Parametric thermal analysis of hollow Beryllium Retroreflector

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In this paper equations have been derived for making order of magnitude estimates of the thermal gradients in a hollow Beryllium retroreflector due to absorption of solar radiation. The equations show the dependence on factors such as the area, thickness, solar absorptivity, conductivity, and emissivity of the reflecting plates. The performance of the retroreflector can be degraded by thermal warping of the plates or changes in the dihedral angles between the reflecting plates as a result of differential expansion and contraction. The dihedral angles could also change due to failure of the adhesive in the joints between the plates. However, that is not studied in this paper. The two cases that have been studied are conduction through the plate and conduction along the plate.

Because of the high thermal conductivity of Beryllium the temperature difference between different parts of the retroreflector will be small (a fraction of a degree in the cases that have been studied). For the purpose of computing the thermal radiation from different plates the retroreflector is considered to be isothermal.

1. Conduction through the plate.

If we have a plate in free space that is subjected to solar radiation on one side, the side facing the sun will be warmer than the side facing empty space. The thermal expansion of the side facing the sun will be greater than the thermal expansion on the back side. This will result in the side facing the sun being slightly convex and the side facing empty space being slightly concave. The objective of this analysis is to calculate the amount of buckling of the plate in order to see if it will cause a significant distortion of the wavefront reflected from the surface.

Suppose we have a square plate of area l x l and thickness w. The thermal parameters are

α = solar absorptivity
ε_1 = emissivity of the front surface
ε_2 = emissivity of the back surface
S = solar constant = 1412.5 Watts/sq meter
k = thermal conductivity of Beryllium = 225 Watts/m-°K
c = linear expansion coefficient of Beryllium = $11.3x10^{-6}K^{-1}$
σ = Stefan Boltzman constant = $5.6697x10^{-8}Wm^{-2}K^{-4}$

The emissivity of beryllium as a function of temperature is

T (K)	ε
300	.05
800	.20
1100	.41
1200	.49
1300	.57
1400	.67

For generality we can define a parameter *f* as the fraction of the solar radiation that is conducted through the plate. If we consider just a single plate not connected to the other reflecting plates the fraction *f* is

$$f = \frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2} \tag{1}$$

This assumes that the front and back of the plate are at nearly the same temperature. If the three plates in a retroreflector are subjected to the same amount of solar radiation and are at the same average temperature, this analysis should give a reasonably accurate value of the heat flow through the plate. The solar energy absorbed by the plate is αSA where $A = l$ x l is the area of the plate. The basic equation for conduction is

$$\dot{Q} = \frac{k\Delta tA}{L} \tag{2}$$

where $\dot{Q}$ is the heat flow Δt is the temperature difference, A is the cross sectional area along the heat flow and L is the length over which the heat flows.

The heat flow through the plate is related to the temperature difference ΔT by the equation

$$f\alpha SA = \frac{k\Delta TA}{w} \tag{3}$$

Solving equation (3) for ΔT gives

$$\Delta T = \frac{f\alpha Sw}{k} \tag{4}$$

The temperature difference will result in differential expansion Δl between the front and back surfaces given by

$$\Delta l = cl\Delta t \tag{5}$$

Substituting the expression for Δt from equation (4) gives

$$\Delta l = \frac{clf \alpha S w}{k} \tag{6}$$

The differential expansion will cause the plate to deform into a curved shape. A square plate will deform into a somewhat complicated shape. A circular plate should deform into a spherical cap with a certain radius of curvature *R*. A bar would deform into an arc of a circle with some radius of curvature *R*.

For the purpose of this order of magnitude calculation, the exact shape of the plate will be neglected. We will assume that the plate forms some curved shape whose outer surface is of length l and whose inner surface is of length $l - \Delta l$. The arc lengths of the outer and inner surfaces are assumed to be

$$l = R\theta \tag{7}$$

and

$$l - \Delta l = (R - w)\theta \tag{8}$$

where *R* is the radius of curvature of the plate and θ is the central angle.

The radius of curvature can be calculated from equations (7) and (8). Substituting equation (7) into equation (8) gives

$$-\Delta l = -w\theta \tag{9}$$

from which we have

$$\theta = \frac{\Delta l}{w} \tag{10}$$

Substituting $\theta = \frac{l}{R}$ from equation (7) into equation (10) and solving for R gives

$$R = l\frac{w}{\Delta l} \tag{11}$$

Solving for θ by substituting equation (6) into equation (10) we have

$$\theta = \frac{\Delta l}{w} = \frac{clf \alpha S w}{wk} = \frac{clf \alpha S}{k} \tag{12}$$

Suppose one end of the plate is fixed (attached to another plate). What is the displacement *d* of the other end of the plate? The displacement is given by

$$d = R(1-\cos(\theta)) \approx R\left(1-\left[1-\frac{\theta^2}{2}\right]\right) = R\frac{\theta^2}{2} \qquad (13)$$

Substituting equations (10) and (11) into equation (13) gives

$$d = \frac{1}{2} l \frac{w}{\Delta l}\left(\frac{\Delta l}{w}\right)^2 = \frac{l}{2}\frac{\Delta l}{w} \qquad (14)$$

Substituting equation (6) into equation (14) gives the final result

$$d = \frac{lclf\alpha Sw}{2wk} = \frac{cl^2 f\alpha S}{2k} \qquad (15)$$

We can put some numbers in to see whether the deflection of the plate is significant. Let us take .5 for the solar absorptivity of Beryllium. Let us assume the emissivity of the front and back faces is the same. From equation (1) we have .5 for the fraction *f*. For a 2 inch cube corner the plates are 1.4 x 1.4 x .12 inches or .035 x .035 x .003 meters. The other parameters are given at the beginning of this paper. Putting the numbers into equation (15) gives a deflection of about .0108 microns. This is small compared to the wavelength of .5 microns. Therefor the curvature of the plate due to conduction from the back to the front is not a problem.

It is interesting to calculate some of the intermediate quantities. From equation (4) the temperature difference across the plate is .0047 degrees C. From equation (5) the differential expansion is .00186 microns. From equation (12) the angle θ is .62 microradians. From equation (11) the radius of curvature is 56,451 meters. Putting the values of *R* and θ into equation (13) gives .0108 microns the same as using equation (15).

We should note that the deflection is proportional to the square of the length *l* of the sides of the plate and is independent of the width *w* which cancels during the derivation. For large plates the curvature could become a problem because of the dependence on the square of the length of the sides.

2. Conduction along a plate.

For generality let us again define a constant *f* as the fraction of the solar heat conducted along the plate. If all the plates are being illuminated equally by the sun, the average temperature should be the same on all of the plates so that no heat is conducted from one plate to another plate. In this case the fraction *f* of the energy conducted along the plate should be nearly 0.

If only one of the plates is solar illuminated, that heat will be conducted to the other plates so that the temperature is almost the same on each plate. The heat will be radiated almost equally from all three plates. Approximately 2/3 of the heat absorbed by the plate that is solar illuminated will be conducted to the other plates, so *f* should be approximately 2/3.

If two plates are solar illuminated approximately 1/3 of the heat on each illuminated plate will be conducted to the third plate.

The solar radiation will be absorbed all along the plate so that the heat flow will be different at different parts of the plate. For the purposes of this order of magnitude calculation we will assume that the heat is conducted from the center of one plate to the center of another plate which is a distance l. In other words the calculation is the same as if the heat were conducted from one side of the plate to the other side.

Using equation (2), the heat absorbed by the plate is related to the temperature difference along the length of the plate by the equation

$$f\alpha S l^2 = \frac{k\Delta t l w}{l} \tag{16}$$

Solving equation (16) for the temperature difference Δt we have

$$\Delta t = \frac{f\alpha S l^2}{kw} \tag{17}$$

The linear differential expansion d of the plate is

$$d = cl\Delta t \tag{18}$$

Substituting equation (17) into equation (18) gives

$$d = \frac{cf\alpha S l^3}{kw} \tag{19}$$

Suppose we take $\alpha = .5$, $f = .5$, $l = .035$ m, and $w = .003$ m. From equation (17) the temperature difference is .64 degrees C. Using equation (19) we have a differential expansion of $d = .25$ microns. This is half a wavelength if the wavelength is .5 microns. If the expansion of one of the plates by this amount changes the dihedral angles by a comparable amount this will cause a significant distortion of the diffraction pattern.

The reflectivity of Beryllium is about 50 percent which would not be satisfactory for a retroreflector. The front faces will have to be coated with some other highly reflective metal such as aluminum or silver. Gold would not be suitable because its reflectivity drops off significantly at wavelengths shorter than about .7 microns. This gives poor reflectivity at 532 nm. Gold also has a solar absorptivity of about .3 as a result of the poor reflectivity at short wavelengths. Aluminum has a solar absorptivity of .12 which is much better than Beryllium. However, silver has a solar absorptivity of only .07 and a reflectivity of about 93 percent. Silver would probably be the best choice for a reflective coating both from the point of view of good reflectivity at 532 nm and low solar absorptivity.

Suppose we have α = .07 with the other parameters the same as the previous calculations. From equation (17) the temperature difference would be .09 degrees C. From either equation (18) or (19) the differential expansion is .035 microns. For a wavelength of .532 microns this is .066 wavelengths or 1/15 of a wave. This would be acceptable. However, this is only an order of magnitude calculation.

Note that the expansion of the plate is proportional to the cube of the length of a side. This assumes the width w of the plate is constant. If the width w is proportional to the length l then the expansion is proportional to the square of the length of a side. This is the same dependence as for conduction through the plate as shown in equation (15).

Since the expansion of the plate computed from equation (19) appears to be just below what is acceptable, it may not be feasible to use cubes larger than about 2 inches unless some solution can be found to the problem of differential heating of the plates by solar radiation. I do not know of any reflecting surfaces with a solar absorptivity less than that of silver. Recessing the cubes would reduce the angles over which there is direct solar heating but also decrease the viewing angle for laser ranging. If there is some way to filter out some of the solar energy while allowing the laser wavelength to pass that would also help. However, that would also restrict the wavelengths that can be used for laser ranging.

Since the calculation of the expansion of the plates is only approximate and gives a value that is just below what is acceptable, further analysis and/or testing is needed to verify that thermal effects will not cause significant distortion of the diffraction pattern.

3. Temperature of the retroreflector array.

The solar energy absorbed by the cube corner depends on the ratio of the absorptivity to the emissivity (the α/ε ratio). Suppose the solar absorptivity of each of the reflecting surfaces of the retroreflector is .07. At normal incidence on a retroreflector, the solar radiation is reflected from all three faces and is retroreflected. If the reflectivity is .93 on each reflection, the total reflectivity is .93 x .93 x .93 = .8 for all three reflections. The effective solar absorptivity is .2 at normal incidence.

Suppose it is desired to keep the temperature of the retroreflector array at about 300 deg K. What emissivity would the back surface of the array need to have? Let us assume the area of the back of the array is the same as the area of the front of the array. The basic equation for the temperature is

$$\alpha SA = \varepsilon\sigma AT^4 \qquad (20)$$

Solving this for the emissivity gives

$$\varepsilon = \frac{\alpha S}{\sigma T^4} \qquad (21)$$

If the effective absorptivity on the front is .2, the emissivity of the back of the array needs to be .61. This should be possible. Another possibility would be to have a larger surface on the back such as by using some kind of fins or a hemispherical surface since the area is twice the cross section.

If the solar energy is incident of the back of the array, the α/ε ratio needs to be less than .2/.6 = 1/3 in order to keep the array around 300 deg K. Since silver has a low emissivity of about .02, the front of the array will not be able to get rid of much heat collected on the back surface. The back surface will have to radiate most of whatever solar heat is collected.

In principle the backs of the retroreflectors could be given some kind of low α/ε coating. However, there would still be the problem of uneven heating of the plates of the cube corners. This is marginal for solar radiation hitting the front of the retroreflector. It would be better to have some kind of heat shield to provide a more isothermal environment on the back of the array and prevent uneven heating of the plates. In this way the performance should be good whenever the sun is not hitting the front of the array. The heat shield would not have to be heavy. All it has to do is have enough conductivity to reduce uneven heating of the back surfaces of the retroreflectors. The back surface of the cubes should be radiatively coupled to the heat shield using a high emittance surface on the back of the cubes and on the inside of the heat shield.

4. The joints.

No analysis has been done of possible thermal distortion of, or damage to the joints. Probably the only way to be sure the joints are not a problem is with thermal vacuum testing to make sure that the dihedral angles are not changed temporarily or permanently by thermal cycling.

5. Conclusions.

The use of a hollow cube corner for laser ranging appears to be feasible from these calculations as long as there is good thermal control and the cube is not too large. Since these calculations are only order of magnitude and the results are just under what is acceptable, further analysis and/or testing will be needed to confirm the tentative conclusions. Also, the stability of the joints under thermal cycling which is not considered in this report needs to be determined. The reflecting surfaces will need to be coated with a good reflecting material such as silver to improve the signal strength for laser ranging and reduce thermal distortion of the cube due to absorption of solar radiation. A heat shield with a low a/e ratio should be included on the back of the array to avoid excessive temperatures when the array is sunlit and reduce uneven heating of the back surface of the cube corners by solar radiation.