Single Photon Tracking under difficult condition

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Abstract. *Modern Satellite Laser Ranging Systems apply "Single Photon Tracking" to avoid systematic biases from intensity fluctuations in the measurement. The term "Single Photon Tracking" usually means that the return rate is controlled to a level below 10 %. At these rates, the probability to receive higher photon numbers is negligible in the laboratory. However, considering imperfect conditions, e. g. a lot of small clouds, detector dark noise, beam pointing problems or low repetition rate Satellite Laser Ranging Systems, an estimation of the true signal level based on the return rate can be misleading. In this paper we want to show the relation between the return rate and the mean photon number in Satellite Laser Ranging measurements of the Wettzell Laser Ranging System during 2014. In addition we show an attempt of how to extend the common model of the intensity distribution of the received signal from spherical satellites at the single photon level to be able to be applied to a Boltzmann distributed signal by looking at the mean photon number of the distribution.*

Introduction

The reconstruction of the satellite response function is a challenge in satellite laser ranging (SLR) for many years, now. First investigations in the 1970's were made to describe the optical transfer function of spherical satellites based on the satellite's geometry by [ARNOLD, 1979]. The focus in the 1990's was concentrating on the influence of the laser pulse width [NEUBERT, 1994] and the receiver response [KIRCHNER, 1994] [SCHREIBER, 1994]. All these studies led to the result, that satellite laser ranging should be performed in single photon mode to reduce the systematic error below the 10 mm level. In single photon mode the response signal of the satellite is controlled to a level below 10% during the measurement. In the laboratory a variation of the signal level in this region does not introduce any biases since the operating conditions remain constant. Other recently published models [OTSUBO, 2013] of the satellite response function lead to good agreement with experimental data of kHz satellite laser ranging systems, tracking in single photon mode. However, these models are still based on empirical parameters and lack a complete theoretical description of the processes involved. This is of special interest, when it comes to the need of more accurate ranging data, which is required for a global geodetic observation system [PLAG, 2009] and especially for time transfer, where constant offsets need to be avoided. Here we will analyze the photon statistics of the signal response of spherical satellites for a large number of passes, which were tracked in single photon mode with the low repetition rate Wettzell Laser Ranging System. In addition, we will show, that in satellite laser ranging under difficult conditions the assumption of receiving single photons only, is not valid. So far the term "difficult" condition is not well defined. For the moment we shall consider the telescopes pointing properties in combination with fast moving objects or the atmosphere covered with a lot of small clouds or a low repetition rate of the satellite laser ranging systems as a difficult condition. For our micro channel plate detector, type PHOTEK PMT210 MCP, we can recover the number of photo- electrons for each satellite return from the electrical signal response [Eckl, 2013]. Based on the derived photon statistics we will show an attempt of how to improve the current model of the satellite response function to also include photon statistics.

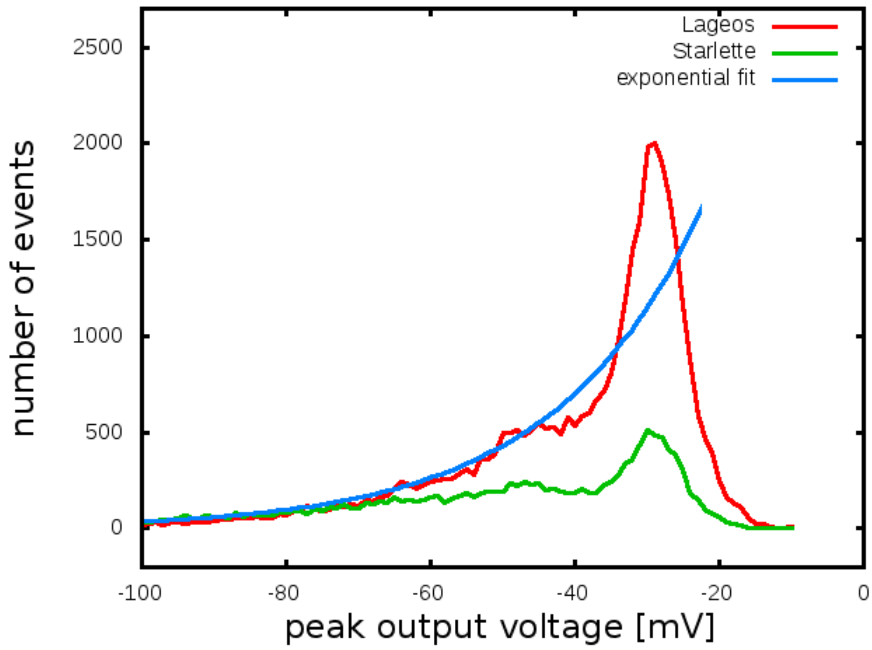

Figure 1. Histogram over the accumulated signal returns over several month of data of the peak output voltage of a micro channel plate receiver, installed at the Wettzell Laser Ranging System.

Photon statistics in satellite laser ranging

Data of several month of the peak output voltage of a single photon sensitive micro channel plate receiver, operated at the Wettzell Laser Ranging System, shows an exponential decay in the peak output voltage for LAGEOS and STARLETTE satellites (Figure 1). Since the signal of the laser source in satellite laser ranging is degraded during the measurement, it can be considered as incoherent. This can be seen from the distinct single photoelectron multiplication peak and washed out peaks at higher seeding photo-electron numbers in Figure 1. To relate the found decay rate to the signal return rate each normal point of the LAGEOS measurements and triple of three normal points of the STARLETTE measurements were evaluated for their respective return rate and their decay rate (Figure 2). It can be seen, that even for return rates below the 10% level, the decay rate, is never below 0.5 with the exception of some outliers. In other words the signal never reached the single photon level, instead there was always a mixture of single- and multi-photon returns. On top of that one can see a linear dependence of the decay rate on the return rate, although the slope is quite small and the scatter in the data is high. To extract constant biases caused by a non-vanishing decay rate, and varying delays caused by the linear dependence of the return rate on the decay rate, we tried to establish a model for a mixed state signal.

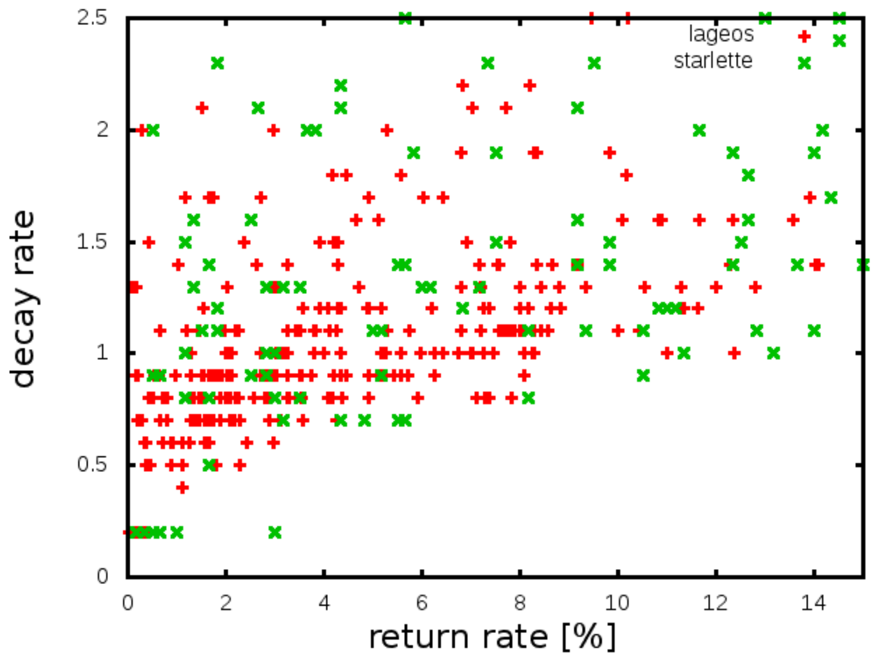

Figure 2. Decay rate as a function of the return rate for each normal point of Lageos measurements and each triplet of normal points of Starlette observations for the data of Figure 1.

Our model of the satellite response function

The current approach of modeling the satellite response is to use the response of a zero signature satellite and to do a convolution with the single photon satellite response function, derived from geometrical satellite properties:

$$\boldsymbol{System\ Noise * Satellite\ Signature_{single} = Residual\ Histogram}$$

The residual histogram here is the probability distribution of the satellite response. To gain better agreement to the experimental data, the resulting residual histogram is usually corrected by means of a parameter estimated from [OTSUBO, 2013]. Our approach in contrast is making allowance for the mixed signal states. It introduces a mixed state receiver signature and also extends the single photon optical transfer function of the satellite to a mixed state transfer function:

$$\boldsymbol{Receiver\ Signature_{mixed} * Satellite\ Signature_{mixed} = Residual\ Histogram}$$

It can be seen from both equations that the laser pulse width is missing. Thanks to modern passively mode locked and therefore almost bandwidth limited lasers the pulse-width is below 10 ps and normal distributed. This leads to a negligible effect of the laser pulse width on the residual histogram.

The model of the receiver signature, at least in principle, for a single photon avalanche diode was introduced in [ECKL, 2013]. The only difference is, that now time is the parameter of interest instead of the peak output voltage. Therefore the probability distribution of a time interval measurement can be written as:

$$p(t) = \sum_n e^{-\frac{n}{f}} \cdot e^{\left(-(t+e\cdot\log(n))^2/2\cdot b^2\right)}$$

The equation basically describes a summation over 1 to N seeding photo-electrons, with each summand regarding the decay in the amount of seeding photoelectrons known from the above mentioned decay rate. Whereby f is this decay rate, e is a parameter describing the intensity dependent detection delay of the diode and b is the jitter in the multiplication process of a specific seeding photo-electron number. The benefit of this model is, that it depends on measureable and intrinsic parameters of the diode only. Therefore, there is no need of estimating parameters. Since a theoretical description of the timing behavior of our micro channel plate receiver is hard to establish (this is caused by the discriminator) we use an empirically derived model. This was done by filtering the detection times of a specific region in the peak output voltage of the device and adjust the probability density function of a normal distribution to each dataset. Afterwards, similar to the model for the SPAD, the decay in the amount of seeding photo-electrons was evaluated.

The model for the satellite signature in the mixed photon state can be derived from the satellite signature in single photon state by applying methods of combinatorics. The principle can be explained considering two identical corner cube reflectors mounted with an offset in the direction of the incoming laser pulse. Performing a range measurement, the probability of receiving an event from the first reflector is equal to the probability of receiving an event from the second. Therefore, the average in the range measurement is exactly the distance to the first plus the distance to the second reflector divided by two for the centroid of the measurement. Considering now exactly two photons returning, the probability of receiving a signal from the first reflector is a combination of the three first of the overall four possible combinations: two photons reflected from reflector 1, the first photon reflected from reflector 1 the second from reflector 2 and vice versa and both photons reflected from reflector 2. This leads to a probability of 0.75 to receive a photon from the first reflector and a probability of 0.25 for the second, respectively. The reason for this result lies in the fact that for single photon sensitive receivers only the arrival of the first photon is detected, while the second photon is lost in the dead-time of the detector. Because of that, a shift of the centroid in the range measurement can be observed, which results in a bias for the range measurement in satellite laser ranging. This principle can be extended to higher photon and reflector numbers or to a discrete probability distribution, which may be the single photon satellite signature.

Figure 3 shows the residual histogram of a LAGEOS satellite in combination with our empirical micro channel plate model for different decay rates. The histogram is a convolution of the above introduced receiver signature with the satellite signature in the mixed state, which was derived from the single photon satellite signature found in [DEGNAN, 1993]. It can be seen, that the centroid is shifting towards earlier detection times and the distribution becomes narrower, when the decay rate increases. This is also expected from the theory as well as a typical experimental observation. To get an idea of the amount of the shift in time of the centroid, it was calculated from the probability distribution, applying iterative 2.2 sigma clipping and a correction for the satellites geometry. This leads to values for the center of mass correction as a function of the decay rate (Figure 4). For a decay rate of 0.1 the data is almost single photon and the value for center of mass correction for LAGEOS is close to the standard of 251 mm used by [OTSUBO, 2014]. However, for the experimentally derived mean value of 1.2 for the mean decay rate of the Wettzell Laser Ranging System the center of mass correction is shifted by 7 mm towards a higher value. In other words, based on the above analysis, the systematic offset in the range measurement to the LAGEOS satellite is 7 mm, which is caused by receive signal fluctuations only, although the measurement was performed in conditions defined as the single photon detection mode by the ILRS.

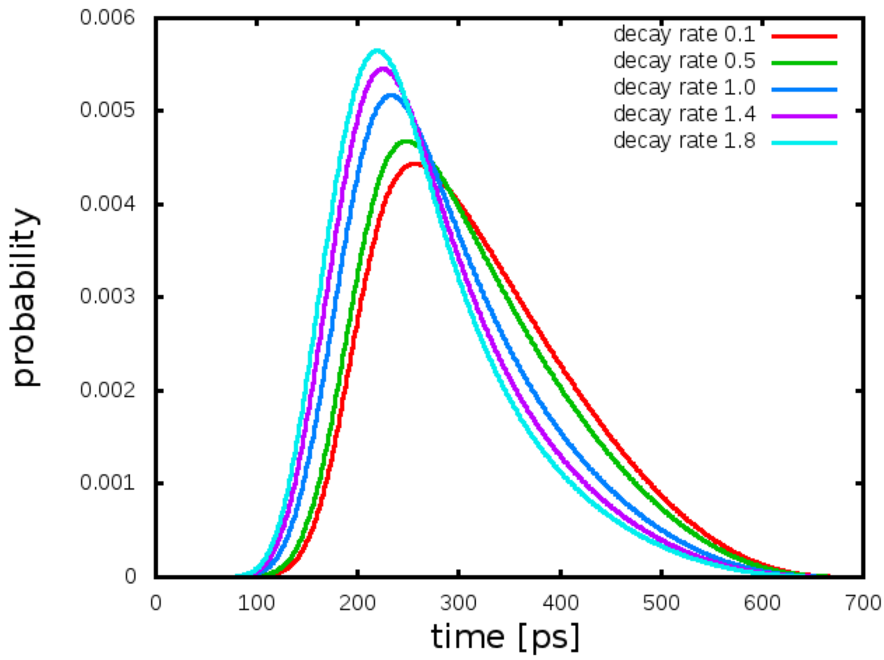

Figure 3. Residual histogram, derived from a convolution of our micro channel plate receiver model with the satellite signature of a LAGEOS satellite in the mixed state for different decay rates in the seeding photoelectron statistics.

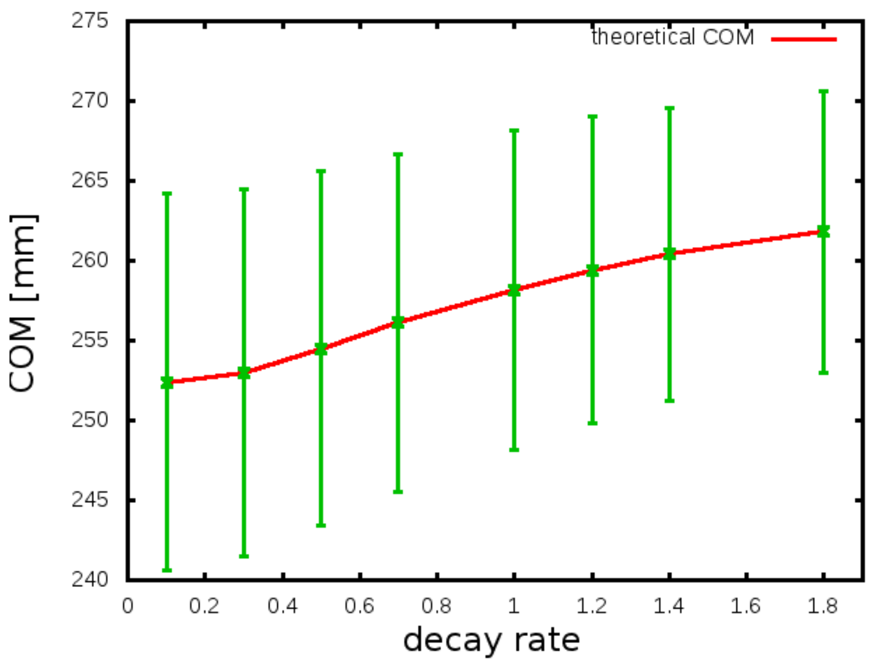

Figure 4. Values for center of mass correction derived from the probability distributions of Figure 3, applying iterative 2.2 sigma clipping.

Experimental data

To verify our model it has to be compared to experimental data. A plot of the residual histogram of the accumulated LAGEOS passes along with our theoretical model is shown in Figure 5. It can be seen, that the agreement is good but not perfect. This may be caused by the accumulation process, where each single passage has to be treated separately in the adjustment of the satellite orbit. This can introduce small offsets between the histograms of different passages in the accumulation process. Therefore a higher repetition rate would be desirable in order to examine the historgrams for the observations of a single satellite pass.

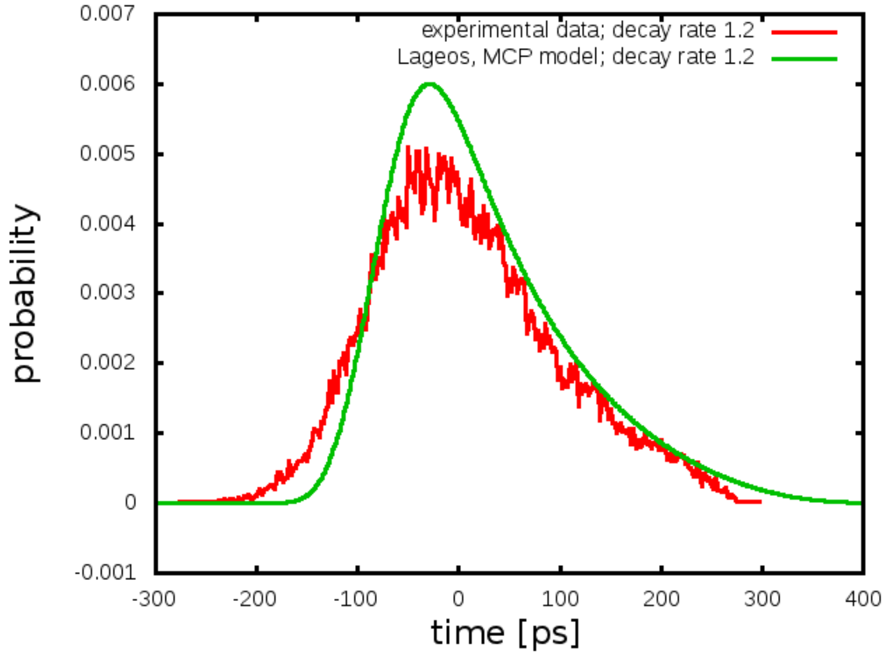

Figure 5. Residual histogram of accumulated data of distance measurements to LAGEOS satellite passages, performed at the Wettzell Laser Ranging System with our theoretical model (green) of the satellite response.

Conclusion

The investigation of the photo-electron statistics of satellite laser ranging measurements with the Wettzell Laser Ranging System in single photon tracking mode showed, that the recorded data never was just single photons although the return rate was adjusted to not exceed 10%, corresponding to the ILRS recommendations. Based on these findings we have established a model of the satellite response in mixed states and derived a receiver signature. The analysis of our model suggests a range bias of 7 mm for the ranging to LAGEOS under conditions of a low return rate.

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