. Generally, the error in satellite mean anomaly is the only significant one in satellite ephemerides, so that when this error has been determined by field personnel, they can update predictions from the Computations Center in Cambridge by applying a simple correction to the firing time of their look angles.

Figure 1 demonstrates how the error in mean anomaly is determined. The satellite is predicted to be at position 1, azimuth, altitude, and range from the station, at epoch 1, and at position 2 at epoch 2. Suppose that at epoch 1 the observed and the predicted range differ by \(\Delta r\). (It is feasible to obtain returns in this case, provided the major component of the satellite's motion is toward the observer; this is true very
early in the pass for passes with high culmination.) We can assume that the predicted time interval between epochs 1 and 2 is correct, although the epochs themselves are in error by \(\Delta t\); and similarly \(s\), the range interval between positions 1 and 2 , is correct, although the range to position 1 is wrong. Then, \(\Delta t=(\mathrm{d} / \mathrm{s}) \Delta \mathrm{r}\), approximately.


Figure 1. Satellite-station geometry at a low elevation angle.

Actually, for convenience, \(\Delta \mathrm{r}\) and s are expressed in terms of propagation times rather than of distances. The time interval between successive predicted points, \(d\), is almost always 15 sec ; s varies, but it is typically around 0.5 msec for low elevation angles. In order to determine \(\Delta t\) quickly, station personnel use graphs of the linear relationships between \(\Delta t\) and \(\Delta r\) for the various values of \(s\) frequently encountered. The value \(\Delta \mathrm{r}\) can be resolved to 1 nsec , so that, for example, when \(\mathrm{s}=0.5 \mathrm{msec}\) and \(\mathrm{d}=15 \mathrm{sec}\),
\[
\Delta t=\frac{30 \mathrm{msec} \text { mean-anomaly correction }}{1 \mathrm{nsec} \text { observed range error }}
\]

Figure 2 is the station graph for determining the error in mean anomaly. Given \(\Delta \mathrm{r}\) and \(s, \Delta t\) can be quickly found.


Figure 2. Typical station graph for determination of mean-anomaly error: \(s=\) prop-agation-time differences between successive predicted points; \(\Delta r=\) (observed - predicted) range propagation time; \(\Delta t=\) mean-anomaly correction; \(\mathrm{d}=15 \mathrm{sec}\).

\section*{3. ATMOSPHERIC-DRAG EFFECTS}

The Peole satellite ( 701090 l ) has an apogee of 730 km and a perigee of 500 km . Predictions of it are difficult because not only is it subject to considerable atmospheric drag but also the drag is highly variable. Figure 3 shows the effects of drag varying by about an order of magnitude.
 Figure 3. The effects of geocentric angle and the \(10.7-\mathrm{cm}\) solar flux on the orbital acceleration of Peole. (a) \(\psi\), the geocentric angle between the perigee of Peole and the atmospheric hot spot; (b) \(\mathrm{F}_{10.7}\), \(10.7-\mathrm{cm}\) solar flux; and (c), n, the orbital acceleration of Peole.

For predictions to be useful for laser ranging, we estimate that the acceleration in mean anomaly (the empirically determined term that represents the effect of drag) ought to be correct within 5 or 10 parts per million. As can be readily seen from Figure 3, the term frequently changes by several times this amount in just a few days. The problem amounts to predicting the future state of the atmosphere, for which there is no satisfactory procedure.

An additional problem is that of obtaining an adequate orbital determination in the first place. Orbit determination was frequently weak because of insufficient observations.

We attempted to correlate the orbital acceleration of Peole with two parameters that are readily obtainable. The first is \(\psi\), the geocentric angle between the satellite perigee and the center of the atmospheric "hot spot, "i.e., the subsolar "bulge" in the atmosphere. We set the hot-spot center at the subsolar point delayed by \(30^{\circ}\) in longitude. The angle \(\psi\) is plotted in Figure 3, along with preliminary values of our second parameter, the \(10.7-\mathrm{cm}\) solar flux, which gives a rough indication of the solar influence on atmospheric activity.

It seems apparent that the shapes of both the \(\psi\) and the flux curves are reflected in the acceleration of Peole, although not in any quantitatively consistent manner.

We conclude that the only feasible way to solve the prediction problem caused by Peole's high drag is to increase the frequency of the prediction-observation-orbitdetermination cycle. Yet, a cycle faster than the present twice-weekly one is impractical. Possibly, an orbit-computation capability on location at the laser sites would permit a rapid and accurate enough iteration of the cycle for the drag problem to be overcome.

\section*{}

SIGNAL STRENGTH AND OBSERVABILITY

\section*{J. Latimer}

One problem that the laser-ranging technique continually presents is that of observability; that is, how can the observer know if a particular satellite and laser geometrical configuration is such that he is likely to receive a sufficiently strong return signal?

Although this question applies to all satellites, an especially striking situation is that of the four magnetically stabilized satellites (BE-B, BE-C, D1C, and D1D) when they are observed from stations in the southern hemisphere. It can be very difficult to observe them successfully because the retroreflector arrays, on the northseeking ends of these satellites, tend to face away from southern stations.

To see the effect of this problem, we selected at random 26 passes ( 342 observations) of BE-C observed from station 7907 (Arequipa, Peru), at latitude \(-17^{\circ}\). The observations were made between August 31 and November 5, 1971, and all give acceptably small orbit-fit residuals. Figure 1 is a plot of the passes, in the station's altitude-azimuth coordinate system. Indeed, when the satellite is in the northern half of the sky, it cannot be observed from this station. (Attempts were made to observe all passes.)

If we assume that the satellite is always oriented along the lines of the earth's magnetic field, we can calculate the aspect angle at the satellite between the symmetry axis (North) and the line of sight to the station. We used the spherical-harmonic representation (up to degree and order \(4^{*}\) ) of Cain, Hendricks, Langel, and Hudson (1967) as the geomagnetic model and derived aspect angles (see Appendix A) for each

\footnotetext{
*Although Cain's field is represented to degree and order 10, we found that the truncated field yielded aspect angles differing only by about \(1^{\circ}\). This is sufficient, considering that the satellite is likely to oscillate about the field direction with an amplitude of a degree or more.
}
of the 342 observations. Figure 2 is a plot of the reflective area versus the angle of incidence for the \(\mathrm{BE}-\mathrm{C}\) satellite. Cases 1 and 2 are measured from different radial angles. We used mean values, since there is no way to determine the radial angle, and the difference is slight. The histogram in Figure 3 shows the observed frequency of occurrence of each aspect angle. The significance of the aspect angle is its relation to the effective area of the satellite retroreflector array.


Figure 1. 26 passes of BE-C in the altitude-azimuth coordinate system of the Peru station.

We could avoid generating laser predictions for magnetically stabilized satellites that are impossible to observe by calculating the aspect angle, choosing a suitable limit for the angle, and suppressing all predicted points exceeding that limit. Although the
histogram of Figure 3 serves to confirm Figure 2 (there were no returns from aspect angles greater than \(110^{\circ}\), the Figure 3 cutoff), the slow fall-off of returns for large aspect angles suggests that we ought to consider the range equation if we wish to determine whether particular satellite-station geometries are likely to be observable.


Figure 2. Reflective area vs. angle of incidence of BE-C (from Minott, 1963).


Figure 3. Histogram of the aspect angles of satellite BE-C.

We estimated average signal strengths for all the data by using the range equation of Appendix B and the retroreflector area function of Figure 2. The results are displayed in Figure 4. The large population at low signal strengths is disturbing, but we must consider the following factors:
A. Error in the assumed beam divergence. Owing to the method of recording the transmitted beam divergence, some estimates are surely too high; therefore, some signals are greater than indicated.
B. Scintillation. We estimated only the average signal - the actual signal may vary by more than an order of magnitude (Jaffe, 1971), and some of the low average signals probably yielded high actual signals.
C. Weak-signal conditions. There are many more opportunities to attempt observation under weak-signal conditions than under strong-signal conditions. Thus, although the probability of success diminishes, the number of opportunities greatly increases, yielding a significant number of weak-signal returns.


Figure 4. Histogram of relative signal strengths for Peru BE-C data.

Point C is easily verified. We constructed a uniform distribution of 706 satellite positions at a height of 1150 km (typical for BE-C) over Peru such that all points were above the elevation angle limit of \(10^{\circ}\). By using a typical beam divergence ( 3.5 mrad ) and the same values for the magnetic field that we used in Figure 3, we computed the expected average signal strengths. From the histogram in Figure 5, it is clear that the population of weak-signal situations is large.

In conclusion, we can see that satellite configurations yielding aspect angles greater than \(110^{\circ}\) can be suppressed without loss of possible observations. We will attempt to measure signal strengths directly in order to have a better idea of both the minimum useful signal and the statistical behavior of scintillation. In addition, we intend to extend this analysis to the gravitationally stabilized satellites Geos 1 , Geos 2, and Peole. For these, the principal problem is that of obtaining returns at low station-elevation angles.


Figure 5. Histogram of estimated signal strengths for a uniform distribution of positions of BE-C from Peru.

\section*{REFERENCES}

CAIN, J. C., HENDRICKS, S. J., LANGEL, R. A., and HUDSON, W. V. 1967. A proposed model for the International Geomagnetic Reference Field 1965. NASA-TM-X-55845, 47 pp .

JAFFE, R. M.
1971. Signal strength fluctuations in a laser ranging system due to optical interference between the many reflectors on a satellite. JPL Technical Memorandum 391-218 to P. M. Muller, July 28.
MINOTT, P.O.
1963. Monthly research and advanced technology development activity report for April 1963. Optical Systems Branch, NASA/GSFC, Memorandum to H. J. Goett, April 23.

\section*{APPENDIX A}

\section*{COMPUTATION OF ASPECT ANGLE a}

Station position: \(\overrightarrow{\mathrm{S}}\)
Observation vector (line-of-sight direction and range): \(\overrightarrow{0}\)
Satellite position: \(\overrightarrow{\mathrm{P}}\)
Magnetic field at \(\overrightarrow{\mathrm{P}}: \quad \overrightarrow{\mathrm{F}}\)

\section*{Clearly,}
\[
\vec{S}+\vec{O}=\vec{P}
\]
and
\[
\cos a=-\frac{\overrightarrow{\mathrm{O}} \cdot \overrightarrow{\mathrm{~F}}}{|\overrightarrow{\mathrm{O}}||\overrightarrow{\mathrm{F}}|},
\]
where \(\vec{F}=\vec{F}(\vec{P})\); i. e.,
\[
\overrightarrow{\mathrm{F}}=-\nabla \mathrm{V},
\]
where
\[
\mathrm{V}=\mathrm{a} \sum_{\mathrm{n}=1}^{\mathrm{n}_{\max }}\left(\frac{a}{r}\right)^{\mathrm{n}+1} \sum_{m=0}^{n}\left(g_{n}^{m} \cos m \phi+h_{n}^{m} \sin m \phi\right) P_{n}^{m}(\theta)
\]
\(r, \theta\), and \(\phi\) are polar coordinates for \(\vec{P}\), a is the earth's mean radius, \(P_{n}^{m}\) are Schmidt normalized spherical functions, and \(g_{n}^{m}\) and \(h_{n}^{m}\) are the coefficients of the magneticfield representation.

\section*{APPENDIX B}

\section*{RANGE EQUATION}

We use the range equation from Lehr (1966):
\[
\mathrm{S}=\frac{\mathrm{E}}{2.86 \times 10^{-19}}\left(\frac{1}{\mathrm{R}^{4}}\right)\left(\frac{\mathrm{A}_{\mathrm{s}}}{\Omega_{\mathrm{t}}}\right)\left(\frac{\mathrm{A}_{\mathrm{r}}}{\Omega_{\mathrm{S}}}\right) \mathrm{T}^{2} \text { photons },
\]
where E is the output laser energy ( \(\approx 7.2 \mathrm{~J}\) ), R is the range to the satellite in megameters, \(T\) is the atmospheric transmission for Peru \(\left(\approx \mathrm{e}^{-0.071 / \sin a}\right.\), where \(a\) is the elevation angle), \(A_{s}\) is the effective area of the satellite reflective surface, \(\Omega_{s}\) is the solid angle of the reflected laser beam \(\left(=2.83 \times 10^{-9} \mathrm{sr}\right), \mathrm{A}_{\mathrm{r}}\) is the effective area of the receiver aperture \(\left(=0.21 \mathrm{~m}^{2}\right)\), and \(\Omega_{\mathrm{t}}\) is the solid angle of the transmitted laser beam. The numerical factor converts joules to photons at a wavelength of \(6943 \AA\).

In addition, we use \(\mathrm{N}=\mathrm{S} / 58\) to represent the photodetector conversion efficiency, where N is the number of photoelectrons generated.

\section*{REFERENCE}

LEHR, C. G.
1966. Satellite tracking with a laser. Smithsonian Astrophys. Obs. Spec. Rep. No. 215, 58 pp.

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\section*{DATA FORMATS}
B. R. Miller

\section*{1. INTRODUCTION}

The observational data formats described are the ones used by SAO in various computer programs.

The optical observation format is the same as that used in the past. It has been reproduced here to facilitate use by ISAGEX members.

The laser format has been revised to provide room for time designation to the nearest nanosecond and range to 0.01 m . Temperature is now given in degrees Celsius, pressure in millibars, and humidity in percent.

\section*{2. SAO OPTICAL OBSERVATION CARD FORMAT AND EXPLANATION}
\begin{tabular}{|c|c|c|}
\hline Field & Column & Description \\
\hline \multirow[t]{5}{*}{1} & 1-7 & Satellite identification \\
\hline & 1-2 & year of launch from 1900 \\
\hline & 3-5 & number of launch in that year \\
\hline & 6-7 & particle number \\
\hline & & Satellite 1959 al, for example, would be designated 5900101. \\
\hline \multirow[t]{6}{*}{2} & 8-12 & Observation number - Each observation of a satellite in a given year is designated by a different number. The source of an observation is also indicated by the observation number. \\
\hline & & 1-9999 miscellaneous \\
\hline & & 10000-19999 Baker-Nunn, field-reduced \\
\hline & & 30000-39999 Moonwatch \\
\hline & & 50000-59999 miscellaneous \\
\hline & & 70000-79999 photoreduced Baker-Nunn \\
\hline
\end{tabular}

\section*{Description}

3

4

5

6

24-25 hour
26-27 minute
28-29
30-33

34-52

34
35-36
37-38
39-40
41-43
44
45-46
47-48
49-50
Blank
second
blank
hours of a
sign of \(\delta\)

Station number - In the COSPAR numbering format, e.g., 9039 is Natal, Brazil.
\begin{tabular}{ll}
\(\frac{18-23}{18-19}\) & Date of observation \\
\(20-21\) & year, from 1900 \\
\(22-23\) & month \\
day
\end{tabular}

24-33 Time designation - Different types of observations are made using different time systems. Different times used in reporting SAO observations are as follows:
a. Field-reduced Baker-Nunn observations - generally WWV received before 1966, UTC(USNO) after.
b. Photoreduced Baker-Nunn observations - A.S

Note: A.S is a time scale with a fixed relation to NBS(A)
before April 1968 and to A. 1 after then. Values of (A.S-WWV emitted) are available in tabular form.
fraction of seconds, to 0.1 msec

The interpretation of the following field depends on the code in column 56. If column 56 is 0 , then the observation is right ascension and declination \((a, \delta)\).
minutes of a
seconds of a
fractions of seconds to 0.001 sec
degrees of \(\delta\)
minutes of \(\delta\)
seconds of \(\delta\)

\section*{Description}

51-52 fractions of seconds to 0.01 sec
If column 56 is 1 , the observation is altitude and azimuth corrected for atmospheric refraction. Altitude and azimuth observations not corrected for atmospheric refraction have 3 in column 56.

34-36
degrees of azimuth; 999 indicates azimuth is in mils

37-38
39-40
41-43
37-41

44
45-46
47-48
49-50
51-52
45-51

34
35-42
43
44
45-52

53-58
53
minutes of azimuth
seconds of azimuth
fraction of seconds to 0.001 sec
mils to nearest tenth if azimuth is in mils; decimal point assumed before column 41
blank
degrees of altitude; 999 indicates altitude is in mils
minutes of altitude
seconds of altitude
fractions of seconds to 0.01 sec
mils to nearest tenth if altitude is in mils; decimal assumed before column 51
If column 56 is 4 , the observation is direction cosines ( \(\ell, m\) ), corrected for refraction; a 5 in column 56 indicates the observation is in direction cosines uncorrected for refraction.
sign of \(\ell\) (blank or minus)
\(\ell\) to 8 decimal places (decimal point implied before column 35)
blank
sign of \(m\) (blank or minus)
m to 8 decimal places (decimal point implied before column 45)
\[
\mathrm{n}=\ell^{2}+\mathrm{m}^{2}
\]

Index codes
time-precision index
Code Standard error in timing \(\sigma_{t}\)
\(0 \quad\) No estimate
1
\(\sigma_{t} \leq 0.0003 \mathrm{sec}\)
\(20.0003<\sigma_{t} \leq 0.002\)
\(30.002<\sigma_{t} \leq 0.005\)
\begin{tabular}{llll} 
Code & \multicolumn{2}{c}{ Standard error in timing \(\sigma_{t}\)} \\
4 & 0.005 & \(<\sigma_{t} \leq 0.02\) \\
5 & 0.02 & \(<\sigma_{t} \leq 0.05\) \\
6 & 0.05 & \(<\sigma_{t} \leq 0.2\) \\
7 & 0.2 & \(<\sigma_{t} \leq 0.5\) \\
8 & 0.5 & \(<\sigma_{t} \leq 2.0\) \\
9 & \multicolumn{4}{c}{\(\sigma_{t}>2.0\)}
\end{tabular}
\begin{tabular}{|c|c|}
\hline Code & \(\underline{\text { Standard error in direction } \sigma} \mathrm{D}\) \\
\hline 00 & No estimate \\
\hline 01 & \(\sigma_{D} \leq 1!5\) \\
\hline 02 & \(1: 5<\sigma_{D} \leq 2: 5\) \\
\hline 03 & \(2!5<\sigma_{\text {D }} \leq 3!5\) \\
\hline 04 & \(3!5<\sigma_{\mathrm{D}} \leq 4!5\) \\
\hline 05 & \(4!5<\sigma_{\mathrm{D}} \leq 5!5\) \\
\hline 06 & \(5!5<\sigma_{\mathrm{D}} \leq 6!5\) \\
\hline 07 & \(6!5<\sigma_{\mathrm{D}} \leq 7!5\) \\
\hline 08 & \(7: 5<\sigma_{D} \leq 8: 5\) \\
\hline 09 & \(8!5<\sigma_{\mathrm{D}} \leq 9!5\) \\
\hline 10 & \(9!5<\sigma_{\mathrm{D}} \leq 10!5\) \\
\hline 11 & \(10!5<\sigma_{\mathrm{D}} \leq 11!5\) \\
\hline 12 & \(11: 5<\sigma_{\mathrm{D}} \leq 12: 5\) \\
\hline 13 & \(12.5<\sigma_{\mathrm{D}} \leq 13!5\) \\
\hline 14 & \(13.5<\sigma_{\mathrm{D}} \leq 14.5\) \\
\hline 15 & \(14!5<\sigma_{\mathrm{D}} \leq 15!5\) \\
\hline 16 & \(15!5<\sigma_{\mathrm{D}} \leq 16!5\) \\
\hline 17 & \(16!5<\sigma_{\mathrm{D}} \leq 17!5\) \\
\hline 18 & \(17!5<\sigma_{\mathrm{D}} \leq 18.5\) \\
\hline 19 & \(18!5<\sigma_{\mathrm{D}} \leq 19: 5\) \\
\hline 20 & \(19: 5<\sigma_{\mathrm{D}} \leq 20: 5\) \\
\hline 21 & \(20 \cdot 5<\sigma_{\mathrm{D}} \leq 22^{\prime \prime}\) \\
\hline 22 & \(22^{\prime \prime}<\sigma_{\mathrm{D}} \leq 23: 5\) \\
\hline 23 & \(23!5<\sigma_{\mathrm{D}} \leq 26^{\prime \prime}\) \\
\hline 24 & \(26^{\prime \prime}<\sigma_{\mathrm{D}} \leq 29^{\prime \prime}\) \\
\hline 25 & \(29^{\prime \prime}<\sigma_{\mathrm{D}} \leq 33^{\prime \prime}\) \\
\hline
\end{tabular}
\begin{tabular}{|c|c|}
\hline Code & Standard error in direction \({ }^{\text {d }} \mathrm{D}\) \\
\hline 26 & \(33^{\prime \prime}<\sigma_{\mathrm{D}} \leq 38^{\prime \prime}\) \\
\hline 27 & \(38^{\prime \prime}<\sigma_{\mathrm{D}} \leq 45^{\prime \prime}\) \\
\hline 28 & \(45^{\prime \prime}<\sigma_{\mathrm{D}} \leq 54^{\prime \prime}\) \\
\hline 29 & \(54^{\prime \prime}<\sigma_{\mathrm{D}} \leq 1!1\) \\
\hline 30 & \(1: 1<\sigma_{\mathrm{D}} \leq 1: 3\) \\
\hline 31 & \(1: 3<\sigma_{\mathrm{D}} \leq 1: 7\) \\
\hline 32 & \(1: 7<\sigma_{\mathrm{D}} \leq 2: 1\) \\
\hline 33 & \(2: 1<\sigma_{\mathrm{D}} \leq 2: 7\) \\
\hline 34 & \(2!7<\sigma_{\mathrm{D}} \leq 3: 5\) \\
\hline 35 & \(3: 5<\sigma_{\mathrm{D}} \leq 4: 4\) \\
\hline 36 & \(4: 4<\sigma_{\mathrm{D}} \leq 5.8\) \\
\hline 37 & \(5!8<\sigma_{\mathrm{D}} \leq 7: 5\) \\
\hline 38 & \(7: 5<\sigma_{D} \leq 9.7\) \\
\hline 39 & \(9: 7<\sigma_{\mathrm{D}} \leq 13^{\prime}\) \\
\hline 40 & \(13^{\prime}<\sigma_{\mathrm{D}} \leq 17^{\prime}\) \\
\hline 41 & \(17^{\prime}<\sigma_{\mathrm{D}} \leq 22^{\prime}\) \\
\hline 42 & \(22^{\prime}<\sigma_{\mathrm{D}} \leq 28^{\prime}\) \\
\hline 43 & \(28^{\prime}<\sigma_{\mathrm{D}} \leq 37^{\prime}\) \\
\hline 44 & \(37^{\prime}<\sigma_{\mathrm{D}} \leq 49^{\prime}\) \\
\hline 45 & \(49^{\prime}<\sigma_{\mathrm{D}} \leq 1.1\) \\
\hline 46 & \(1: 1<\sigma_{\mathrm{D}} \leq 1: 4\) \\
\hline 47 & \(1.4<\sigma_{\mathrm{D}} \leq 1.8\) \\
\hline 48 & \(1: 8<\sigma_{\mathrm{D}} \leq 2.4\) \\
\hline 49 & \(2: 4<\sigma_{\text {D }}\) \\
\hline
\end{tabular}
\begin{tabular}{cl}
\begin{tabular}{c} 
Code
\end{tabular} & \multicolumn{1}{c}{ Explanation } \\
\cline { 1 - 2 } & \(\quad\)\begin{tabular}{l} 
right ascension, declination \\
1
\end{tabular} \\
2 & altitude, azimuth (corrected for refraction) \\
3 & not used \\
4 & \begin{tabular}{l} 
altitude, azimuth (uncorrected for refraction) \\
\\
5
\end{tabular} \\
\begin{tabular}{l} 
tion) (direction cosines, corrected for refrac- \\
tim (direction cosines, uncorrected for refrac- \\
tion)
\end{tabular}
\end{tabular}

This index refers to the date of equator and equinox to which the observation is referred. (Meaningful for right ascension and declination only.)
\begin{tabular}{|c|c|}
\hline Index & Date \\
\hline 0 & Date of observation \\
\hline 1 & 1855.0 \\
\hline 2 & 1875.0 \\
\hline 3 & 1900.0 \\
\hline 4 & 1950.0 \\
\hline
\end{tabular}
instrument description index

Code
0
1 telescope, aperture less than 5 inches
2 apogee telescope, astronomical refractor or reflector, theodolite, visual
3 Baker-Nunn camera, photographic
4 small missile tele-camera, tracking cameras with focal length 20 inches or greater, photographic

5 cinetheodolite, tracking cameras with focal length less than 20 inches, photographic

6 Harvard meteor camera (Super-Schmidt), photographic

7 stationary telescope or camera with focal length equal to or less than 10 inches, photographic
8 direction observation associated with a laser instrument

9 other instruments

65-70 Conversion from the UTl to the A. 1 time system, i.e., A. 1-UT1

65 minus if A. 1 - UTl is negative, or tens digit if positive and necessary

66 units digit of A. 1 - UT 1 in seconds
67-70 decimal fraction A. 1-UT 1

71-80 Identification information
71-75 film number
76 contains an \(S\) if observation is simultaneous

77-78 Passive or flash information
a. If the satellite is a flashing one, column 77 will contain an F and column 78 will contain the number of the flash as it actually occurred. (This does not apply to ANNA flashes.)
b. If the satellite is passive, columns 77 and 78 will contain the frame number.

13
79
Contains the letter associated with the film number if any; otherwise it will be blank.

80 Used for balloon satellites to indicate a precision reduction correction for satellite size has been added; otherwise blank.

11 71-80 Moonwatch - used for apparent magnitude information.

Precisely reduced Baker-Nunn observations are given in the coordinate system of the SAO Star Catalog (equator and equinox of 1950.0). The positions have been corrected for annual aberration, and the star positions, for proper motion to the year of observation. No corrections have been applied for diurnal aberration or parallactic refraction.

The time of the observation is given in A.S (Smithsonian Atomic Time), defined by the expression
\[
\text { A. } \mathrm{S}-\mathrm{UTC}(\mathrm{USNO})=6 . \mathrm{S}^{\mathrm{S}} 140768+0.002592000(\mathrm{~T}-39856.0)
\]
for the time period February 1, 1968, to the present; \(T\) is the Universal Time in Modified Julian Days (MJD), and 39856 is January 1, 1968:
```

MJD = Julian Day - 2400000.5 .

```

\section*{3. SAO LASER OBSERVATION FORMATS AND EXPLANATION}
\begin{tabular}{|c|c|c|}
\hline Field & Column & Description \\
\hline \multirow[t]{5}{*}{1} & 1-7 & Satellite identification \\
\hline & 1-2 & year of launch from 1900 \\
\hline & 3-5 & number of launch in that year \\
\hline & 6-7 & particle number \\
\hline & & Satellite 1964 64A, for example, would be designated 6406401 \\
\hline \multirow[t]{4}{*}{2} & 8-12 & Observation number \\
\hline & & 20000-29999 uncorrected observation \\
\hline & & 70000-79999 corrected observation \\
\hline & & 90000-99999 GOCC laser and direction observation \\
\hline 3 & 13 & Blank \\
\hline 4 & 14-17 & Station number - In the COSPAR numbering format, e.g., 7921 is SAO laser site at Mt. Hopkins, Arizona. Station designations in the 7000 series include laser sites. \\
\hline \multirow[t]{4}{*}{5} & 18-23 & Date of observation \\
\hline & 18-19 & year from 1900 \\
\hline & 20-21 & month \\
\hline & 22-23 & day \\
\hline \multirow[t]{5}{*}{6} & 24-35 & Time designation - Different types of observations are made using different time systems. Time systems used are indicated by the code in column 57. \\
\hline & 24-25 & hour \\
\hline & 26-27 & minute \\
\hline & 28-29 & second \\
\hline & 30-35 & fraction of seconds to \(1 \mu \mathrm{sec}\) \\
\hline \multirow[t]{2}{*}{7} & 36-52 & Interpretation of the following field depends on the codes in columns 56 and 57. \\
\hline & 36 & blank \\
\hline
\end{tabular}

37-46

47-48
49-52

8

57

58
\(9 \quad \underline{59-64}\)
59
60
56
range in meters (decimal implied before column 45 allows range observations to be specified to 0.01 m )
blank
value of refractivity correction to 0.01 m - code 1 in column 57

54-55 standard deviation of the range \(\sigma_{r}\) in meters and tenths of meters
observation type index
\begin{tabular}{cc}
\(\frac{\text { Code }}{1}\) & \\
8 & \\
8 & Explana \\
& altitude, azi \\
& laser range
\end{tabular} code to indicate time system and corrections applied

Code Explanation
\(0 \quad\) UTC emitted at transmission of laser pulse no corrections applied to range
1 A.S time at reception of laser pulse - refractivity correction given in columns 49-52 but not applied to range
2 A.S time at reception of laser pulse - refractivity correction applied to range
3 UTC time at the satellite (GOCC observations refractivity correction applied to range)
Instrument description index
\begin{tabular}{cc} 
Code & Explanation \\
Laser observation
\end{tabular}

Range correction - pulse shape and size - codes 0 and 1 in column 57
sign
meters

\section*{Description}
\begin{tabular}{|c|c|c|}
\hline & 61 & 10 centimeters \\
\hline & 62 & centimeters \\
\hline & 63-64 & blank \\
\hline 11 & 65-77 & Pressure, humidity, temperature-uncorrected observa \\
\hline & & only, code 0 in column 57 \\
\hline & 65-66 & blank \\
\hline & 67-70 & barometric pressure in millibars \\
\hline & 71-72 & humidity in percent \\
\hline & 73 & sign of temperature \\
\hline & 74-76 & temperature to tenths of degrees Celsius \\
\hline & 77 & blank \\
\hline & 65-77 & Conversion from the UT1 to the A. 1 time system \\
\hline & 65 & i.e., A. 1-UT1 (actually A.S), code 2 in column 57 minus if A.S-UTl is negative, or tens digit if positive and necessary \\
\hline & 66 & units digit of A. S - UT1 in seconds \\
\hline & 67-72 & decimal fraction of A.S-UT1 \\
\hline & 74-77 & blank \\
\hline 12 & 78-80 & Identification information \\
\hline & 78 & blank \\
\hline & 79 & type of laser pass \\
\hline & & Code Explanation \\
\hline & & 0 night pass, satellite illuminated \\
\hline & & 1 night pass, satellite in shadow \\
\hline & & 2 daylight pass \\
\hline & 80 & blank \\
\hline
\end{tabular}

\author{
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}

Each observing station performs the following calibration exercises:
A. Determination of the fundamental system delays to be applied to measured time intervals.
B. Precision check of the fundamental time-interval measurement.
C. Determination of the reliability of an individual measurement.
D. Calculation of the reproducibility of an individual measurement.

Even with this elaborate procedure, further checking and information are necessary; two approaches are employed:
A. Pass analysis, which depends on trends, noise, and consistency of the data.
B. Comparison of observations with precision orbit computation.

This latter approach, the primary tool used, is the subject of this section.

Laser data have a precision of 1 m and an accuracy that is probably somewhat better. On the other hand, our current set of geodetic parameters is based primarily on \(20-\) to \(40-\mathrm{m}\) camera data. From the determination of these geodetic results, Gaposchkin and Lambeck (1970, 1971) estimated the station coordinates to an accuracy of 5 to 10 m . The accuracy of our orbit computation is no better than 5 to 10 m for optimum satellites (Geos 1) and significantly worse for others (Peole). It is clear that we cannot obtain an unambiguous validation of \(1-\mathrm{m}\) laser data with a \(10-\mathrm{m}\) tool.

Orbits were computed with 4 days of data every 2 days, i.e., with 2 days of overlap. Four days was usually sufficient to compute a reliable orbit, yet short enough to minimize the effects of errors in the gravity field and station coordinates.

Owing to the paucity of laser data, we have also incorporated other tracking data, including minitrack and field-reduced Baker-Nunn. As the main computation center, SAO received laser data from CNES and GSFC to be used for predictions. We included these data in our reference orbits, although we recognized that such data did not have the benefit of refinement and validation by the originating agency. The supplementary laser data used were from stations 7050 (GSFC), 7060 (Guam), 7815 (moved to 7809, Haute Provence), and 7804 (San Fernando). The data from station 7820 (Dakar) were not included because of uncertainties in the coordinates and timing. It was not our purpose to validate the data from other agencies, but our success in validation indeed hinged on having these supplementary laser data.

Very bad data were easy to detect. The detection of poor data proved to be very difficult. Ultimately, three rules were applied:
A. The successive orbits had to be consistent. Nonuniform evolution of the mean orbital elements indicated poor data had been included in the orbit determination.
B. Orbital residuals had to be consistent (i.e., reproducible) in the two computed orbits. Using a conservative estimate of the orbital accuracy, observations that had residuals greater than 50 m ( 500 m for Peole) were rejected.
C. The run of residuals in a pass had to be smooth. A large variation in residuals ( \(\geq 50 \mathrm{~m}\) ) from point to point ( 1 sec apart) must be an observational error as there is no unmodeled orbital perturbation of that magnitude. The run (trend or signature) of the residuals is a very powerful device and hinges on having more than 10 observations per pass. Many passes early in ISAGEX did not have sufficient data for this test to be applied, and these were consequently very difficult to validate.

The applications of these rules had varying degrees of success. The confidence we can put in the validated data varies considerably from satellite to satellite and from period to period. Some of the data in periods II and III are questionable. Such data are distributed so that only when they are combined with laser and precisionreduced camera data can a final evaluation be made.

A few data were analyzed on a pass-by-pass (short arc) basis. This involved use of the orbit-computation program as an interpolation device, determining the parameters \(I\), e, \(\mathrm{M}_{0}\), and \(n\). The other orbital elements were held fixed at the values computed from long-arc computation. This interpolation will reject the groww outliers and will provide a measure of the noise (i.e., the precision) of the data. We have found this noise to be 50 to 100 cm . Figure 1 gives the typical residuals from a short-arc orbital fit.


Figure 1. Residuals for short-arc fit of satellite 6508901 from station 7907.

It became apparent that the trend of residuals for 4-day arcs would provide the same information when plotted as the short-arc residuals do. Further, bad points not discarded by the short-arc procedure often resulted in poor short-arc fits. The longer arcs discarded these bad points. In addition, many passes had so few points (<5) that short arcs were not possible. Finally, short-arc computation provided no estimate of the accuracy. We therefore abandoned this approach and proceeded to use 4-day arcs.

The SAO and the French laser data were input to our processing program in A. \(S\) time at reception of the laser pulse and without the refraction correction given. The refraction correction was computed following Tsiang and Lehr (this volume),
and time was converted to A.S at the satellite for orbital computation. The GSFC laser data were input in UTC time at the satellite and with refraction applied to range; again, time was converted to A.S at the satellite. We also used as inputs the tesseral harmonics determined in Standard Earth (II) (Gaposchkin and Lambeck, 1970), the station coordinates listed in Table 1 below, and the polar-motion values published by BIH. In Table 1, the station coordinates are given in megameters and the weight in arcseconds for all but the laser stations, which are in meters. Photoreduced weights where applicable are in parentheses.

Figure 2 gives the residuals of the same pass as in Figure 1 as it appears in a 4 -day arc. The interpolation curve has been drawn to illustrate the short-arc fit. This particular pass was chosen because it was representative of SAO's ISAGEX laser data. It has a noise level of \(0.93-\mathrm{m}\) rms and an accuracy of \(2.2-\mathrm{m} \mathrm{rms}\).

Figure 3 gives the history of the semimajor axis for Geos 1 (6508901). The scatter appears to be less than 1 m , which indicates that the data are good, welldistributed, and well-understood. Geos 1 is a well-behaved satellite with an inclination of \(59^{\circ}\). It is visible to many stations, thus giving rise to a nicely distributed data set. Its eccentricity of 0.071 presents no problems in the determination of the argument of perigee, and if we look at M2 as an indication of the magnitude of the modeled drag, we see that it is about \(5 \times 10^{-7} \mathrm{rev}^{2} \mathrm{day}^{-2}\), or \(0.65{\operatorname{arcsec} d y^{-2}}^{-2}\).

Figure 4 gives the semimajor axis for Peole (7010901), with a scatter of less than 10 m . With an inclination of \(15^{\circ}\), an eccentricity of 0.016 , and M 2 of \(3 \times 10^{-5} \mathrm{rev}\) day \({ }^{-2}\) ( 0.65 arcmin day \({ }^{-2}\) ), orbit computation for Peole is very challenging. Its inclination made it visible to very few stations (7907, 7929, 7060, 4492, 4800), four of which are in South America, giving rise to a badly distributed view of the satellite's orbit. Satellites with small inclinations make computation of both the argument of node and the argument of perigee difficult. The modeling of the drag and the solving for \(\dot{\omega}\) and \(\dot{\Omega}\) became impossible. We assumed values for \(\dot{\omega}\) and \(\dot{\Omega}\), held them fixed, and solved for the drag modeling. This worked satisfactorily, but not so well as we had expected for laser data.

Table 1. Station coordinates used in the validation.
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{2}{|l|}{Station} & \multirow[b]{2}{*}{X} & \multirow[b]{2}{*}{Y} & \multirow[b]{2}{*}{Z} & \multirow[b]{2}{*}{Weight (arcsec)} \\
\hline Location & \begin{tabular}{l}
COSPAR \\
Number
\end{tabular} & & & & \\
\hline \multicolumn{6}{|c|}{Baker-Nunn Stations} \\
\hline San Fernando, Spain & 9004 & 5. 105588 & -0.555228 & 3.769667 & 34 (4) \\
\hline Naini Tal, India & 9006 & 1.018203 & 5.471103 & 3. 109623 & 34 (4) \\
\hline Maui, Hawaii & 9012 & -5.466053 & -2. 404282 & 2.242171 & 34 (4) \\
\hline Dakar, Senegal & 9020 & 5.886264 & -1.845649 & 1.615282 & 34 (4) \\
\hline Mt. Hopkins, Arizona - & 9021 & -1.936782 & -5.077704 & 3.331916 & 34 (4) \\
\hline Olifantsfontein, South Africa & 9022 & 5.056125 & 2.716511 & -2.775784 & 34 (4) \\
\hline Island Lagoon, Australia & 9023 & -3.977765 & 3.725101 & -3. 303034 & 34 (4) \\
\hline Dodaira, Japan & 9025 & -3. 910438 & 3. 376362 & 3.729219 & 34 (4) \\
\hline Arequipa, Peru & 9027 & 1. 943040 & -5.804207 & -1.796491 & 34 (4) \\
\hline Debre Zeit, Ethiopia & 9028 & 4.903750 & 3. 965201 & 0.963872 & 34 (4) \\
\hline Dionysos, Greece & 9030 & 4. 595200 & 2.039446 & 3. 912606 & 34 (4) \\
\hline Natal, Brazil & 9039 & 5. 186461 & -3.653856 & -0.654325 & 34 (4) \\
\hline Rosamund, California (AF) & 9113 & -2.450011 & -4.624421 & 3.635035 & 34 (4) \\
\hline Cold Lake, Canada (AF) & 9114 & -1. 264838 & -3. 466884 & 5.185467 & 34 (4) \\
\hline Johnston Island (AF) & 9117 & -6. 007402 & -1. 111859 & 1. 825730 & 34 (4) \\
\hline Mt. John, New Zealand (AF) & 9119 & -4. 533650 & 0.761590 & -4.407772 & 34 (4) \\
\hline San Vito, Italy (AF) & 9120 & 4.613757 & 1. 485659 & 4. 132293 & 34 (4) \\
\hline & & Laser Stations & & & (m) \\
\hline GSFC, Maryland & 7050 & 1. 130673 & -4. 831368 & 3. 994112 & 2 \\
\hline Guam Island & 7060 & -5.068960 & 3. 584106 & 1.458756 & 2 \\
\hline Salisbury, Australia & 7803 & -3. 939150 & 3. 467040 & -3.613265 & 2 \\
\hline San Fernando, Spain & 7804 & 5. 105606 & -0.555251 & 3.769633 & 2 \\
\hline Haute Provence, France & 7809 & 4.578352 & 0.457957 & 4.403160 & 2 \\
\hline Haute Provence, France & 7815 & 4.578371 & 0.457950 & 4.403134 & 2 \\
\hline Dakar, Senegal & 7820 & 5. 886271 & -1.845666 & 1.615250 & 2 \\
\hline Olifantsfontein, South Africa & 7902 & 5.056125 & 2.716511 & -2.775784 & 2 \\
\hline Arequipa, Peru & 7907 & 1. 942775 & -5.804081 & -1.796933 & 2 \\
\hline Mt. Hopkins, Arizona & 7921 & -1.936781 & -5.077701 & 3.331921 & 2 \\
\hline Natal, Brazil & 7929 & 5. 186461 & -3.653856 & -0.654325 & 2 \\
\hline Dionysos, Greece & 7930 & 4.595207 & 2.039446 & 3. 912595 & 2 \\
\hline
\end{tabular}


Figure 2. Residuals for 4-day fit for satellite 6508901 from station 7907.


Figure 3. Evolution of semimajor axis for satellite 6508901 during ISAGEX periods IV and V.


Figure 4. Evolution of semimajor axis for satellite 7010901 during ISAGEX period VII.

The scatter for the remaining satellites was about 3 m , except for \(\mathrm{BE}-\mathrm{B}\) (6406401), which had so few data that its scatter was about 7 m .

The number of validated SAO laser points broken down by station is given in Table 2. The ISAGEX periods covered by these data are listed in Table 3, where an X indicates a validated time period (Note: in-between time periods were also validated where possible).

Table 2. Number of validated SAO laser points.
\begin{tabular}{lcrrrrr}
\hline & \multicolumn{7}{c}{ Station } \\
\cline { 2 - 7 } Satellite & 7902 & 7907 & 7921 & 7929 & 7930 & Total \\
\hline 6406401 & - & 12 & 13 & 5 & - & 30 \\
6503201 & 3 & 162 & 3 & 41 & 33 & 242 \\
6508901 & 3960 & 1408 & 164 & 528 & 106 & 6166 \\
6701101 & - & - & 9 & 3 & 88 & 100 \\
6701401 & - & 235 & 412 & 43 & 109 & 799 \\
6800201 & 937 & 466 & 137 & 276 & 42 & 1858 \\
7010901 & - & 35 & - & 230 & - & 265 \\
Total & 4900 & 2318 & 738 & 1126 & 378 & 9460 \\
\hline
\end{tabular}

Table 3. Validated ISAGEX periods.
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline Satellite & \[
\begin{gathered}
\text { II } \\
\text { Feb. 15-Mar. } 8 \\
\text { MJD 40997-41019 }
\end{gathered}
\] & & \[
\underset{\substack{\text { Mar. } \\ 41035-A p r . ~}}{ } 15
\] & & \[
\begin{gathered}
\text { IV } \\
\text { Apr. 29-May } 20 \\
41070-41092
\end{gathered}
\] & & \[
\begin{gathered}
\text { V } \\
\text { June 5-26 } \\
41107-41129
\end{gathered}
\] & & \[
\begin{gathered}
\text { VI } \\
\text { July 14-31 } \\
41146-41164
\end{gathered}
\] & & \[
\begin{aligned}
& \text { VII } \\
& \text { Aug. } 11-31 \\
& 41174-4195
\end{aligned}
\] \\
\hline 6406401 & & & X & & & & & & & & \\
\hline 6503201 & X & & & . & & & & & & & X \\
\hline 6508901 & X & X & X & X & X & X & X & X & X & X & X \\
\hline 6701101 & & & & & & & X & & & & \\
\hline 6701401 & & & & & X & & & & X & & \\
\hline 6800201 & & & X & X & X & K & X & \(\mathbf{x}\) & X & X & X \\
\hline 7010901 & X & & X & & X & & X & & & & X \\
\hline
\end{tabular}

As a result of our validation process, we can conclude the following:
A. The use of error signatures can be successfully employed to validate data. By error signatures, we mean:
1) Large scatter in the data.
2) Inconsistent mean orbital elements.
3) Lack of systematic trends in the residuals.

To observe these signatures, we require more than 10 observations per pass.
B. The overall accuracy of the data is at least 1 m , and the noise level, 60 cm .
C. The overall accuracy of our geodesy is 10 m for Geos-type satellites, as can be seen from the standard error of unit weight for orbital fits as reported by Gaposchkin and Mendes (this volume). This accuracy confirms the evaluation of Gaposchkin and Lambeck (1970, 1971). The weight for laser data was taken as 2 m .
D. With routine validation by use of Geos 1 data, a monitoring of a station's reliability to the \(10-\) to \(20-\mathrm{m}\) level is possible.

\section*{REFERENCES}

GAPOSCHKIN, E. M., and LAMBECK, K.
1970. 1969 Smithsonian Standard Earth (II). Smithsonian Astrophys. Obs. Spec. Rep. No. 315, 93 pp.
1971. Earth's gravity field to the sixteenth degree and station coordinates from satellite and terrestrial data. Journ. Geophys. Res., vol. 76, pp. 4855-4883.

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\section*{ORBITAL ELEMENTS FROM ISAGEX DATA}

\author{
E. M. Gaposchkin and G. M. Mendes
}

The process of data validation hinges on a consistent evolution of orbital elements derived from the data. These elements are also useful for other analyses-e.g., of zonal harmonics and earth tides - and are given here. Only those ISAGEX data available at SAO for validation were used; the orbital elements will be revised when the complete set of data has been processed. However, these orbits are an improvement over previous ones, especially for Peole ( 7010901 ), the first geodetic satellite with such a low inclination. This catalog of satellite data is similar to those previously published by SAO (see, e.g., Miller, 1968). The orbital elements are mean elements in the sense that the effects of the short-period perturbations due to the earth's gravity field have been eliminated.

The SAO mean elements have been computed from observations covering several days and are given in the form of a table. The successive sets of elements are essentially independent of each other. Only entries that are considered satisfactory are given. A missing epoch is due to insufficient data.

The times of epoch in the mean elements are reckoned in Julian Days. For convenience, the number 2400000.5 has been subtracted to provide an abbreviated rotation, which we call "Modified Julian Days" or MJD.

The units of the orbital elements are degrees for angular quantities, megameters for metric quantities, and revolutions for the mean anomaly.

The tabulated values of SAO mean elements are as follows:
line 1 Satellite designation, epoch, first and last dates, standard error of unit weight, number of observations, and date the orbit was computed.

Preceding page blank
line 2 Epoch.
line \(3: \omega\), argument of perigee and secular rate.
line \(4 \Omega\), right ascension of the ascending node and secular rate.
line 5 I, inclination.
line \(6 \quad e\), eccentricity.
line 7 M , mean anomaly; n , mean motion; and higher polynomial term(s) as appropriate.

These elements include the long-period perturbations evaluated with the zonal harmonics as tabulated in Gaposchkin and Lambeck (1970, 1971); the short-period perturbations due to the geopotential computed with the numerical values given in Gaposchkin and Lambeck (1970, 1971); and the lunar perturbations with period \(2 \lambda_{\text {( }}\) computed as given by Gaposchkin (1966). The fundamental constants GM, ae, and the velocity of light are as follows:
\[
\begin{aligned}
& \mathrm{GM}=3.986013 \times 10^{20} \mathrm{~cm}^{3} \mathrm{sec}^{-2} \\
& \mathrm{ae}=6.378155 \times 10^{8} \mathrm{~cm} \\
& \mathrm{c}=2.997925 \times 10^{10} \mathrm{~cm} \mathrm{sec}^{-1}
\end{aligned}
\]

The station coordinates used are given in Gaposchkin and Mendes (this volume).

The reference system adopted results in the inclination and the argument of perigee referred to the true equator of date. The right ascension of the ascending node is reckoned from the mean equinox of 1950.0 along the corresponding mean equator to the intersection with the moving true equator of date, and then along the true equator of date. To transform the right ascension of the node to the mean equinox of date, the following formula is used:
\[
\Omega^{\circ}=\Omega^{\circ}(\mathrm{SAO})+3: 508 \times 10^{-5}(\mathrm{MJD}-33281)
\]

The orbital theory used defines the mean elements. The orbit-computation program employed here is based on a set of formulas due to Aksnes (1970) for the short-period oblateness perturbations. The relationship between the mean elements from Von Zeipel's method as previously published and those by the Lie transform method is given by Aksnes (this volume).

\section*{REFERENCES}

AKSNES, K.
1970. A second-order artificial satellite theory based on an intermediate orbit. NASA TR 32-1507, 34 pp.; also in Astron. Journ., vol. 76, pp. 10661076.

GAPOSCHKIN, E. M.
1966. Orbit determination. In Geodetic Parameters for a 1966 Smithsonian Institution Standard Earth, ed. by C. A. Lundquist and G. Veis, Smithsonian Astrophys. Obs. Spec. Rep. No. 200, vol. 1, pp. 77-183.

GAPOSCHKIN, E. M., and LAMBECK, K.
1970. 1969 Smithsonian Standard Earth (II). Smithsonian Astrophys. Obs. Spec. Rep. No. 315, 93 pp.
1971. Earth's gravity field to the sixteenth degree and station coordinates from satellite and terrestrial data. Journ. Geophys. Res., vol. 76, pp. 4855-4883.
MILLER, B., prepared
1968. Satellite orbital data, No. E-8. Smithsonian Astrophys. Obs. Spec. Rep. No. 290, 27 pp.

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5129931167
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.692087343513 .7537762692
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41047.000006
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\(-2.5582226726\) 95.6434764927
\(-1.0799686443\)
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\section*{ORAIT}

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\(41051.00000 \quad 2,1051327142\)
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87.0023513990
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.772399167 , 70.12534964
\(-.5833702426\)
11.968ก930683
9.8182F-06
\(-6.2535 E-06\) \(40999.00100 \quad 2.8434130479\)
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\(198589=\mathrm{A}\)
PITT \(=\mathrm{PTT1} \mathrm{PTl}\)
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331.6575485335
.6272650946
\(=82.5172397733=-245186191\)

\subsection*{59.3627754591}
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\(.35269149441 .9679759819 \quad 3.0638 E-66 \quad 3.06\)
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\(41503-4001-6=4110506\)
\(z_{-1273}\)
\(94=017!3 / 72\)

\(41003.00000 n=0.000000\)
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.5569786875
78. 2208966415
\(-2.249342835 ?\)
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\ 59,3685275853
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                4007.0000% 2-7273109105
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                        p31:11
    41005,0005n\ - 6000तf
    334.3348794527=6447535275
        73.5249735744 -2. 2476990873
        59.3587055772
            .7716699138=0000039027
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                41205.20n0n 1, 1.4791436917
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    410070000000
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    335.635n317457
                                6579776477
        69.0313633199-2.c47773964n
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            .0716634739 .0000039769
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    41007.00000, 107400517950
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0.070009
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2.2438995551
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1.7289E-07
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2.9556039362
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\(41021.000000 \quad 3.090090\)


59.3713398797
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41 221.00000 1.3104936087
345.769769 37.61351n 59.372083 . 0713634 .708760211.9679215 8.0733968

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\(\begin{array}{ll}41023.000300 & 0.090097 \\ 346.1688220588 & .6592263929\end{array}\) \(33.0816754173 \quad-2.2471847134\)
\(=59.37 n 5948927\)
\(0211543755=900053576\)
\(.647210251=11.9579484932 \quad-4.2779 E-05=-1.9215 E-05\)
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41023.00070 \quad 1.5942996698
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\(31037.000000=0.029500=1.6567932434\)
\[
1.6299975054 \quad-2.2458241773
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\(59.3757640750 \quad-.211337717\)
- \(2716632946=090090135\)

\(41737.0400=2.1450054027\)
\(356.254625-1.66829 \quad 59.375252 \quad .7715865=195846111.9679485 \quad 8.1733847\)
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\(356.5654358791=654719729\)
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\(59.3738101770 \quad-0000277342 \bar{n}\)
\(.0716609257-0.900001275\) ñ

\[
41937-19 n 91 \quad 1-6521561689
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125589.4 4, 41041 4039.0 41043.0 2.1011 % 201001714/72
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- -7.3558717539=-2.2452745261
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.0716593744 0.0000004394
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        41n45.00.071 1.122420272?
        1.500926 -16.393339 59.373286 .0716949 . 9394274 11.9679464 8.07733855
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        .65444477289
        -2.2464421038
    -.00017683055
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        -7811月2-21
        -5.0725E-n9 2.-3993E=077
            41047.10001# = 1.3105409110%
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    -25.32748.18035
        0.0090002
        .0553729270!
                            -2.2465865975
        59.3729093005 -.0002029073
        .0716530377 =.0000009575
        .8140170231: FE=9679506693
            41249.00000
                1.375:233697
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\(.685821382 \quad 1.9879493798 \quad 1.1799 E-07 \quad 1-2221 E-08\)
\(41353.00010 \quad 2.8445 \overline{114693}\)

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41055.000300
.6513 A63594
\(-38.9 n 77941344\)
59.368589977
\(-0.92363 \ln 39\)
\(-2716531573=\) - 0 27n917n32

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\(8.036787-38.770104 \quad 59.768154\).0718311 . 618964511.9679587 8.0733804
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\(-43.3002054204\) 59.3709814115
-2.2446 ก26509
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.0008316603
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\(.5576571687 \quad 11.0579531378\)

\(41057.00090 \quad .9512889971\)
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\(12.2536141751 \quad .6532763847\)
\(-56.791773531=-2.246393735\)
\(59.3703470704 \longrightarrow 0002194369\)
\(.071647940=00003791375\)

\(41.663 .00900 \quad 2.4937655317\)
\(13.233839-56.744787 \quad 59.369643 \quad .7719363 \quad .362633511 .9679526 \quad 8.0733828\)
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\(41065.000000 \quad 2.0100003\)
\(13.5647201921 \quad .6535373477\)
\(-61.2756336349-2.247: 4417347\)
59.3701072907 =-6901218492
\(.7154724993=-0000005621\)

41.765 .0000 2.1592126239
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\(.071347597 \quad 40000095021\)

\(=417.57 .00097 \quad 16939247634\)
\(15.833782-65,733137-59.369116 \quad .7710996 \quad .23449181109679531\) 8.0733827 OREIT

. 0002597156
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41071.0000073 .000000

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\(.0716476415 \quad=.900017717\)
.1099665653 IT．967955437？1．262ÑE－06 7．4587E－07
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PFTL 41080.00000 ？
.5579662419
\(-2.2467010974\)
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\(1.9574480598 \quad-3.2603 \mathrm{E}-07 \mathrm{O}=204 \mathrm{AE}-09\)



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25.9527467192
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\(41085.000000 \quad 0.000000\)

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P11 \(=\) 217t 4509200090 ？ 31.182547979 238.0619261293 59．3696？ 29375 .17716589295 .4358793353
.6514 .883099
－2．2465632919
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\(-234672 \mathrm{~F} 1126\)
－0100713975
\(11.9679484826 \quad-4.52077 E-07 \quad \overline{1} . \overline{1} 58 \overline{1} E-0 \overline{7}\)

\(-0000000\)
.6525234849
-2.24673273 ñ
0009020357
\[
11.9679444686=-1 \cdot 3559 \mathrm{E}-07 \quad \mid 1.5556 \mathrm{~F}-07
\]
27.2598496955
251.547865779
\(=59.3704789175\)
.0716623352
27.2598496955
251.547865779
\(=59.3704789175\)
.0716623352
27.2598496955
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\(=59.3704789175\)
.0716623352
27.2598496955
251.547865779
\(=59.3704789175\)
.0716623352 ． 6282 n68in8
ORAIT
P171 89 － 111
P111
41088.00011
28.573691647
247.0433161918
59.3701739774
\(=\)
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P1 11
41088.00011
28.573697647
247.0433161918
59.3701739774
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Pl11
41088.000911
28.573691647
247.0433161918
59.3701739774
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P111
41088.0011
28.573691647
247.0433161918
59.3701739774
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.0716799945 \\
.2435296961 & \(11.967941587 ? ~\) & \(2.32 ? 7 E-08\) & \(1.90063 E-09\)
\end{tabular}

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41100.000097 36.4n.32852544 220.0890595557 59.3686979938 느=.07668.77999 .1794099015
ORAII
\(1965-89 \mathrm{~A}\)
P111
PT11
\(41102-41100.0 \quad 4110490001.4889 \quad 2001171\)
41102.000007
37.7088618946
215.5956562132

59,3682542945 .0716831044 .1152877996
ORATT
 41104,000000 39,1141344415 211.1024844213 59.3580476758
- 71686 TR24
.0511633641
n. 000000
.5536746337
\(-2.2467686225\)
11.9679383609
pal pili p31111
\(=6522520023\)
\(-2.2465352357\)
6.000090
.6534962328
\(-2.2457940053\)
.0000726975
11.9679366831
-3.4426E-07 \(6.4888 \mathrm{E}-08\)

9000092
.6510500197
\(-2.2455778435\)
30000003859
11.9679409199
7. \(3636 E=19\)
1. \(1548 \mathrm{~F}=01\)

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.0716884567 .9973377457 9.

P1II 4108.000901 41.6288031357 202.1159029239 59.3677597374 1716913891 9279737916
pत̈ PT11 P3I11
7. 000000 .6548664617
\[
-3.2467684777
\]
.0000014473
\(11.9679327079-1,2909 E-07 \quad-3.7 \overline{50} 1 \varepsilon-09\)

0000011
.6552610941
\(-2.2464361853\)
2000915234
\(4109-41107.0 \quad 411100299\)
\(154 \quad 1271171\)
P111 P11.1

ค31111
0.000000
\(-2,2457538426\)
-19144961533
.0000006251
\(11.9679392937 \quad-5.933\) n̄E-07 \(7.1863 E-09\)

\(202-171 / 71\)
(.0010)
.6507330261
000
- 09415215


P!11 p̄̄1 \(\quad\) P3й111
0.000000
-2512977127
-22459227975
\(-205929997\)
\(11.9679447677 \quad-5.0989 E-07 \quad-9.497 \overline{0} E-077\)

211 0111
2010907
.6534743624
-2.2467646364
.0000302444
17957936157
\(4.7651 E-98\)

-!11 PTll p3ill
5.300050

6525417739
2.2457327796

0
11.7579364493 -6.2644E-07 3.9011E-07


41119.000007
-
\(-2.2467790053\)
-. 0000132545


0111 pill p3111
0.000000
-2. 2469387595
\(-2002120756\)
11.9679362935

0.000000
.5544772376
\(-2.2468239918\)
- 0007967926
11.967932 nラ72 \(-6.5165 \mathrm{En7}\) -
pl1] pili p3ill1
0.300000
\(-2.246978=0646\)
\(-0000213629\)
- 0000013247
11.9579362355
. \(0392 E-09\)
\(291-12<174\)

9.00299の
\(-2.246755: 427\)
-0000521010
11.9679377468
\(-2.7955 \mathrm{E}=\mathrm{n} 8\)
-3.9659 F-n 8


OREITI
\(1965 \cdot 89.4\)
41129.000090
0.000000
54.932855785?
- 77171384 23 . 2495629177

0111 151.000000 56.6515121903 59.3672522123 0717155549 1854332433 PTī 41133.000000 57.9554957n35 45.9461495527 .0717 ? 10085 .1213113955

1965-89-4
41735.000009

141 452363 กำ 59.36792 27925
- 1717252371

60571779091
\(1965-89\)
Plil pili 41137.000000 \(=6\). 5 द2 333.9 3.95B-13 .36 -
 .9930464621
OREIT
\(19 \in 5=89=A\)
P111 4139.000901 61.8789242872
132.4659932253

4685064215
. 717317157
9289211778
\[
\begin{array}{r}
-2246725533 \mathrm{l} \\
=0.00039-7414
\end{array}
\]
11.9579352474

4131 - 4129 \(019=0111=03111\)
.6528508132
-2.2467345643
- 00020896 ñ4
\(10-67030\)
\(41133-41131.0\) P3111!
\(\overline{0} .000000\)
\(-2.2468798174\)
-. 03001067632
17.9679422951
\(4135-41330-4137.9\) \(.5595 \quad 309271771\)

15.009009
\(-2.2459232894\)
. 10000359774
11507933633
4137 -4135.0
p31111
3.000000
\(-2.2468334369\)
-0市20785512
11.9679353259
\(-5.5362 F-08\)
1. -989
\(75-125171\)
317 0171
0.900009
.5524536304
\(-2.2466035295\)
- 0000935001
. 1000022378
11.9679395736
3. 5935 - 07
3. \(0424 \mathrm{E}=07\)

\(0 R 1 T T\)

ORBET

ORQIT
 Plil plī plil pili poilil 41155.000000
0.000000
\(-2.2469353972\)
.00000022464
\(11.9679446459 \quad 5.3899 E-07 \quad-4.5837 E-08\)

\(0.00000 ?\)
.6543071763
\(-2.246989787^{7}\)
-. 00014417578

\(0114160 \quad 1.8940 \quad\) IT8 \(41158.0 \quad 411162.01\)
0.000000
.65197737 포
2.2464553547

03005723497
\(17.9679541040 \quad 1.0629 E-06 \quad-6.00941 E-07\)

PEIE 5.000000

24
- 5013908160

0000079177
\(11.7679339791 \quad 3.3441 \mathrm{~F}-\mathrm{n7} \quad-5.6575 \mathrm{E}-09\)
\(000=.5263948739\) \(77.132127-81.795315=59.363799 .0729594 \quad 191003111.9679330-8.0733913\)

ORgIT
\(1955: 89\)
Plil
○门！

41164.000000
0.000000
778.2269270077
76.297219429 n

E． 6555709925
59.369517218
\(\cdot 9717836917\)
2.2469894626
\(1274835021 \quad 11077036465\)
\(41154.00000 \quad 1.5021823137\)

OROIT
196589 A
\(411664164 \%\) ． \(41168 \% 0=1114\)
\(26312716+71\)
P1il
41165.000000 79.53309 .90517 \(71.90-30798071\) 59.369289499 ？
0.0909942795 .06339 ว2563
0.000050
.5568957114
2． 2 车 68933134
.00000607571
17.9679385875

PIT1 PTIt p3It1
\(4 \overline{1} 169\)
41170.0

之． 6150
\(3 \overline{4} 6\) 12／16／71

196589 A

0.209000

5530626218
4158.000000
\(=80.844691 \pi 167\) 67.3197219338 59．36945กワ？27 ． 0717936569 \(.0717936569 \quad .0009002311\)
\(.9992775292 \quad-11.9679532954\)
\(2.2469 \pi 37135\)
－．\() 001551678\)
\(2.48435-07\) 41574.000000

ה．000000
84.775116251 ？
.6511062596 53.82757 かス753 59.3690063225
.0717975599 .906974270
\(2-247844389\)
－7917589275
.0002013217
1.9679623898
－9－2726F－019
1.5592 ） 344 －12／16／II

P1T1 P1T P31tI
0.000000
． 654633478 ；
\(-247297605\)
0900213397
.000009672

\subsection*{11.9679504751 \\ \(3.5427 E-07=-4.6053 E-08\) \\ }
\(.935187297 ?\)

\section*{ORGIT}

196589 A
41172
PILE Plit Tl 41172.00 नीnत 87.462321970 ？ 58.3208692359 59.3592 त17530
.0717966458 \(0.871081+67\) 196589 A
\(\mathrm{PTHF}=\mathrm{PTO}\)
0.090007

5548546454
2． 2473523947
－． 10000117250
.0000007382
\(11.967949 \times 997 \quad-16553-06 \quad=-7738-07\)

196589 A
P1II＝pill
41170.000000
\(.8 \overline{1} 29\)
2̄9 12／16／71


ORBII

.7512002971
.523079974
\(.000002359 ?\)
13.9602809476
4.6ñ73E-06
\(4.9439 \mathrm{E}-07\)
ORBIT

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ORFIT

```

```

P1i1 Piji pil pili pulill
41123.00000n 0.000000
290.9557243466=5=5.9844722425
174.4347652172 - 4.7427801671
39.958>377753
. 512031808-0.0000002239
.4055737n58 13.8.503383859 3.694ñE-06 -7.3818E-07
41123.00000 2.2057914277

```

```

OREIT
196711FA
4125 盾1230, 41127.0 =2. 1489
99-02\&07/72
P1II -PIT
07T P11! P31111
41125.000900 -0.000000
302.9307629393 5.9919821858
164.9491135279 = - 4-7428707273

```

```

            .1252545565 13.8503369703 % 6.5994E-05% 7.1384E-07
                41135.00000 2.17999314531
    303.245598 i64.95年46 39.959752 .0507459 . 1253749 13.86033770 7.3207050
    ```


```

    4127.000000 0.000000
    314.9010183468 5.9852755295
    \55.45898677?9 -4.7437775564
        39.9593859587
        0511999217= - 000n016276
        8469983191 = 13-8603787505 = 8.01550-06 \ = 5964E-06
            41727.0000n-2.5896n71776
        315.313500.755.465988 39.960759 .0509089 . 8458460 13.86ñ3789 7.3200903
    ```



OREIT

\(67 \overline{10} 1401\)
P1LI \(\quad\) PII
41088,000000
\(238.1727+19942\)
100.0605732508 39.4322378316 .0839778125 .9411145165 OROIT
\(67 n 14 \pi t\)
\(\mathrm{PTII}=\mathrm{PtIt}\)
41090.000000 248.9958839257 92.213940535 39.433527399 - 8839791294 - 288322436

ORE IT
\(67514 \pi 1\)
PII \(=\) PIT \(41149.00099 n\)
\(202.933344954 n\)
207.3126234934 39.4314439179 .0838999996 ORET
\(67 n 154\)
\(\mathrm{P11}=011\) 41150.000000 213.5549020564 198:867279354: 39.431515789 .n. 838937154 .6503630765
pit -pil o3ill
0.900000
. 4105059598
-.0000514595
\(13.0935745152 \quad-1-0572 E-06 \quad-4.497 \mathrm{E}-07\)

大तl PIt
0.000030
\(5.410430458 \overline{1}\)
\(=0000011586\)
\(41988 \quad 41086.0 \quad 41090.0\)
2.9464
\(8 \overline{1} 05 / 04 / 72\)
0.000009
5.4293113816
-4.221 .937541 ?
-. \(000051548 ?\)
13.1935958753
3.9亲27E=06
9.3i3if=07

\(011=40=0.0111\)
0.000000
5.4107959175
\(-4.3212769856\)
.0000015346
\(13 \cdot 0936139487\)
\(4 \overline{1148}-41146.0\)
41150.0
1. 73

22̄3 12/28/71

A 100000
5.150391846
\(-4 \cdot 2231986041\)
.9000018741
103093796955
\(9-395 \mathrm{~F}=97\)
\(415 \%=41148.1\). 4152.0 - 12
\(200-1272971\)
Pl
?.000000
5.4111086027
\(-2,22705752 \pi\)
\(-2099050707\)
\(13.1937978914==4.0251 \mathrm{E}=06\)

\(\qquad\)
\(\qquad\)

ORETT

 OREIT
 PIHE——PT

P11－Pll
41039．00070
0.070000
\(67.3342694598-1.5251825507\)
F244．4998491656＝F．3999495948
\(105.8102362651=-921116896\)
\(=.031977 n 677=-.07043936589\)
\(.3617727561 \quad 1 ? .3305554249 \quad-6.8598 \mathrm{E}-08 \quad-9.8375 \mathrm{E}=09\)
\(41139.00039 \quad 1.5576300815\)
\(67.936364 \quad 244.498448 \quad 105.811954 \quad .0327910 \quad .360100012 .8306554 \quad 7.7073076\)
OREIT


41041.0000000000000
\(64.0824103159 \quad-1.6261944378\)
247.29933282 ה6 \(\quad 1.4000384736\)
\(175-9094641594 \quad\) eno9034 3699
31969879 － 101090187 A

\(4174 T .00000=1.2975326434\)
\(64.765472 .247,297745\) 195．908971 ．0327534 ．0212094 12．83076999 7．7073059 ORAIT


41043.000000 人

6 6． \(8214587189 \quad-1.532171142 ?\)
خे50．1019213372 1．4F14846411
\(155.8128717912 \quad .0919313604\)
\(-931955971=0004076201\)

\(41443.00 n 01=2.2157920317\)
 ORDIT
6800201
41345
\(41043 . n\)
41947.0
₹．\(\cdot \overline{3} \overline{4} 27\)
ミフ8

PI若 PT
PNI
41045.000020
0.000009
55.5929597658
－1．6265393527
252.932682636

1． \(42044942 \pi 7\)
.2006768120
105.8100226195
\(.0319792633-.0000008005\)
\(.3457484679 \quad 12.8305710563=-3.3444-07 \quad 3-8313 E-68\)
\(41345.00007=1,1477438684\)
\(58.430303 \quad 252.919735\)


\section*{OREIT}
 \(41349.00000-1.6834144756\)

6800201
\(410251.1049 .7-41053.012307\)
T57 02703FEZ2
Pl|
41051.0000000 .000000
47.8596429412
\(-1.6209658863\)
261.3078714 Sn5 \(\quad 1.4003926778\)
105. \(812222 n 130 \quad 000048545\)
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\(4105 \% .00000 \quad 1.836659100\)


PLI PTH \(=41053.000000\)
\(=47053-4051,00550\)

44.5153335446
15.007000
-1. \(623964894 ?\)
\(264.1592246254 \quad 1.4010530075\)
105.8121731913
\(\square 0319724931\)
\(-9917233 \times 60\)

\section*{41753.0 nins 1.9434511755}
.17000805399

OROIT
6800201
41055
41053.0 41057.0
2.4572

「ラ2 02/03/72
\(-41055,00007 \pi\) 41.3771432133 266.911329335 105.9125728922
. 7319744125
.5523323936
\(4 \pm 155.00000=2.4571745047\)


ORDI


P111
0111
p111
p31111
41057.000007
7.090000
\(38-144015533\) F 5153744764
569.71?8585532 \(\quad 4476216253\)
\(105.813899250, \quad 100893902 ?\)
\(.0319895193 \quad 000091129\)
\(.3136095429 \quad 12.3305421620\)
\(4.04725-06\)
\(\overline{1.4962 E-06 ~}\)
41057.00000
- 65 574556079

ORFET

OREIT

Plul \(\leq\) P位
P3T11
\(41061.0000 \pi=00000\)
\(\begin{array}{rr}31.64812394 .77 & -1.6347491429 \\ 275.3167780399 & 1.4011181055\end{array}\)
\(155.415371251 \quad 0014266976\)
\(\qquad\) \(-101977654=-0007137794\)


OREIT
\(68=002=0\)
P1ET
e111 Q111 \(41063=41051.0\) 4196500 091 "3111I
41063000001 3.002001
\(29.4292796249=7.5215690035\)
\(278.1139864792 \quad 1.4007731138\)
\(10 \boxed{5} .8142535 .984 \quad \cdot 3000534125\)
\(031987617=00009982\)
\(\equiv .277696277 \quad 122836589954\) \(\qquad\) 24. \(4789 E-97\) - \(7841 E-07\)
\#
45063.00971 \(989 F 20637\)
 ORA I T
6800201

\(41065.001000-001907\)
\(=25.1646917173=-1923719172\)
\(290.9206347 n 47 \quad 1.4012211528\)
\(105.8145899679-0002147132\)
\(.031983238 \bar{n} \quad .000002293 \bar{n}\)

\(\qquad\)

\[
41969.00090 \quad 1.8625005083
\]
20.159525 286.5218i1 105.815084 . 7322632 . 2774599 12.8306609 7.7073094 OROIT Pl1
41071.0000010
15.4467424799
289.3272771 ñ28 \(\quad .6153859884\)

E195.8169384532 \(=097359632\)
\(0319871275=-0002029151\)
 \(41771.0000 \% 1.7992080272\)

OREIT
196892
\(111=\frac{0111}{1072,00000}\)
13.8750735975
290.7280908759
105.3155077341 \(=.0319971514\) \(=.2764723959\)
ORAIT
1968 त2
PIII Pī1
41074.000000
100.5855345732
293.5310797656 175.8157544 .548 .0319876481
.4348428748
ORGIT
1268 ก2 \(A\)
PLIT P1
41076.000009 7.3449949349 296.3325777784 105.8154842189 1 1319837192
1965606990
p. 1

231 211070 - 410
E20000:
\(-1.519284753 ?\) 1.4010563797
 3.300070
\(-1.5184721979\) 1.4003560467

5-90000322233
12.930552658 n
\(-6.37045-07\)

ORCIT
P111 \(\quad\) P111
41079.000907
4896862771
299.134969849
Din Pili p3lili
0.0000 0n
-15-6194-138976
1.4510947351
.131393700 .7574652497 12,8396561187
-3.11315-07
-2.1909E-07
OREIT
\(1968=02 \mathrm{~A}\)
PIF 01 2110710.031111
\(41080.00000 n\)
.8751571383
1.210901
\(301.9369725115 \quad 1.4511893775\)
105.8163392926
-2319928499 -0.091974704

ORPII
1969 万2 A 41092.000000
5.000000
-3697645429
-345
\(105.9151+32+31\)

ORgIT
1968 27 4

\(=41084.006 \ln 9\)
\(-5.5424153549\) 347.5399185082 \(105.8178 \% 67653\)
\(\ldots-19959 \times n 2\)

ORPIT

41086.000000
0.000000
\(8-9443677937\)
310.3435514435
\(105.817012 n 497\) -3 319979327
\(=.0000051096\)
.4027587798
12.3306400176
\(-2 \cdot 3275 E-06\)
\(4.3363 E-077\)
ORAIT
\(1968-924\)
P111 -6T1
\(0 n t\)

\(2=000000\)
\(-17.0789567919\)
\(-1.6787218794\)
313.1455922367 105.9167960701
\(=0319956943\)
.0646123782

1-516235年

\subsection*{1.46nst 34578}
\(41084=41082.0 \quad 10.6841\)
3.030000
-T. 6215657256
1.4974521795

1.4012142024
=-gnnoño4392
12.8306556139
4.4 E12E-0
\(4.45595=09\)







1968 ? \(A\)
    41166.000000
    \(\because .00000^{\circ}\)
        62.4383644328
        15458764
        -
- 8

\(5.43445-06\)
OREII
P1968 \(\square^{-A}\) PII 41170.000000
0.000000
\(-15.5773316722\)
1-4 00033634 ?
\(=01933324271\)
-.0000285471
12.8305250684
1.9849E-05
- 7 3
 Q11
\(=0.000000\)

P1969 192
41174.000000
208.5416325537
\(-73.6357726523\)
105.8129573726
.0319895985 .5008034090
5.1793E-09

OREIT
PR11
41176.006909
205.3087329652
76.43734ํ.. .16209559 E
12.8306478948
7.4588E-09
plll pili P3ill
0.000000
1.5159943633i"
-4nत9995492
\(=0006596048\)
.0000035173

41764174 - 41780
F-2000009
1.6167144160
1.4005366797
-.0004578974
-0.0.00 0911489
12.0366464157
\(=3.7554 \mathrm{E}=\mathrm{n} 7\)
. \(1329 E=07\)

ORPIT

    \(41178.000000 \quad 3.000000\)
\(=202.0584506244-1.6295925653\)
ORRIT



OROIT
\(1970-109=\mathrm{A}\)
Pl10 Fita
口й
\(41009.000000 \quad-1.000000\)
\(113.997515174=13.06978000\)
\(40.1425252113-5.9858900000\)
15.2044497333
\(.0156053 n 33=.0001033948\)
-. 9252147943 14.8162110555 4.7799E-05
41038.00000
\(5.5 \overline{9} 950305 \geq 9\)
 ORETF
 41010.000000
.000000
\(\overline{140.1243560064 \quad 13.06997800007 ~}\)
\(26.1696582165=-6.9858999000\)
150037254564
\(0163619383=2041211353\)
.7074090758 14.02163574633 ? 9465 - 25
\(41010.00000 \quad 4.821552016\) न
\(139.524257 \quad 26.16557715 .003213 \quad .0165082 \quad .709086414 .8163575 \quad 7.0023071\) orgIT


166.029219. 12.244923 15.0.j2410 .0165589 . 340868414.9184993 7.00022624 OREIT
19701094

\(41014.001000=.000009\)
192.974229729? 13.0699780002
-1.594416941 - -6.9858900000
i5,0017593659
ㄷN164334n44 - -.conn355397
\(\div 578856103\) 14.2166391445 \(=3.06375-05\)
41014.00001 = -519193677
 ORGIT


OFETI


\section*{GREET}

1979109 A
Plio piln

\(41044.00090 n\)
.000000
229.92265815 n
43.2353020091
149.0432009806
55.9791030007
14.995841761
\(.0163243337=-.0000780984\)
\(.4916972316 \quad 14.8180543923\) 3.25019E-05
\[
41044.00000 \quad 1.0541568058
\]
 OREIT
 \(.1171541725 \quad 14.8182676545 \quad 4.9134 E-05\)
\[
41046.00000 \quad 1.3560215758
\]
\(256.515836 \quad 135.073642 \quad 14.995530 \quad .0150829 \quad .117669814 .8192677 \quad 7.0017053\)
ORRTI


\begin{tabular}{ll}
41048.000009 & .000000 \\
283.2032190914 & 13.231300009 \\
121.1217506988 & -5.9791030000
\end{tabular}

IT4-9926911854

\(41049.06000 \quad 1.3737 n 49449\)
 ORRIT
\(1970=109 \mathrm{~A}\)
1110
P177
031 20. 2111048.021
\(41050.000000 \quad 0000001\)
309.715920 .970913 .2313000000
\(107.1522774642-6.9791030000\)
15.011282n179
\(-0152762959=0000195378\)
\(.3907837941=14.8186956512 \quad 6.44376-15\)
4145 r.00nen \(\quad .5999923624\)



 ORET
 OREFF


\section*{ORRIT}
 \(41090.00910 n=1.0000410\)
\(119.2477330917=13.237300092\)
187.9916531 ก̄73 -6.97798000 ก̄
14.9982512750
\(0.0163426456=0000012096\)
 ORGIT



Plio pijn
PE1
pili
p2ī1
\(41 \overline{21}, 000000\) .030030
\(169.34952+3695\) 13 ? 131302000 n
\(-28.477373992 \quad-6.979151000\)
14.9989256649
\(.0162357170=0001759 ? ?\)
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.7115657046 14.8232.340778 2.5247E-05

```
41127.00000
1.4927365808
 ORRIT
1910109.4
\(\mathrm{PITO}=\mathrm{PFTn}=-\mathrm{O}\)

.000000
\(196.3032519744 \quad 13.2313000090\)

\(\overline{1} 95.534675 \quad-42.379812 \quad 14.908904 \quad .0151322 \quad .360292414 .8233139 \quad 7.000101161\)


41775.000107
\(-1.000000\)
165.156101579 הु
13.2313000007
314.5214490847
\(-6.9843190000\)
\(=14.9957529698\)
- 161879298 - 007259425

\(41775.0000 \quad 1.7165674713\)
\(164.398138 \quad 314.517653 \quad 14.995545 \quad .1763476 \quad .216276514 .8249989 \quad 6.9995857\) ORRIT
 P110
41177.000000
.049009
\(191.59 \pi 736788913.3313000 \pi 01\)
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\title{
A NOTE ON THE RELATIONSHIP AND AGREEMENT BETWEEN TWO SATELLITE THEORIES
}

\section*{K. Aksnes}

We shall refer to the two theories under consideration as theory A (Gaposchkin, Cherniack, Briggs, and Benima, 1971; Kozai, 1962) and theory B (Aksnes, 1970). The main purpose here is to introduce some simple theoretical relations by means of which, when the elements of one theory are given, we can predict those of the other.

Theory B differs from theory A in that the former makes use of l) a reference orbit that is a rotating ellipse (intermediate orbit) instead of a fixed ellipse, 2) Hill variables instead of Delaunay variables, and 3) Hori's method in Lie series rather than Von Zeipel's method in Taylor series.

In both theories, the periodic perturbations are expressed as deviations from a \(\underline{\text { mean }}\) orbit whose elements are semimajor axis a, eccentricity e, mean anomaly M, inclination \(i\), argument of perigee \(\omega\), and right ascension of the ascending node \(\Omega\). To distinguish between the two sets of elements, we shall attach a subscript " 0 " to those of theory A. Although, strictly speaking, the formulas introduced below are valid only if the mean elements are constants or linear functions of time, they should also be sufficiently good approximations if the mean elements are allowed to contain long-period terms, as is the case with the SAO mean elements.

For convenience, \(M, \omega\), and \(\Omega\) have been so defined that
\[
\begin{equation*}
\mathrm{M}=\mathrm{M}_{0}, \quad \omega=\omega_{0}, \quad \Omega=\Omega_{0} . \tag{1}
\end{equation*}
\]

However, internally, theory Butilizes a set of elements \(g\) and \(h\) that differ from \(\omega\) and \(\Omega\) by the amount of rotation that the intermediate orbit is undergoing, viz.,
\[
\begin{equation*}
\omega=\mathrm{g}+\mathrm{g}_{21}(\mathrm{~g}+\mathrm{M}) \quad, \quad \Omega=\mathrm{h}+\mathrm{g}_{32}(\omega+\mathrm{M}) \tag{2}
\end{equation*}
\]
where
\[
\begin{align*}
& g_{21}=-\frac{3}{4} \gamma\left(1-5 c^{2}\right)-\frac{1}{64} \gamma^{2}\left(41+30 c^{2}-135 c^{4}\right)+0\left(\gamma^{3}\right) \\
& g_{32}=-\frac{3}{16} c\left[8 \gamma+\gamma^{2}\left(7-33 c^{2}\right)\right]+0\left(\gamma^{3}\right) \tag{3}
\end{align*}
\]

Here and in the following, we have used the notation
\[
\begin{aligned}
& \mathrm{c}=\cos \mathrm{i}, \quad \mathrm{n}=\sqrt{\mu / \mathrm{a}^{3}}, \quad \eta=\sqrt{1-\mathrm{e}^{2}}, \quad \mathrm{G}=\eta \sqrt{\mu \mathrm{a}} \\
& \mathrm{\gamma}=\frac{\mathrm{J}_{2}}{\mathrm{a}^{2} \eta^{4}}, \quad \gamma_{4}=\frac{\mathrm{J}_{4}}{\mathrm{~J}_{2}^{2}}
\end{aligned}
\]
where a is measured in earth radii. The mean rates of change of \(M_{0}, \omega_{0}\), and \(\Omega_{0}\) are given by
\[
\begin{align*}
\dot{M}_{0}= & n_{0}-\frac{3}{4} n_{0} \gamma_{0} \eta_{0}\left\{1-3 c_{0}^{2}-\frac{1}{32} \gamma_{0}\left[10\left(1-6 c_{0}^{2}+13 c_{0}^{4}\right)\right.\right. \\
& -5\left(5-18 c_{0}^{2}+5 c_{0}^{4}\right) e_{0}^{2}+16 \eta_{0}\left(1-6 c_{0}^{2}+9 c_{0}^{4}\right) \\
& \left.\left.-15 \gamma_{4}\left(3-30 c_{0}^{2}+35 c_{0}^{4}\right) e_{0}^{2}\right]\right\}+0\left(\gamma^{3}\right)  \tag{4}\\
\dot{\omega}_{0}= & -\frac{3}{4} n_{0} \gamma_{0}\left\{1-5 c_{0}^{2}+\frac{1}{32} \gamma_{0}\left[2\left(5+43 c_{0}^{2}\right)\left(1-5 c_{0}^{2}\right)\right.\right. \\
& +\left(25-126 c_{0}^{2}+45 c_{0}^{4}\right) e_{0}^{2}-24 \eta_{0}\left(1-8 c_{0}^{2}+15 c_{0}^{4}\right) \\
& \left.\left.+20 \gamma_{4}\left(3-36 c_{0}^{2}+49 c_{0}^{4}\right)+45 \gamma_{4}\left(1-14 c_{0}^{2}+21 c_{0}^{4}\right) e_{0}^{2}\right]\right\}+0\left(\gamma^{3}\right), \tag{5}
\end{align*}
\]
and
\[
\begin{align*}
\dot{\Omega}_{0}= & -\frac{3}{2} n_{0} \gamma_{0} c_{0}\left\{1-\frac{1}{16} \gamma_{0}\left[4-40 c_{0}^{2}-\left(9-5 c_{0}^{2}\right) e_{0}^{2}\right.\right. \\
& \left.\left.+12 \eta_{0}\left(1-3 c_{0}^{2}\right)-5 \gamma_{4}\left(3-7 c_{0}^{2}\right)\left(2+3 e_{0}^{2}\right)\right]\right\}+0\left(\gamma^{3}\right) \tag{6}
\end{align*}
\]

These are invariant rates, and although expressed differently, they must be equal individually to \(\dot{\mathrm{M}}, \dot{\omega}\), and \(\dot{\Omega}\), which are given by
\[
\begin{align*}
\dot{\mathrm{M}}= & \mathrm{n}+\frac{3}{128} \mathrm{n} \gamma^{2} \eta\left[8\left(1-6 \mathrm{c}^{2}+5 \mathrm{c}^{4}\right)-5\left(5-18 \mathrm{c}^{2}+5 \mathrm{c}^{4}\right) \mathrm{e}^{2}\right. \\
& \left.-15 \gamma_{4}\left(3-30 \mathrm{c}^{2}+35 \mathrm{c}^{4}\right) \mathrm{e}^{2}\right]+0\left(\gamma^{3}\right),  \tag{7}\\
\dot{\omega}= & \dot{\mathrm{g}}+\mathrm{g}_{21}(\dot{\mathrm{~g}}+\dot{\mathrm{M}}) \\
\dot{\mathrm{g}}= & -\frac{1}{128} \mathrm{n} \gamma^{2}\left[44-300 \mathrm{c}^{4}+\left(75-378 \mathrm{c}^{2}+135 \mathrm{c}^{4}\right) \mathrm{e}^{2}+60 \gamma_{4}\left(3-36 \mathrm{c}^{2}+49 \mathrm{c}^{4}\right)\right.  \tag{8}\\
& \left.+135 \gamma_{4}\left(1-14 \mathrm{c}^{2}+21 \mathrm{c}^{4}\right) \mathrm{e}^{2}\right]+0\left(\gamma^{3}\right)
\end{align*}
\]
and
\[
\left.\left.\begin{array}{l}
\dot{\Omega}=\dot{\mathrm{h}}+\mathrm{g}_{32}(\dot{\omega}+\dot{\mathrm{M}}),  \tag{9}\\
\dot{\mathrm{h}}=\frac{3}{32} \mathrm{nc} \mathrm{\gamma} \\
\\
2
\end{array} 2-10 \mathrm{c}^{2}-\left(9-5 \mathrm{c}^{2}\right) \mathrm{e}^{2}-5 \gamma_{4}\left(3-7 \mathrm{c}^{2}\right)\left(2+3 \mathrm{e}^{2}\right)\right]+0\left(\gamma^{3}\right) \quad,\right\}
\]

Aksnes (1970) has shown that the elements a, e, and i relate to \(a_{0}, e_{0}\), and \(i_{0}\) through the following equations:
\[
\begin{align*}
\frac{1}{\mathrm{a}}= & \frac{1}{\mathrm{a}_{0}}\left\{1-\frac{1}{2} \eta_{0} \gamma_{0}\left(1-3 \mathrm{c}_{0}^{2}\right)+\frac{1}{32} \eta_{0} \gamma_{0}^{2}\left[1+6 \eta_{0}-\left(6+36 \eta_{0}\right) \mathrm{c}_{0}^{2}\right.\right. \\
& \left.\left.+\left(45+54 \eta_{0}\right) \mathrm{c}_{0}^{4}\right]\right\}+0\left(\gamma^{3}\right),  \tag{10}\\
\mathrm{G} & =G_{0}\left[1+\frac{1}{4} \gamma_{0}\left(1-3 \mathrm{c}_{0}^{2}\right)\right]+0\left(\gamma^{2}\right), \tag{11}
\end{align*}
\]
and
\[
\begin{equation*}
c=c_{0}\left[1+\frac{3}{4} \gamma_{0}\left(1-c_{0}^{2}\right)\right]+0\left(\gamma^{2}\right) \tag{12}
\end{equation*}
\]

While the third-order parts of equations (3) to (9) are available in the cited literature, the terms beyond the second order in equation (10) and the first order in equations (11) and (12) are not known.

For conversion between the two sets of mean elements, equations (1) and (10) to (12) will suffice. In view of the importance of these equations, we have tested them on the orbits of three actual satellites (Table 1). The first two sets of elements have been derived by fitting theory A (line 1) and theory B (line 2 ) to a series of mostly very accurate laser observations in the manner described by Gaposchkin and Mendes ("Orbital Elements from ISAGEX Data," this volume). The third line shows the elements predicted for theory \(B\) by means of the above-mentioned equations and the elements on line 1. The agreement between the last two sets of elements is very good and well within standard errors. The two theories agree on the computed ranges to a few tenths of a meter.

Table 1. Comparison of mean elements for three satellites at epoch 41080.0.
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline Satellite & Theory & a & e & i & M & \(\omega\) & \(\Omega\) \\
\hline \multirow{3}{*}{6701401, 261 observations} & ( A & 1. 1925238 & 0.0838280 & 39: 44949 & 33: 64344 & 194: 54493 & 134\% 43624 \\
\hline & , B & 1. 1921616 & 0.0838340 & 39.43323 & 33.64344 & 194.54491 & 134.43625 \\
\hline & (B (Pred.) & 1. 1921616 & 0.0838329 & 39.43321 & 33.64344 & 194.54493 & 134.43624 \\
\hline \multirow{3}{*}{\begin{tabular}{l}
\[
6508901,
\] \\
241 observations
\end{tabular}} & ( A & 1. 2656913 & 0.0721680 & 59.38145 & 294.47892 & 24.26520 & 265.05593 \\
\hline & S B & 1.2657869 & 0.0721670 & 59.36859 & 294.47892 & 24.26519 & 265.05594 \\
\hline & ( B (Pred.) & 1. 2657869 & 0.0721664 & 59.36859 & 294.47892 & 24.26520 & 265.05593 \\
\hline \multirow{3}{*}{\begin{tabular}{l}
\[
6800201
\] \\
138 observations
\end{tabular}} & ( A & 1. 2080431 & 0.0320070 & 105. 80768 & 149. 20308 & 2. 42648 & 301.93324 \\
\hline & \& \({ }^{\text {B }}\) & 1. 2083920 & 0.0320040 & 105.81605 & 149. 20308 & 2.42648 & 301.93324 \\
\hline & ( B (Pred.) & 1. 2083920 & 0.0320045 & 105.81605 & 149.20308 & 2.42648 & 301.93324 \\
\hline
\end{tabular}

\section*{REFERENCES}

AKSNES, K.
1970. A second-order artificial satellite theory based on an intermediate orbit. NASA TR 32-1507, 34 pp.; also in Astron. Journ., vol. 75, pp. 1066-1076.
GAPOSCHKIN, E. M., CHERNIACK, J. R., BRIGGS, R., and BENIMA, B.
1971. Third-order oblateness perturbations for an artificial satellite. Presented at the 52nd American Geophysical Union Meeting, Washington, D.C., April; to be published as Smithsonian Astrophys. Obs. Spec. Rep.
KOZAI, Y.
1962. Second-order solution of artificial satellite theory without air drag. Astron. Journ., vol. 67, pp. 446-461.

\section*{ABBREVIATIONS USED IN THIS REPORT}
\begin{tabular}{ll} 
BIH & Bureau International de l'Heure \\
CNES & Centre National d'Etudes Spatiales \\
EPSOC & Earth Physics Satellite Observation Campaign \\
GOCC & Geodetic Operations Control Center \\
GSFC & Goddard Space Flight Center \\
IGY & International Geophysical Year \\
ISAGEX & International Satellite Geodesy Experiment \\
NASA & National Aeronautics and Space Administration \\
NTU & National Technical University, Athens, Greece \\
PMT & Photomultiplier Tube \\
SAO & Smithsonian Astrophysical Observatory \\
USNO & United States Naval Observatory
\end{tabular}```

