



Federal Agency for  
Cartography and Geodesy



# Single Photon tracking under difficult condition

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## What is a difficult condition?

Consider e. g.:

- Telescope pointing in combination with fast moving objects
- The SLR station covered by a lots of small clouds and plane trails
- Low repetition rate SLR systems

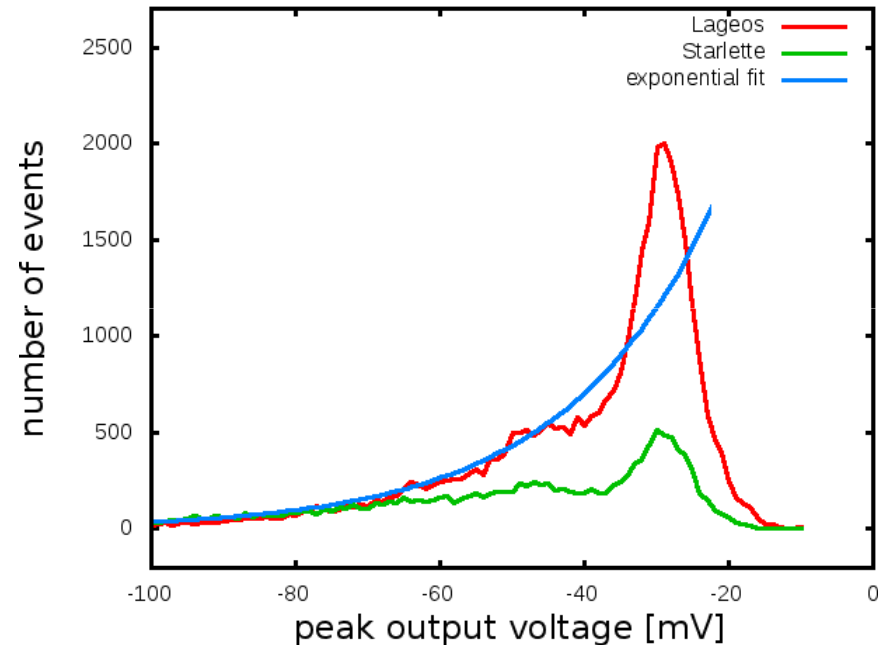
Problem:

**Is single photon really single photon?**



Evaluate 4 month of single photon sensitive MCP data of 20 Hz WLRS (unfiltered),  
to get intensity profile.

Distinct single photoelectron peak, washed out  
multi-photoelectron events  
(incoherent signal)



Determination of decay rate possible (gives information  
about probability of each photoelectron number)

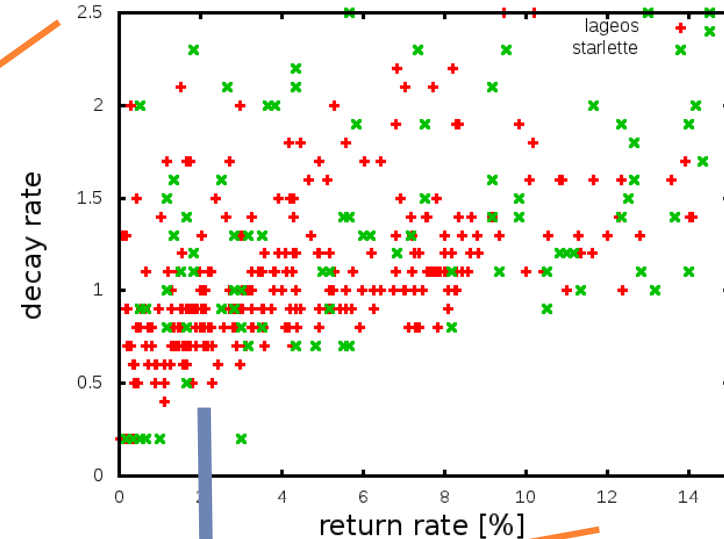
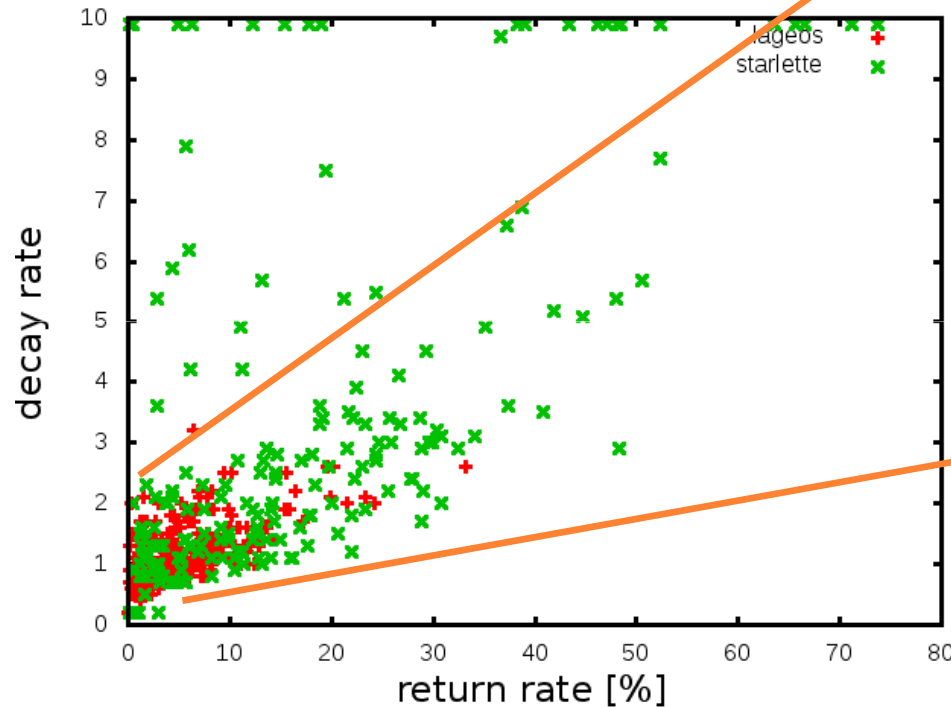
Conclusion: Data is not single photon, consider return rate!



# Correlation return rate – decay rate



Use each NP for Lageos and 3 NPs for Starlette to calculate RR & decay rate



Is there a station dependent  
lower limit for the decay rate?  
What is the range bias for a  
specific decay rate?

→ **Build model to find COM as a function of decay rate**



## Common model:

Tracking of zero signature target to get system noise,  
convolution with single photon satellite answer  
(satellite signature)

$$\textit{System noise} * \textit{Satellite Signature}_{\textit{single}} = \textit{Residual Histogram}$$

## Our idea:

Extend single photon satellite answer for mixed state  
answer and do convolution with receiver model (receiver  
signature), **decay rate is the only free parameter**  
**(measurable)**

$$\textit{Receiver Signature}_{\textit{mixed}} * \textit{Satellite Signature}_{\textit{mixed}} = \textit{Residual Histogram}$$



## SPAD:

Introduced in Fujiyoshida:

$$p(t) = \sum_n \exp(n \cdot f) \cdot \exp\left(\frac{(t + e \cdot \log(n))^2}{-2 \cdot b^2}\right)$$

n: different photon numbers, evaluation from 1 to 5  
suffizient for decay rate up to ~ 1.5

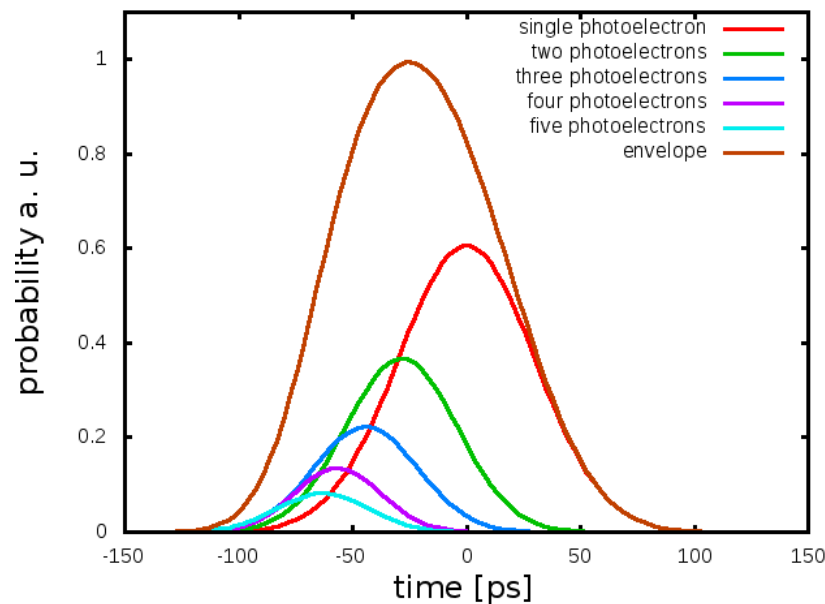
## MCP:

Empirical model behaviour of Discriminator hard to describe  
by theory, build by filtering different peak voltage intervals in  
laboratory measurement.

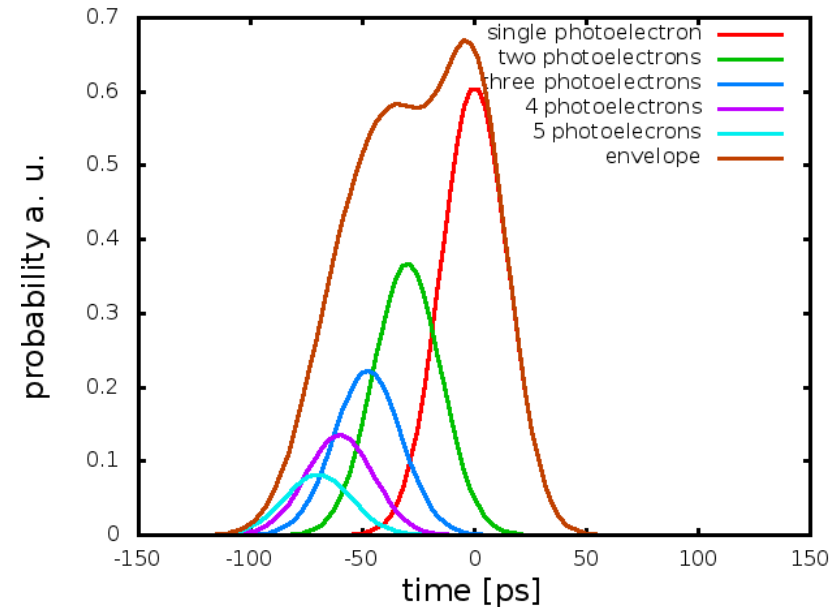


Combine single and multiple photoelectron multiplication together with decay rate to receiver signature e. g. for:

### MCP, decay rate 2



### SPAD, decay rate 2

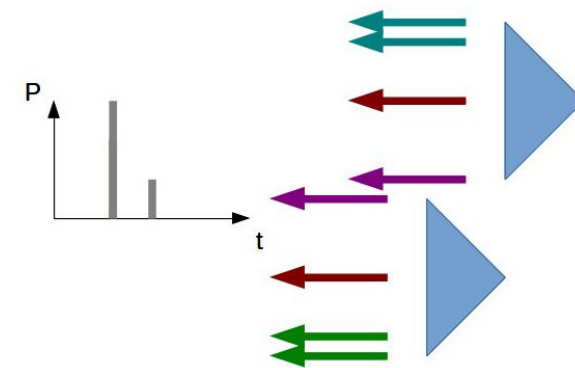
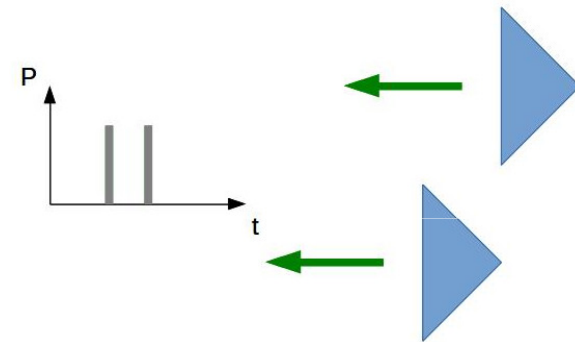




Need to find multiple photon satellite answer, principle:  
Consider 2 identical CCR with Single Photon Detector (first photon counts)

Two peaks for single photon returns, with same Probability

Now: two returning photons  
Apply methods of Combinatorics  
 $\rightarrow P(CCR1) = 3 \cdot P(CCR2)$

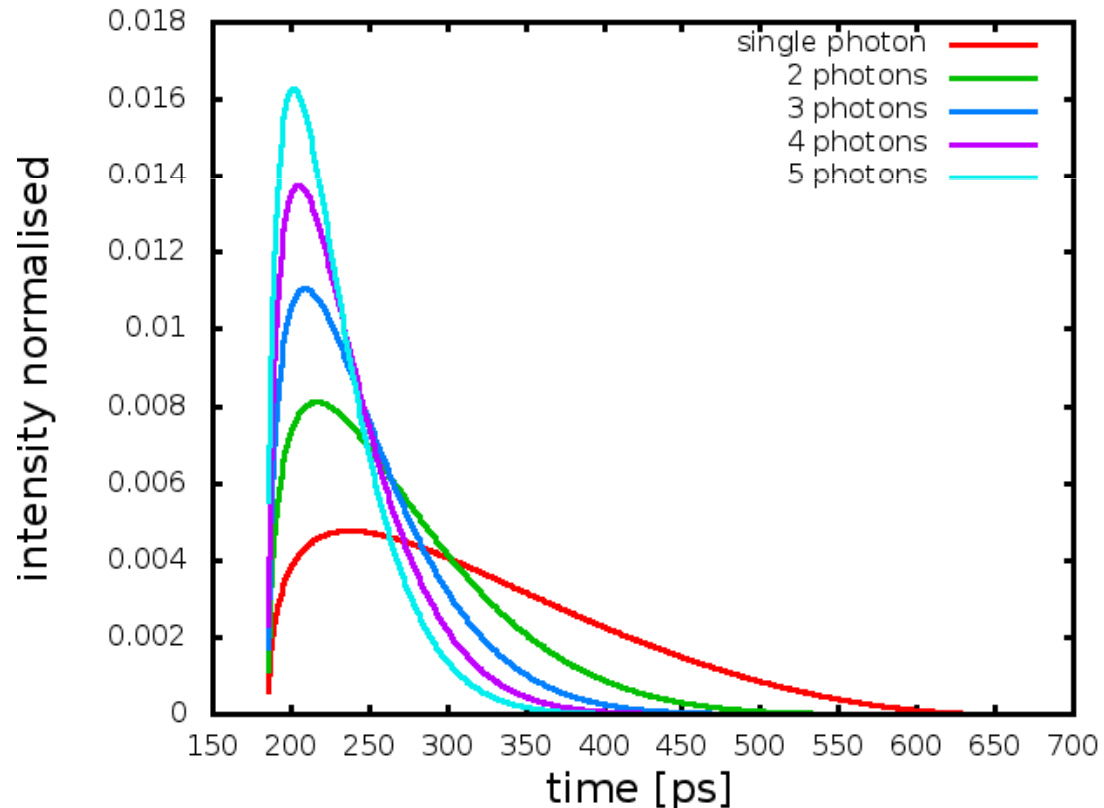






Use single photon satellite response from John Degnan [Millimeter Accuracy SLR] for single photon data and compute multi photon distributions, e. g. Lageos:

Apply decay rate  
& convolute  
with receiver  
signature



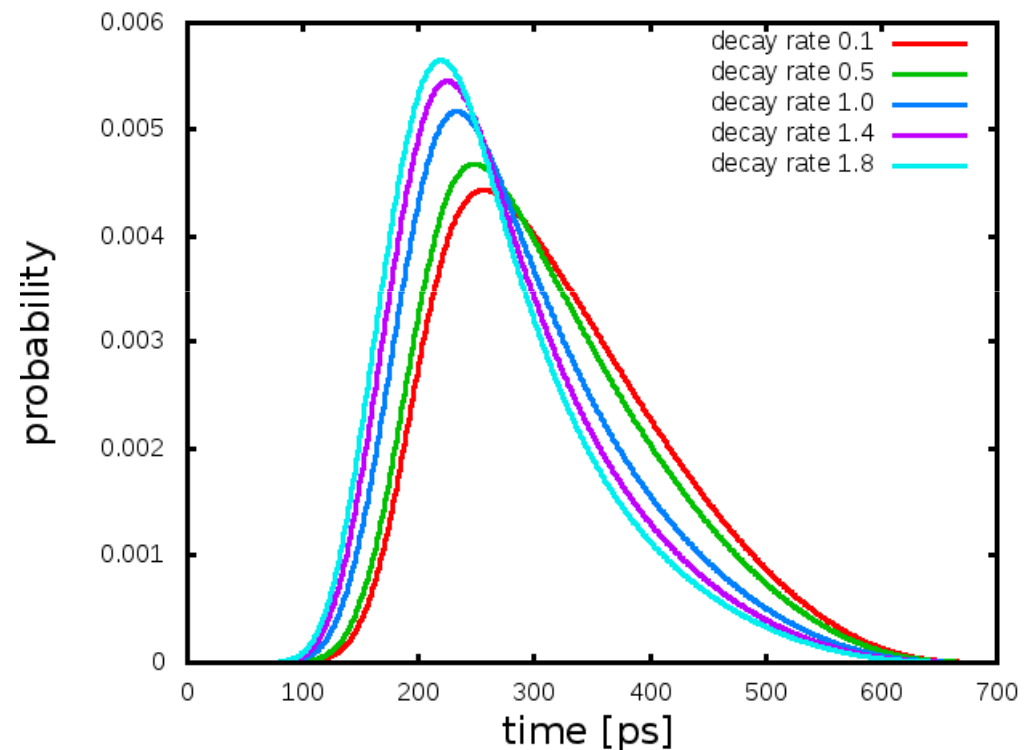


For Lageos tracked with MCP and different decay rates

decay rate increasing

=>

Distribution becomes  
peaky and (of course)  
shifts towards satellite  
front

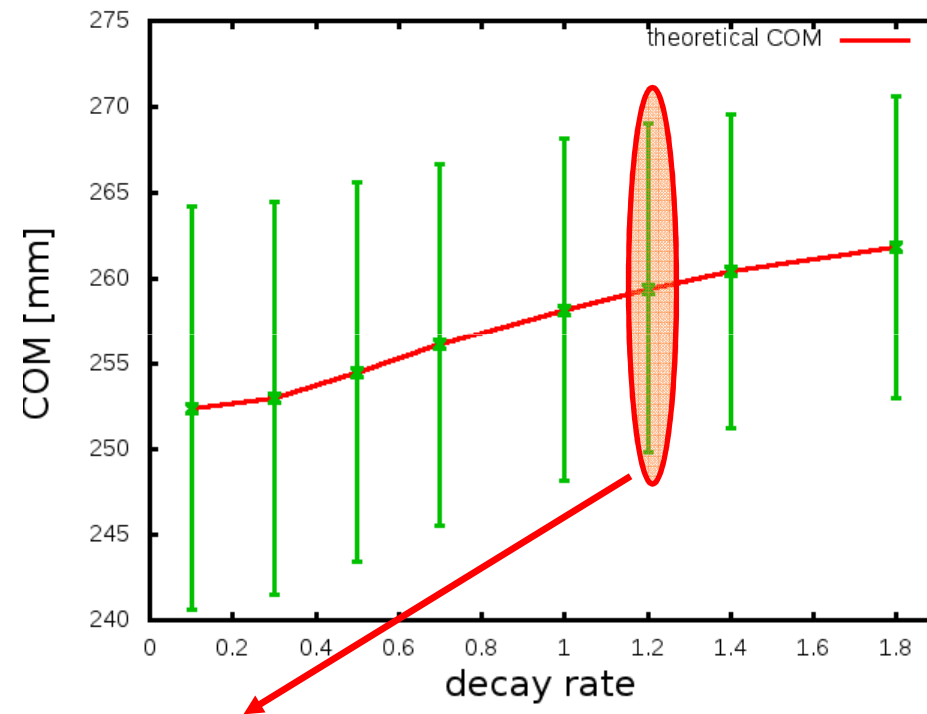


Calculate mean to get COM (decay rate)



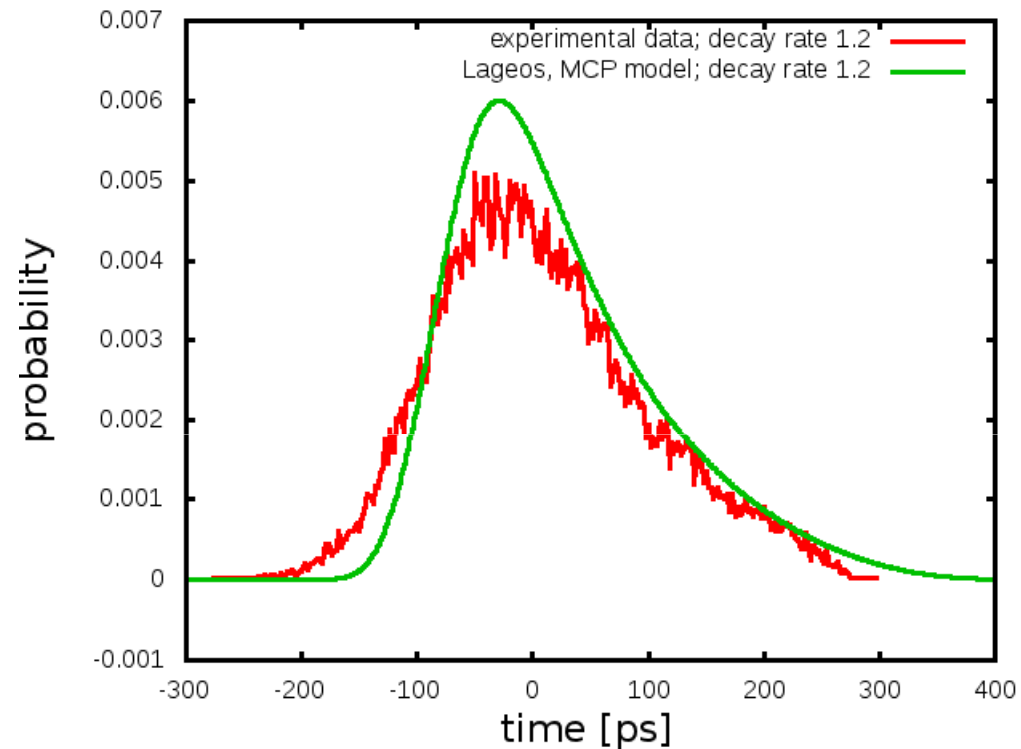
## COM and SDEV (Lageos, MCP)

Range Bias  $> 1\text{mm}$   
for decay rate  $> 0.4$



**Mean Decay Rate WLRS 1.2  $\Rightarrow$  Range Bias  $\sim 7\text{ mm}$  !**

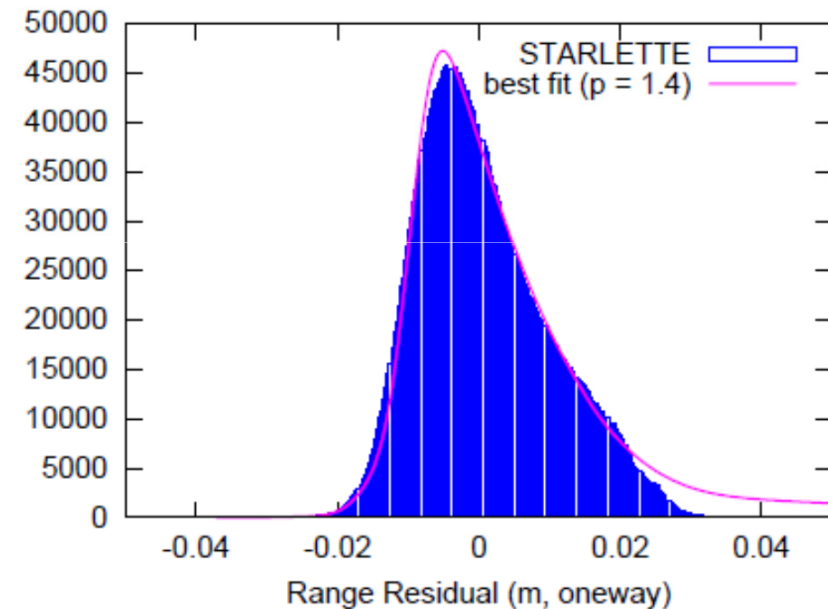
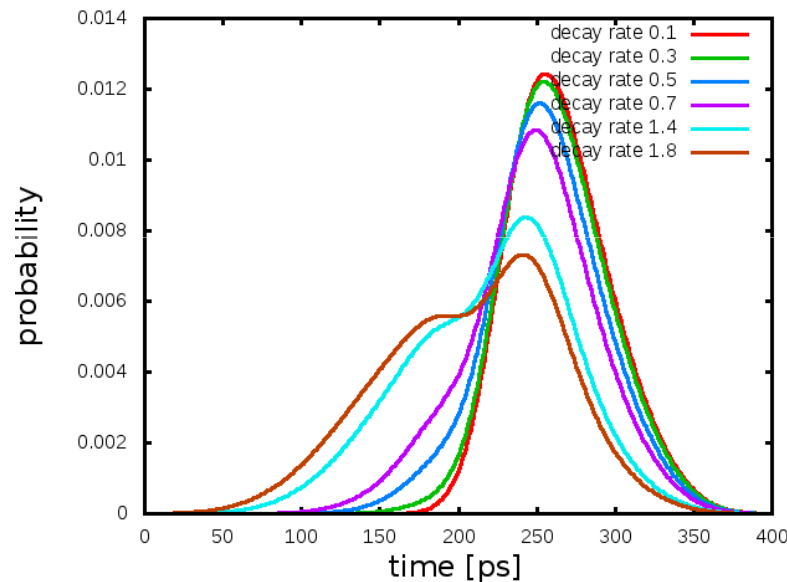
However: verification of model still pending



Agreement not perfect, accumulation of different passages.  
Find agreement during 1 passage → kHz data necessary



Considering data published by Toshi [Annapolis, 2014]  
And our model for Starlette & K14 SPAD



Our model might be able to explain behaviour of rising edge  
Trailing edge is more complex, SPAD diffusion tail not modelled, yet



- Unfortunately model verification still pending
- Introduced receiver signature
- Chance to identify constant (SCALE, lower limit of decay rate) and varying range biases (NP RMS)
- We have developed a model that is able to handle mixed states, which may arise in difficult tracking conditions with all of the parameters measurable