# Secular drifts of low degree zonals obtained from SLR geodetic satellites

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#### Abstract

Within the space geodesy, only Satellite Laser Ranging is able to continuously monitor the long period variations of the zonal components of the Earth gravity field. The geodetic satellites are used as "gravitational sensors" to recover the low degree coefficients. In this study we are interested in estimating reliable time series of the zonal coefficients  $J_2$ ,  $J_3$ ,  $J_4$ ,  $J_5$  and  $J_6$  using all the implicit information available in the SLR observations of LAGEOS-I, LAGEOS-II, Stella and Starlette. The best satellite constellation for the estimation of each zonal coefficient is defined paying attention to the orbital characteristics of the satellites and to the temporal and spatial distribution of their acquired data. The time series values are used to derive the secular drifts of the coefficients. The analysis results are showed and compared with those computed by other analysis groups.

### Introduction

Monitoring the geodetic parameters and their variations is one of the fundamental activities to study the geophysical processes of the dynamical Earth system. The variations of the global gravity field is one of the most evident effects of the complex interaction among the geophysical units of the system; they are the evidence of a large mass redistribution within the solid earth involving the oceans and the atmosphere. They can be caused by episodic events such as earthquakes, by periodical events such as tides and seasonal effects or by quasi-regular secular variations; hence, the direct measurement of these variations is a necessary condition to validate the geophysical models of the Earth. One important process, responsible of the geopotential secular variations, is the so-called post-glacial rebound; the mass redistribution following the Pleistocene de-glaciation broke the isostatic equilibrium and the recovering of the lost equilibrium is still under course. This process is strongly linked to the viscosity of the Earth mantle and an estimation of the radial viscosity profile can be obtained through an inversion analysis using the estimates of the secular drifts of the geopotential zonal components [Vermeersen et al., 1997].

Geodetic SLR satellites, customary used for precise geodetic measurements, has been proven to be useful in measuring the time evolution of the long wavelength part of the gravity field [Cheng et al. 1989; Nerem et al. 1993; Nerem and Klosko, 1996; Eanes, 1995; Cazenave et al., 1996]. In particular the long observation history of Starlette and LAGEOS-I is important to properly extract the secular drift from the higher frequency variations in the zonal harmonic coefficients of the gravity field. On the other hand further observations from their twin satellites Stella and LAGEOS-II are important to de-correlate the low degree coefficients from the higher degree terms.

## **Estimation strategy**

The problem of finding the optimal estimation rate and the best satellite constellation capable to decorrelate the zonal coefficients has been faced studying the normal matrix product  $(A^{T}A)^{-1}$ , where A is the matrix of the partial derivatives of the real observations (design matrix). The partials are extracted after the last iteration in the least squares reduction of SLR observations (see next paragraph).

The Dilution of Precision (DOP) factor of each estimated parameter is defined as the square root of the corresponding diagonal element of the normal matrix. Given the required precision, the DOP factors, computed as a function of time, help in defining the optimal estimation rate and the offdiagonal factors represent the statistical correlation between the estimates. It is important to choose the satellite constellation in such a way that the correlation coefficients will be close to zero, otherwise the estimates will be biased. Figure 1 shows the correlation of  $J_2$  and  $J_4$  as a function of time using three satellites: LAGEOS-I, LAGEOS-II and Starlette. The dashed line indicates that these two zonals cannot be statistically de-correlated analysing only LAGEOS-I and LAGEOS-II observations. This theoretic result is confirmed by the analysis results: the SLR estimates reveal a high correlation between the estimates in almost all the arc solutions. One can overcome this problem by adding Starlette observations allowing a more robust estimate of  $J_4$ .

When choosing the data to be analysed, the use of the DOP factors must necessarily be supported by other considerations that involve the characteristics of the satellite, its orbit and the length of its observational history (i.e. Etalon are too high, GFZ-1 is too low and too sensitive to higher degree zonals). It is now proved that the orbital motion of both LAGEOS satellites is strongly affected by thermal forces that are highly correlated with the odd zonal terms of the gravity field [Eanes, 1995; Ries et al., 1997; Ries et al. 1998]. Therefore, because of the unreliability of the current non-gravitational models, in this study LAGEOS is not used to provide the odd degree zonal variations. The even zonals could be regarded as independent from the thermal perturbation because they excite mostly the ascending node of the orbit that is not corrupted by thermal forces. Strong unmodeled thermal perturbations have not been observed on the low orbiting satellites Starlette and Stella because of their different orbital and physical characteristics and therefore can be suitable candidates to recover the odd degree zonal harmonics.



Figure 1 : The correlation coefficient between J2 and J4 zonals as a function of time. The two curves show the different correlation functions using only the two LAGEOS observations (dashed line) and adding Starlette observations (solid line).

## **Data Analysis Reduction and Secular Drifts Estimation**

The data reduction is made with two sequential steps: the performing of many arc solutions, using the NASA/GSFC software Geodyn-II, and their combination in a unique global solution to estimate the zonal time series, with the NASA/GSFC software Solve.

The complete orbit and force model is defined in the first step together with the analysis approach (i.e. arc length, type of estimated parameters) but only the arc dependent parameters are estimated, namely those related to the orbit (state vector and non-gravitational forces) and the observations (measurement bias). We adopted the JGM3 gravity model and its own tide model, considered the secular variation and the dynamic polar motion of the  $C_{21}$  and  $S_{21}$  coefficients, realized the terrestrial reference frame using the *a priori* ITRF94 site coordinates and velocities, and constrained the Earth Orientation Parameters to the IERS homogeneous series. The SLR observations (normal points) have been analysed using the worldwide network of tracking stations which differ both in terms of quality and quantity of acquired data. To take into account these two aspects, each normal point has been weighted by the quantity *n/bin\_variance* where *n* is the number of single range data that have been used to construct the normal point and *bin\_variance* is the quadratic bin standard error.

All normal matrices from the different arc solutions are then combined in order to estimate the global and the arc parameters together.

The coefficient time series obtained from the data reduction process are then used to estimate the secular drifts. The zonal coefficients are affected by seasonal oscillations originating principally in the ocean and atmosphere; therefore, together with the estimation of the secular rate, a residual seasonal signal is filtered out fitting an annual and semestrial sinusoid. The frequency dependent correction of  $J_2$ , due to the Earth anelasticity, has been implemented through the classical relationship and tide amplitudes indicated in the IERS conventions [1996].  $J_2$  is corrected for all tidal terms that contribute more than 2  $10^{-13}$  and these include periods from 1 month up to 18.6 years.

#### **Discussion of results**

Different time series of the zonal coefficients have been estimated using different satellite constellations. Figure 2 shows, as an example, a typical solution of zonal time series. The estimated zonal rates up to degree six are shown in Table 1.

As far as the odd zonals are concerned we have analysed only Starlette observations and therefore we get the lumped value  $\dot{J}_{3L} = \dot{J}_3 + 0.9 \dot{J}_5$  that is a linear combination of the odd degree zonals.

The even zonal rates in Table 1 are averaged values obtained from a number of different solutions. Each solution has a common modeling but differs in the adopted satellite constellation and in the number of estimated zonal coefficients. In the following we will outline the estimated zonal drifts comparing them with the values of other analysis groups.

- The  $J_2$  drift is significantly greater than the values reported by other groups however, if the time series is not corrected for the mantle anelasticity effects, the value of  $J_2$  is  $-2.6 \cdot 10^{-11}$  yr<sup>-1</sup> that is in agreement with the published values. After applying the anelastic response correction we observe an increase of the  $J_2$  drift on the order of  $0.5 \cdot 10^{-11}$  yr<sup>-1</sup> in all  $J_2$  solutions (see Table 2). Therefore this anelastic tide correction can be a critical point in estimating secular drifts especially if the observation history does not cover an integer number of tidal cycles.
- Currently, using only Starlette observations, we are not able to separate the odd zonal drifts and only a linear combination of  $J_3$  and  $J_5$  can be obtained,  $\dot{J}_{3L} = \dot{J}_3 + 0.9 \dot{J}_5$ , which is inferred from the observations themselves.
- $J_4$  has a slightly positive drift (positive in three different solutions) that could be in agreement with the other published values with the exception of the solution obtained by *Cheng et al.* [1997] that

shows a slightly negative drift. However, in our solutions, the  $J_4$  estimates are highly correlated with the  $J_2$  zonals and hence the observed differences may be ascribed to a partially unresolved lumped estimation of  $J_4$ .

-  $J_6$  drift is less stable in our solutions and therefore it is not yet a robust estimate but nevertheless our mean value agrees fairly well with the unique published value [Cheng et al., 1997].



Figure 2 : Time series of the estimated zonal coefficients from degree two up to degree six. The quoted secular drifts are obtained from a weighted mean of three different solutions using a different combination of the observed satellites. The odd degree zonals were not well separated in our Starlette solution and only the lumped odd zonal estimate is shown.

(Units: 10 <sup>-11</sup> /year)	J <sub>2</sub>	J <sub>even</sub>	J <sub>3</sub>	$\dot{J}_{odd}$	$\dot{J}_4$	У <sub>5</sub>	J <sub>6</sub>
<i>Yoder et al.</i> [1983] L1 (1976-1981)	-3						
Rubincam [1984] L1	-2.6±0.6						
Cheng et al. [1989] Str	-2.5±0.3		-0.1±0.3		0.3±0.6		
<i>Gegout &amp; Cazenave</i> [1993] L1 (1985-1989)	-2.8±0.4						
Nerem et al. [1993] L1 (1980-1989)		-2.6		-1.9			
Nerem & Klosko [1996] L1,L2,Aj,Str (1986-1994)	-2.8±0.3			1.6±0.4	0.2±1.5		
Cazenave et al. [1996] L1,L2 (1984-1994)	-3.0±0.5		-1.7±0.1		-0.8±1.5		
<i>Eanes</i> [1995] L1 (1977-1995)	-2.4±0.2						
<i>Cheng et al.</i> [1997] L1,L2,Stl,Str,Aj,Et1,Et-2, BE-C (1975-1996)	-2.7±0.4		-1.3±0.5		-1.4±1.0	2.1±0.6	0.3±0.7
This study L1,L2, Str, Stl (1987-1997)	-3.3±0.3				1.7±0.8		0.6±1.2
This study Str (1987-1997)				-1.1±0.3			

Table 1: Comparison of SLR zonal rates computed from different analysis groups

$\dot{\mathbf{J}}_{2}$ Solutions (Units: $10^{-11}$ yr <sup>-1</sup> )	Anelastic correction applied	Anelastic correction not applied
L1, Str $(J_2-J_6)$	$-3.1 \pm 0.1$	$-2.4 \pm 0.1$
L1, L2, Str, Stl (J <sub>2</sub> -J <sub>6</sub> )	$-3.2 \pm 0.2$	$-2.7 \pm 0.2$
L1, Str $(J_2-J_4)$	$-3.3 \pm 0.2$	$-2.7 \pm 0.2$
L1, L2, Str $(J_2-J_4)$	$-3.7 \pm 0.2$	$-3.2 \pm 0.2$

Table 2: Summary of estimated J<sub>2</sub> drifts

# Summary

Based on 11 years of SLR observations we obtain a reliable estimate for  $\dot{J}_2$ . Its value differs from the solutions of other analysis group by an amount of  $10^{-11}$  and this difference can be entirely related to the tidal frequency correction caused by the anelasticity of the Earth. The differences among all the values shown in Table 1 for  $\dot{J}_3$ ,  $\dot{J}_4$  and  $\dot{J}_6$  are a bit larger and sometimes exceed the reported uncertainties.

Our solution will be further improved considering a longer period, using the data from Ajisai and optimizing the weight to be assigned to each satellite in the global estimation process.

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